



Overview of the **SiStER** code

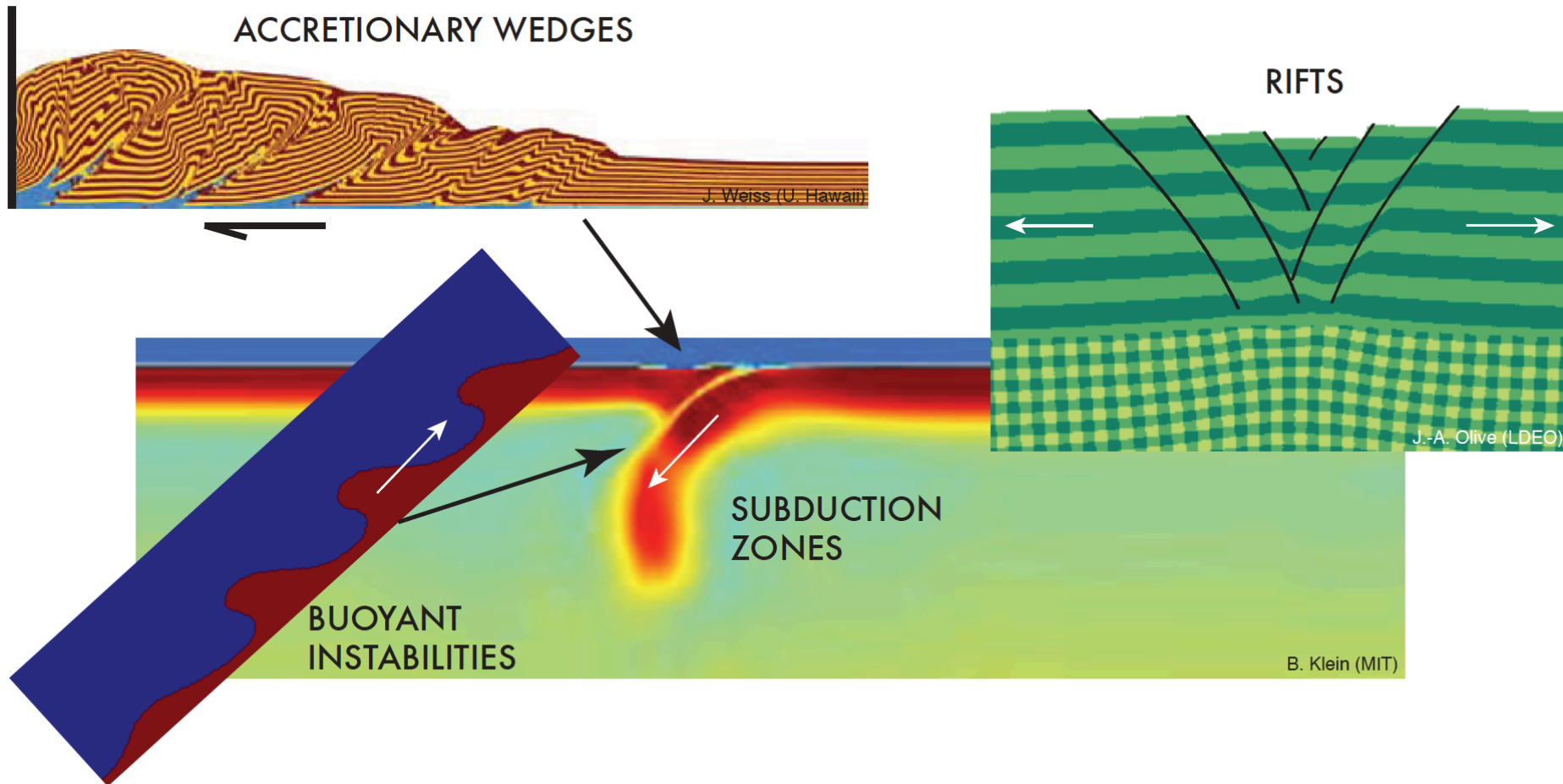
Simple Stokes solver with Exotic Rheologies

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Objectives & Challenges

Modeling lithosphere and mantle deformation with continuum mechanics:
Stokes flow with large strains, strain localization, non-linear rheologies, sharp contrasts in material properties, complex BCs.



Governing equations

- Conservation of mass and momentum

$$\partial_k v_k = 0$$

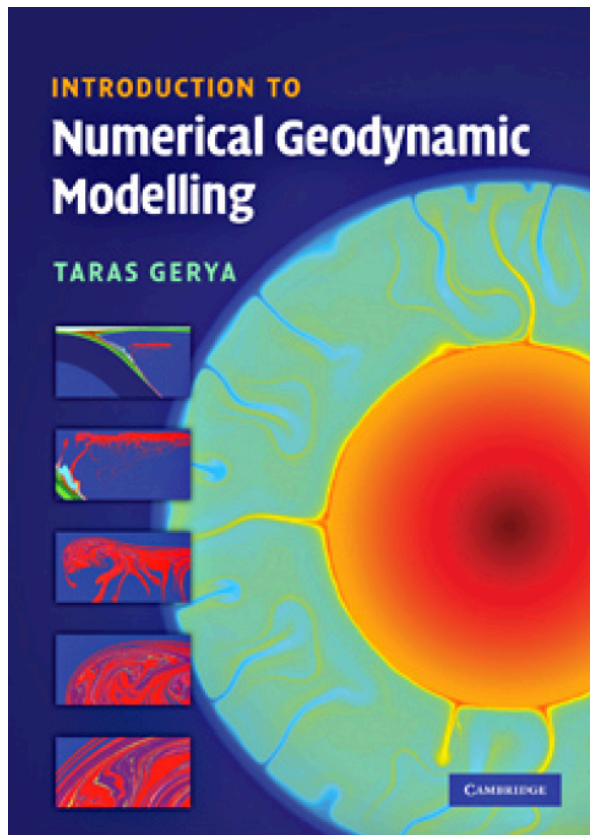
$$\partial_j \sigma'_{ij} - \partial_i P + \rho g = 0$$

- Non linear viscosity law + elasticity + plasticity
- Conservation of energy

$$\frac{\partial T}{\partial t} + v \nabla T = \nabla \cdot (\kappa \nabla T)$$

Numerical methodology / Citations

- Finite Difference / Particle-in-Cell method.
Implementation largely based on *Gerya* [2010].



- Methodology partly described
in *Olive et al.* [2016, GJI]

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The role of elasticity in simulating long-term tectonic extension

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and Benjamin Z. Klein⁵

Finite-difference discretization

Steady-state Stokes flow on Eulerian grid:

Conservative FD scheme
on a staggered grid:

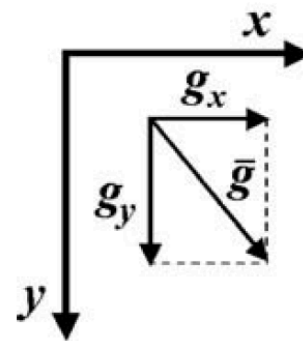
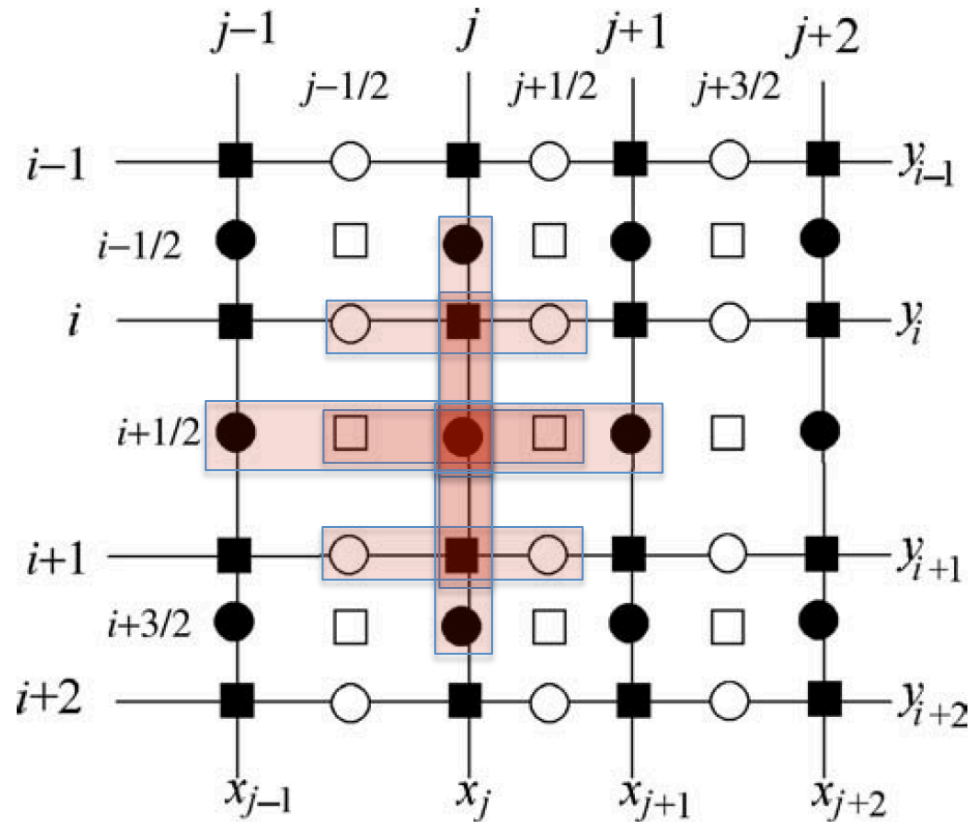
$$\nabla \cdot (\eta (\nabla v + \nabla v^T)) - \nabla P + \rho g = RHS$$

$$\nabla \cdot v = 0$$

Leads to linear system:

$$JX = \begin{pmatrix} K & G \\ G^T & 0 \end{pmatrix} \begin{pmatrix} v \\ P \end{pmatrix} = \begin{pmatrix} rhs \\ 0 \end{pmatrix}$$

Solved with MATLAB's
“backslash” solver.



● v_x, q_y

○ v_y, q_x

■ $\dot{\epsilon}_{xy}, \sigma_{xy}, \eta_s, \mu_s, T, \rho, k, C_p, \alpha, H, \omega$

□ $P, \dot{\epsilon}_{xx}, \dot{\epsilon}_{yy}, \sigma'_{xx}, \sigma'_{yy}, \eta_n, \mu_n, \frac{D \ln \rho}{Dt}$

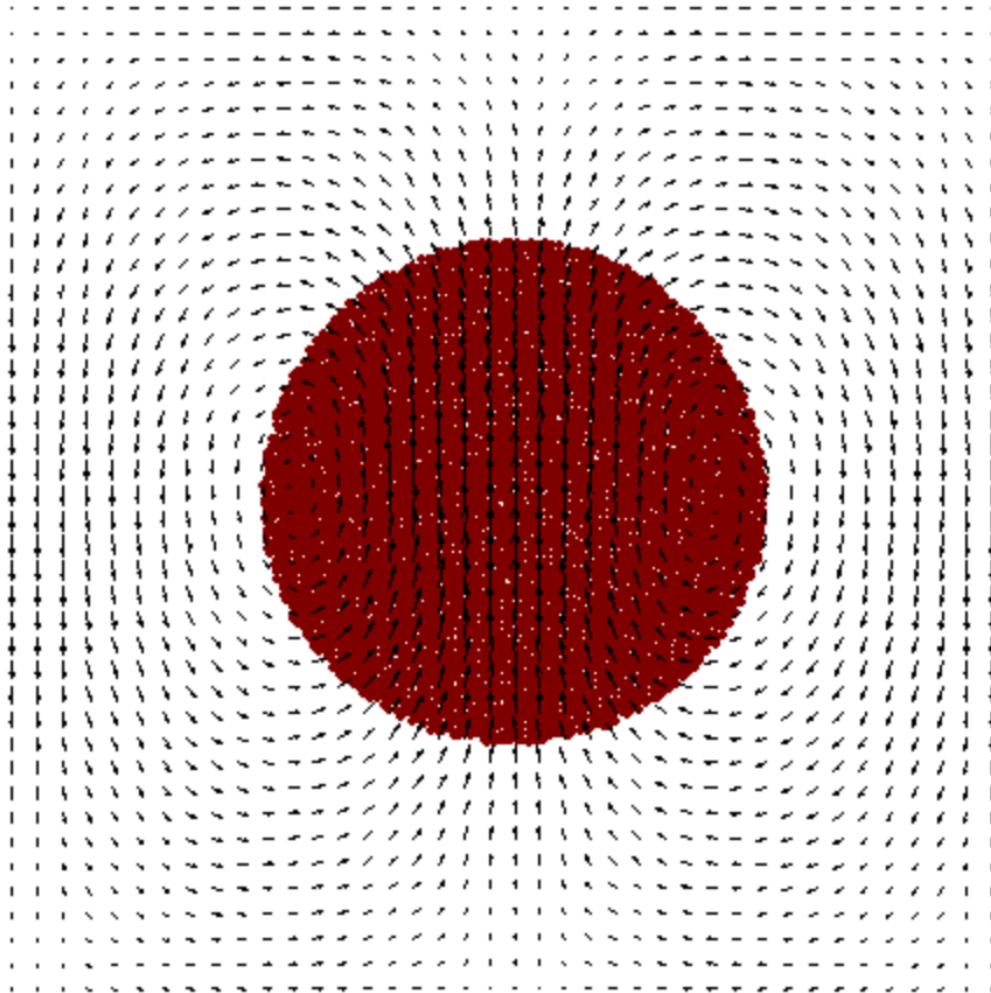
Gerya [2010]

Shear nodes

Normal nodes

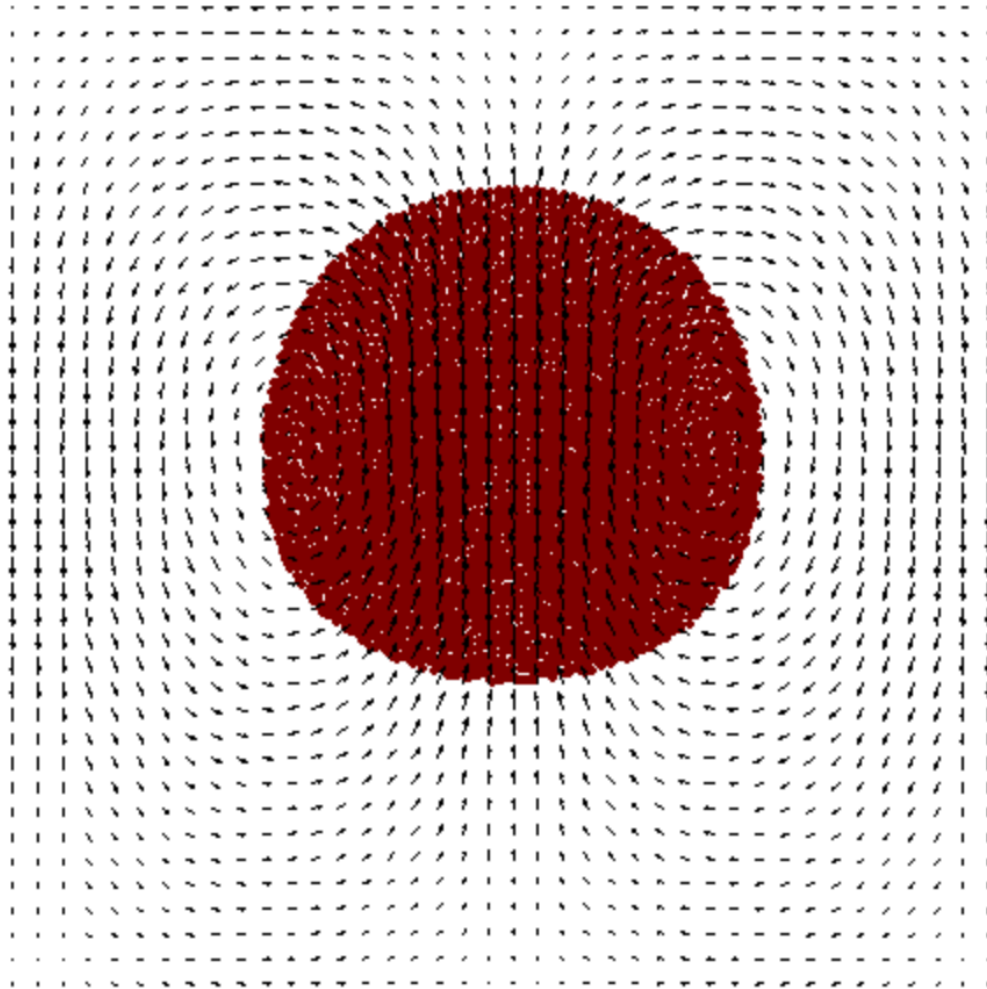
Particles handle material advection

- low density sphere in dense, low viscosity fluid:

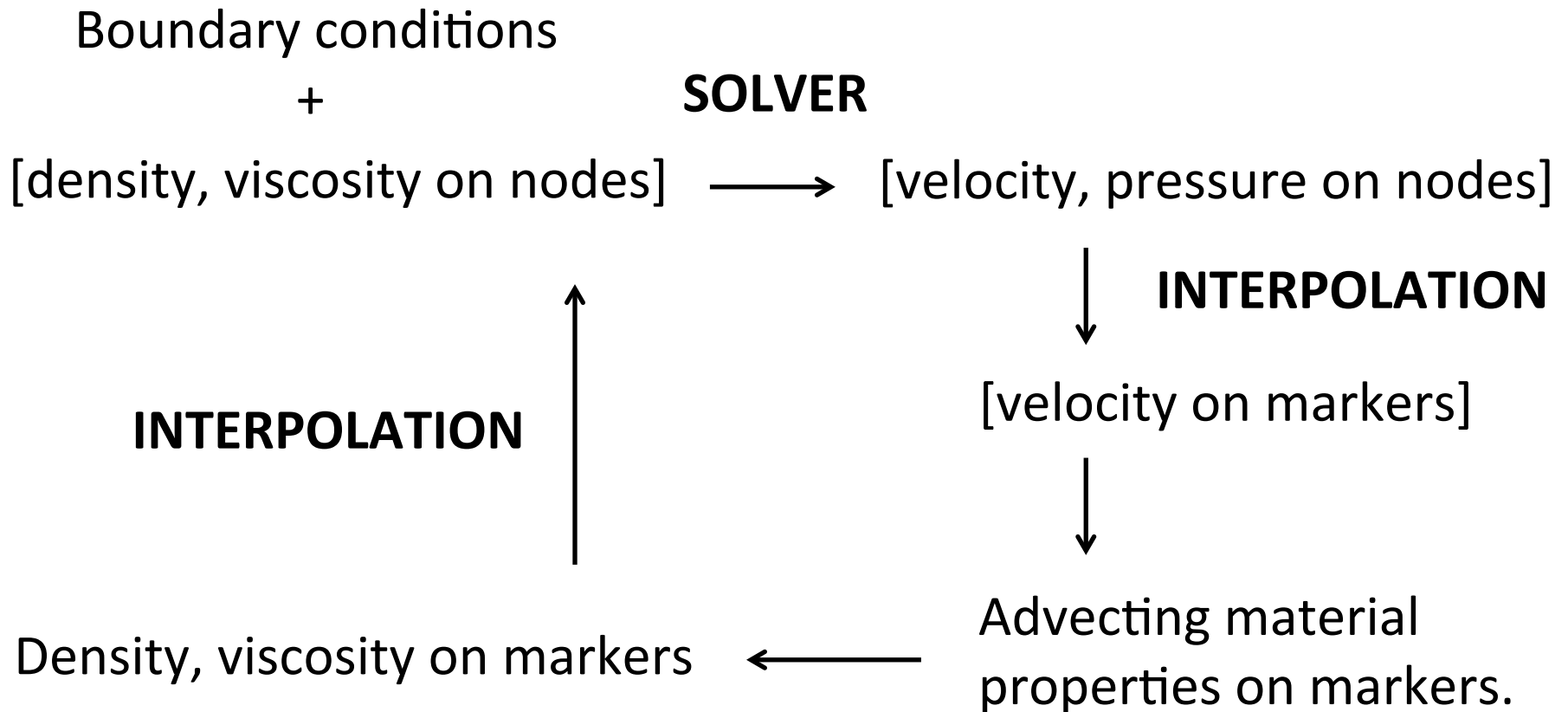


Particles handle material advection

- low density sphere in dense, low viscosity fluid:



Basic code structure



Handling non-linear creep laws

The formulation of viscosity strictly pertaining to non-linear creep laws writes:

$$\eta = \left(\frac{1}{\eta_{DIFF}} + \frac{1}{\eta_{DISC}} + \text{other terms} \right)^{-1}$$

This distinguishes the contributions from diffusion and dislocation creep.

Each creep law can be parameterized as: $\eta = A^{-\frac{1}{n}} \dot{\epsilon}_{II}^{\frac{1-n}{n}} e^{\frac{E}{nRT}}$

Picard iterations (**SiStER_run_Picard_iterations**) consist of:

- 1/ Solving the flow equations using the latest viscosity map
- 2/ Calculating the strain rate from the latest solution
- 3/ Re-evaluating the viscosity from the new strain rate map

Repeat until the solution stops changing (within specified tolerance)

Implementation of visco-elasticity

Maxwell **visco-elastic** rheology:

$$\dot{\epsilon}_{ij} = \underbrace{\frac{1}{2G} \frac{D\sigma_{ij}}{Dt}}_{\text{ELASTIC}} + \underbrace{\frac{1}{2\eta} \sigma_{ij}}_{\text{VISCOUS}}$$



FD approximation

$$\sigma_{ij}^{(t)} = 2 Z \eta \dot{\epsilon}_{ij} + (1-Z) \sigma_{ij}^{(t-\Delta t)}$$

$$\text{with } Z = \frac{G \Delta t}{\eta + G \Delta t}$$

Implementation in a viscous solver:
[Moresi et al., 2003]

$$\nabla \cdot (2 Z \eta \dot{\epsilon}_{ij}) - \nabla p + \rho g = - \nabla \cdot ((1-Z) \sigma_{ij}^{(t-\Delta t)})$$

$$\tau_{\text{Max}} = \frac{\eta}{G}$$

Maxwell time scale:

For **VISCOUS BEHAVIOR**
set very high G so that:
 $G \Delta t \gg \eta$, i.e., **$\Delta t \gg \tau_{\text{Max}}$**
(and $Z \approx 1$)

For **ELASTIC BEHAVIOR**
impose high viscosity.

Implementation of plasticity

Plastic (localizing) rheology

Mohr-Coulomb plasticity is implemented by weakening viscosity to cap σ'_{II} at σ_{yield} :

$$\eta = \left(\frac{1}{\eta_{REF}} + \frac{1}{\eta_{PLAS}} \right)^{-1} \quad \text{with} \quad \eta_{PLAS} = \frac{\sigma_{yield}}{2\dot{\epsilon}_{II}}$$

Yield stress prescribed through cohesion and friction

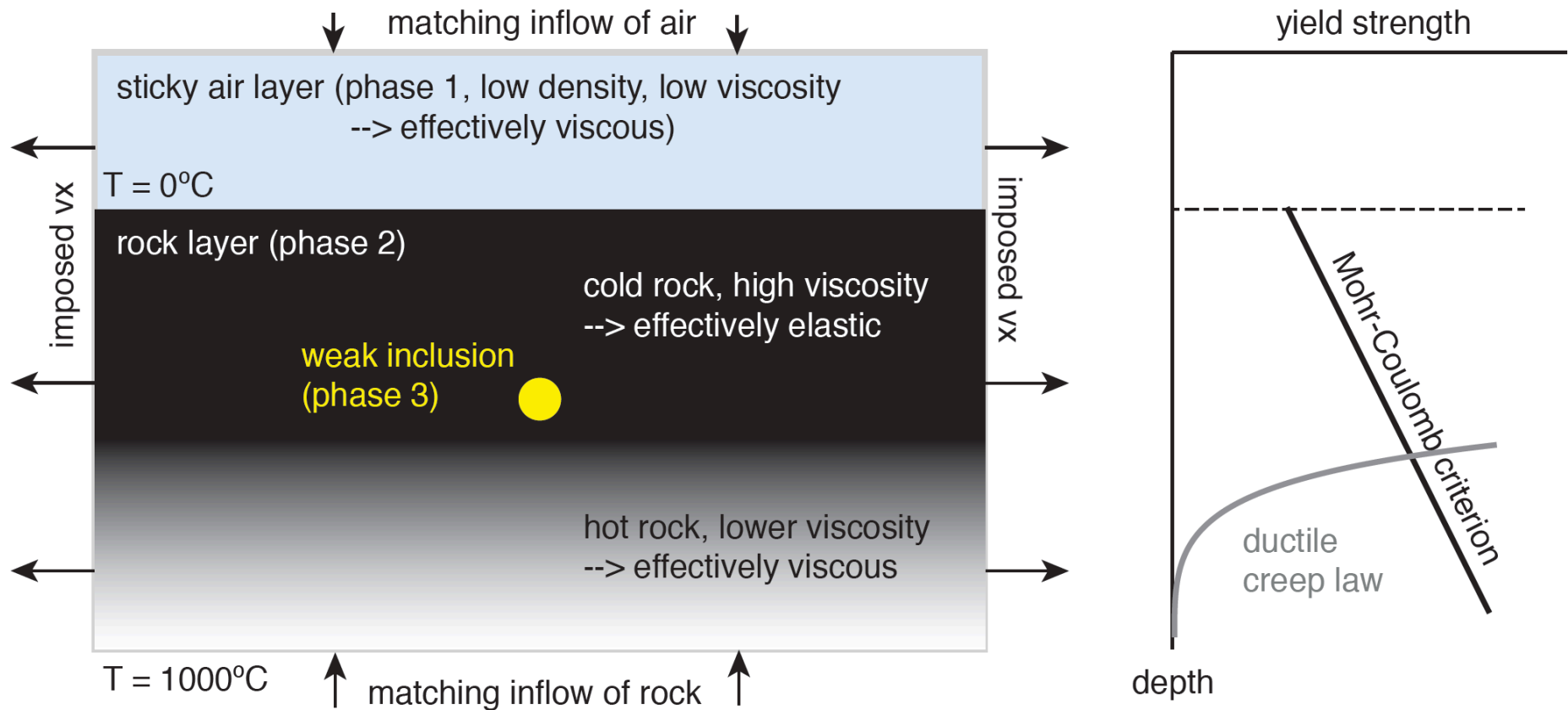
$$\sigma_{yield} = \sin\Phi \mathbf{P} + C \cos\Phi$$

Strain localization is promoted by **dropping cohesion** linearly with accumulated plastic strain ϵ_P . Full weakening occurs when fault offset reaches $h_C \approx 3 \Delta x \epsilon_{P CRIT}$ *Lavier et al. [2000]*

Accumulated plastic strain **heals** on a prescribed time scale

Example case: rifting example

Run **SiStER_MAIN**, then enter **SiStER_Input_File_continental_rift**



Input file structure

Runs for 100 iterations, will output a file every 10 time steps

```
3
4  % DURATION OF SIMULATION AND FREQUENCY OF OUTPUT %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
5 -  Nt=100; % max number of time iterations
6 -  dt_out=10; % output files every "dt_out" iterations
7
8  Top-left corner of the domain at (0,0), x>0 to the right, y>0 down
9  % DOMAIN SIZE AND GRIDDING %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
10 -  xsize=90e3;
11 -  ysize=30e3;
12  % gridding- from 0 to GRID.x(1), grid size is GRID.dx(1)
13  % from GRID.x(1) to GRID.x(2), grid size is GRID.dx(1) etc...
14  % same for y
15 -  GRID.dx(1)=2000;
16 -  GRID.x(1)=30e3;
17 -  GRID.dx(2)=400;
18 -  GRID.x(2)=60e3;
19 -  GRID.dx(3)=2000;
20 -  GRID.dy(1)=2000;
21 -  GRID.y(1)=9e3;
22 -  GRID.dy(2)=400;
23 -  GRID.y(2)=22e3;
24 -  GRID.dy(3)=2000;
25
```

Box size 90 km × 30 km All units SI

Horizontal grid size = 2km between 0 and 30 km, 400m between 30 and 60 km, 2 km between 60 and 90 km.

Vertical grid size = 2km between 0 and 9 km, 400m between 9 and 22 km, 2 km between 22 and 30 km.

Input file structure

Seeding markers to advect material properties:

```
26  
27 % LAGRANGIAN MARKERS %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  
28 - Mquad=6; % number of markers in the smallest quadrant  
29 - Mquad_crit=3; % minimum number of markers allowed in smallest quadrant (for reseeding)  
30
```

The initial marker density is defined such that a quarter of the smallest cell (defined above) has 6 markers, i.e., the smallest cell contains 24 markers. Bigger cells will contain more markers such that the density of markers (number of markers per m^2) is constant.

As markers move around, gaps may form in the marker distribution. New markers will be added to fill in the gaps in cells that have quadrants with less than 3 markers.

Input file structure

Setting up the initial geometry of the phases (material types)

Here there are 3 phases in total: a sticky air layer, a rock layer and a weak rock inclusion

```
30
31 % GEOMETRY %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
32
33 - Nphase=3; % number of phases
34
35 % phase 1
36 - GEOM(1).type=1; % 1 = layer (then specify top and bot) or 2 = circle % 1 = layer (then specify top and bot) or 2 = circle
37 - GEOM(1).top=0;
38 - GEOM(1).bot=10e3;
39
40 % phase 2
41 - GEOM(2).type=1; % 1 = layer (then specify top and bot) or 2 = circle % 1 = layer (then specify top and bot) or 2 = circle
42 - GEOM(2).top=10e3;
43 - GEOM(2).bot=30e3;
44
45 % phase 3
46 - GEOM(3).type=2; % 1 = layer (then specify top and bot) or 2 = circle % 1 = layer (then specify top and bot) or 2 = circle
47 - GEOM(3).x0=xsize/2;
48 - GEOM(3).y0=20e3;
49 - GEOM(3).rad=1e3;
50
```

First phase (air) is a horizontal layer (type 1) spanning the whole width of the box, between $y=0$ (top) and $y=10$ km (bottom).

Input file structure

```
30
31 % GEOMETRY %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
32
33 Nphase=3; % number of phases
34
35 % phase 1
36 GEOM(1).type=1; % 1 = layer (then specify top and bot) or 2 = circle % 1 = layer (then specify top and bot) or 2 = circle
37 GEOM(1).top=0;
38 GEOM(1).bot=10e3;
39
40 % phase 2
41 GEOM(2).type=1; % 1 = layer (then specify top and bot) or 2 = circle % 1 = layer (then specify top and bot) or 2 = circle
42 GEOM(2).top=10e3;
43 GEOM(2).bot=30e3;
44
45 % phase 3
46 GEOM(3).type=2; % 1 = layer (then specify top and bot) or 2 = circle % 1 = layer (then specify top and bot) or 2 = circle
47 GEOM(3).x0=xsize/2;
48 GEOM(3).y0=20e3;
49 GEOM(3).rad=1e3;
50
```

Second phase (rock) is a horizontal layer (type 1) spanning the whole width of the box, between $y=10$ (top) and $y=30$ km (bottom).

Third phase (weak rock) is a circle (type 2) of radius 1 km centered at (45 km, 20 km).

Phase definitions overprint each other in the order they are defined- make sure they cover the entire domain!

Input file structure

Material properties must be specified for each phase defined above, in the following format (shown here for phase 1)

```
51
52 % MATERIAL PROPERTIES %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
53
54 % creep laws of the form: pre^(-1/n)*epsII^((1-n)/n)*exp(E/(nRT))
55 % harmonically averaging diffusion creep, dislocation creep
56 % (and plastic creep to simulate brittle failure)
57
58 % phase 1
59 - MAT(1).phase=1; Phase 1 will be indexed in marker array im with value 1
60 % density parameters
61 - MAT(1).rho0=0.01; Reference density and thermal expansion, see
62 - MAT(1).alpha=0; SiStER_get_density function
63 % elasticity
64 - MAT(1).G=1e18;
65 % diffusion creep parameters Parameters for diffusion creep law
66 - MAT(1).pre_diff=.5/1e18;
67 - MAT(1).Ediff=0;
68 - MAT(1).ndiff=1;
69 % dislocation creep parameters Parameters for dislocation creep law
70 - MAT(1).pre_disc=.5/1e18;
71 - MAT(1).Edisc=0; See SiStER_get_viscosity function
72 - MAT(1).ndisc=1;
73 % plasticity
74 - MAT(1).mu=0.6; Parameters for plasticity (faulting),
75 - MAT(1).Cmax=40e6; see SiStER_get_yield_stress function
76 - MAT(1).Cmin=0.01e6;
77 - MAT(1).ecrit=0.1;
```

Input file structure

Material properties must be specified for each phase defined above, in the following format (shown here for phase 1)

```
51
52 % MATERIAL PROPERTIES %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
53
54 % creep laws of the form: pre^(-1/n)*epsII^((1-n)/n)*exp(E/(nRT))
55 % harmonically averaging diffusion creep, dislocation creep
56 % (and plastic creep to simulate brittle failure)
57
58 % phase 1
59 - MAT(1).phase=1;
60 % density parameters
61 - MAT(1).rho0=0.01;
62 - MAT(1).alpha=0;
63 % elasticity
64 - MAT(1).G=1e18;
65 % diffusion creep parameters
66 - MAT(1).pre_diff=.5/1e18;
67 - MAT(1).Ediff=0;
68 - MAT(1).ndiff=1;
69 % dislocation creep parameters
70 - MAT(1).pre_disc=.5/1e18;
71 - MAT(1).Edisc=0;
72 - MAT(1).ndisc=1;
73 % plasticity
74 - MAT(1).mu=0.6;
75 - MAT(1).Cmax=40e6;
76 - MAT(1).Cmin=0.01e6;
77 - MAT(1).ecrit=0.1;
```

NOTES ON PHASE 1 (sticky air)

Note the low density

+ no thermal dependence of density

Unrealistically high shear modulus ensures
an effectively viscous behavior

Input file structure

Material properties must be specified for each phase defined above, in the following format (shown here for phase 1)

```
51
52 % MATERIAL PROPERTIES %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
53
54 % creep laws of the form: pre^(-1/n)*epsII^((1-n)/n)*exp(E/(nRT))
55 % harmonically averaging diffusion creep, dislocation creep
56 % (and plastic creep to simulate brittle failure)
57
58 % phase 1
59 - MAT(1).phase=1;
60 % density parameters
61 - MAT(1).rho0=0.01;
62 - MAT(1).alpha=0;
63 % elasticity
64 - MAT(1).G=1e18;
65 % diffusion creep parameters
66 - MAT(1).pre_diff=.5/1e18;
67 - MAT(1).Ediff=0;
68 - MAT(1).ndiff=1;
69 % dislocation creep parameters
70 - MAT(1).pre_disc=.5/1e18;
71 - MAT(1).Edisc=0;
72 - MAT(1).ndisc=1;
73 % plasticity
74 - MAT(1).mu=0.6;
75 - MAT(1).Cmax=40e6;
76 - MAT(1).Cmin=0.01e6;
77 - MAT(1).ecrit=0.1;
```

NOTES ON PHASE 1 (sticky air)

To enforce a constant Newtonian viscosity η_0 that does not depend on temperature, set $n = 1$, $E=0$, and prefactor = $0.5/\eta_0$ in both dislocation and diffusion creep laws

Input file structure

Material properties must be specified for each phase defined above, in the following format (shown here for phase 2)

```
79
80 % phase 2
81 - MAT(2).phase=2;
82 % density parameters
83 - MAT(2).rho0=2700;
84 - MAT(2).alpha=0;
85 % elasticity
86 - MAT(2).G=30e9;
87 % diffusion creep parameters
88 - MAT(2).pre_diff=.5/1e40;
89 - MAT(2).Ediff=0;
90 - MAT(2).ndiff=1;
91 % dislocation creep parameters
92 - MAT(2).pre_disc=1.0e-3;
93 - MAT(2).Edisc=167000*2.25;
94 - MAT(2).ndisc=2;
95 % plasticity
96 - MAT(2).mu=0.6;
97 - MAT(2).Cmax=40e6;
98 - MAT(2).Cmin=0.01e6;
99 - MAT(2).ecrit=0.1;
100
```

NOTES ON PHASE 2 (rock)

Very low prefactor in diffusion creep
to select only dislocation creep

Non-Newtonian creep law
with T-dependence

Plasticity parameters follow the formalism
of *Lavier et al.* [2000, JGR]

Input file structure

Material properties must be specified for each phase defined above, in the following format (shown here for phase 3)

```
101
102 % phase 3
103 - MAT(3).phase=3;
104 % density parameters
105 - MAT(3).rho0=2700;
106 - MAT(3).alpha=0;
107 % elasticity
108 - MAT(3).G=30e9;
109 % diffusion creep parameters
110 - MAT(3).pre_diff=.5/1e40;
111 - MAT(3).Ediff=0;
112 - MAT(3).ndiff=1;
113 % dislocation creep parameters
114 - MAT(3).pre_disc=1.0e-3;
115 - MAT(3).Edisc=167000*2.25;
116 - MAT(3).ndisc=2;
117 % plasticity
118 - MAT(3).mu=0.6;
119 - MAT(3).Cmax=0.01e6;
120 - MAT(3).Cmin=0.01e6;
121 - MAT(3).ecrit=0.1;
122
```

NOTES ON PHASE 3 (weak rock)

Identical to phase 2,
but with very low initial cohesion
to promote rapid yielding of this weak seed

Input file structure

More parameters...

```
123
124 % ADDITIONAL PARAMETERS %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
125 - PARAMS.YNlast=1; % elasticity on (1) or off (0)
126 - PARAMS.YNPlas=1; % plasticity on (1) or off (0)
127 - PARAMS.epsII_from_stress=1; % get strain rate from stresses (1, default) or from velocity field (0)
128 - PARAMS.tau_heal=1e12; % healing time for plasticity (s)
129 - PARAMS.gx=0; % gravity along x
130 - PARAMS.gy=9.8; % gravity along y
131 - PARAMS.fracCFL=0.5; % distance by which a marker is allowed to move over a time step, expressed as a fraction
132 - PARAMS.R=8.314; % gas constant
133 - PARAMS.etamax=1e25; % maximum viscosity
134 - PARAMS.etamin=1e18; % minimum viscosity
135 - PARAMS.Tsolve=1; % yes (1) or no (0) solve for temperature
```

Simulation incorporate elasticity and plasticity (brittle strain localization)

Input file structure

More parameters...

```
123
124 % ADDITIONAL PARAMETERS %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
125 - PARAMS.YNlast=1; % elasticity on (1) or off (0)
126 - PARAMS.YNplas=1; % plasticity on (1) or off (0)
127 - PARAMS.epsII_from_stress=1; % get strain rate from stresses (1, default) or from velocity field (0)
128 - PARAMS.tau_heal=1e12; % healing time for plasticity (s)
129 - PARAMS.gx=0; % gravity along x
130 - PARAMS.gy=9.8; % gravity along y
131 - PARAMS.fracCFL=0.5; % distance by which a marker is allowed to move over a time step, expressed as a fraction
132 - PARAMS.R=8.314; % gas constant
133 - PARAMS.etamax=1e25; % maximum viscosity
134 - PARAMS.etamin=1e18; % minimum viscosity
135 - PARAMS.Tsolve=1; % yes (1) or no (0) solve for temperature
```

Strain rate is calculated using the current stress and the stress-strain relation, which limits the number of interpolation. The alternative (set value to 0) is to calculate it directly from the velocity solution.

Input file structure

More parameters...

```
123
124 % ADDITIONAL PARAMETERS %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
125 - PARAMS.YNlast=1; % elasticity on (1) or off (0)
126 - PARAMS.YNplas=1; % plasticity on (1) or off (0)
127 - PARAMS.epsII_from_stress=1; % get strain rate from stresses (1, default) or from velocity field (0)
128 - PARAMS.tau_heal=1e12; % healing time for plasticity (s)
129 - PARAMS.gx=0; % gravity along x
130 - PARAMS.gy=9.8; % gravity along y
131 - PARAMS.fracCFL=0.5; % distance by which a marker is allowed to move over a time step, expressed as a fraction
132 - PARAMS.R=8.314; % gas constant
133 - PARAMS.etamax=1e25; % maximum viscosity
134 - PARAMS.etamin=1e18; % minimum viscosity
135 - PARAMS.Tsolve=1; % yes (1) or no (0) solve for temperature
```

Accumulated plastic strain (in weak yielded zones, i.e., faults) heals exponentially on a time scale specified here, in seconds [Lavie *et al.*, 2000, JGR]. This way, old, abandoned faults where no strain localization is sustained can disappear.

Input file structure

More parameters...

```
123
124 % ADDITIONAL PARAMETERS %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
125 - PARAMS.YNlast=1; % elasticity on (1) or off (0)
126 - PARAMS.YNplas=1; % plasticity on (1) or off (0)
127 - PARAMS.epsII_from_stress=1; % get strain rate from stresses (1, default) or from velocity field (0)
128 - PARAMS.tau_heal=1e12; % healing time for plasticity (s)
129 - PARAMS.gx=0; % gravity along x
130 - PARAMS.gy=9.8; % gravity along y
131 - PARAMS.fracCFL=0.5; % distance by which a marker is allowed to move over a time step, expressed as a fraction
132 - PARAMS.R=8.314; % gas constant
133 - PARAMS.etamax=1e25; % maximum viscosity
134 - PARAMS.etamin=1e18; % minimum viscosity
135 - PARAMS.Tsolve=1; % yes (1) or no (0) solve for temperature
```

Gravity in m/s^2

(both x and y components, i.e., cases with sloping gravity are possible)

Input file structure

More parameters...

```
123
124 % ADDITIONAL PARAMETERS %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
125 - PARAMS.YNlast=1; % elasticity on (1) or off (0)
126 - PARAMS.YNplas=1; % plasticity on (1) or off (0)
127 - PARAMS.epsII_from_stress=1; % get strain rate from stresses (1, default) or from velocity field (0)
128 - PARAMS.tau_heal=1e12; % healing time for plasticity (s)
129 - PARAMS.gx=0; % gravity along x
130 - PARAMS.gy=9.8; % gravity along y
131 - PARAMS.fracCFL=0.5; % distance by which a marker is allowed to move over a time step, expressed as a fraction
132 - PARAMS.R=8.314; % gas constant
133 - PARAMS.etamax=1e25; % maximum viscosity
134 - PARAMS.etamin=1e18; % minimum viscosity
135 - PARAMS.Tsolve=1; % yes (1) or no (0) solve for temperature
```

The time step (used for marker advection) is set such that markers cannot move by more than $\text{PARAMS.fracCFL} \times$ the smallest grid size, at every time iteration (= CFL condition for advection).

Input file structure

More parameters...

```
123
124 % ADDITIONAL PARAMETERS %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
125 - PARAMS.YNlast=1; % elasticity on (1) or off (0)
126 - PARAMS.YNplas=1; % plasticity on (1) or off (0)
127 - PARAMS.epsII_from_stress=1; % get strain rate from stresses (1, default) or from velocity field (0)
128 - PARAMS.tau_heal=1e12; % healing time for plasticity (s)
129 - PARAMS.gx=0; % gravity along x
130 - PARAMS.gy=9.8; % gravity along y
131 - PARAMS.fracCFL=0.5; % distance by which a marker is allowed to move over a time step, expressed as a fraction
132 - PARAMS.R=8.314; % gas constant
133 - PARAMS.etamax=1e25; % maximum viscosity
134 - PARAMS.etamin=1e18; % minimum viscosity
135 - PARAMS.Tsolve=1; % yes (1) or no (0) solve for temperature
```

Hard limits on the viscosity, enforced in **SiStER_get_viscosity**

Here it cannot drop below 1e18 Pa.s, or exceed 1e25 Pa.s.

Input file structure

More parameters...

```
123
124 % ADDITIONAL PARAMETERS %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
125 - PARAMS.YNlast=1; % elasticity on (1) or off (0)
126 - PARAMS.YNplas=1; % plasticity on (1) or off (0)
127 - PARAMS.epsII_from_stress=1; % get strain rate from stresses (1, default) or from velocity field (0)
128 - PARAMS.tau_heal=1e12; % healing time for plasticity (s)
129 - PARAMS.gx=0; % gravity along x
130 - PARAMS.gy=9.8; % gravity along y
131 - PARAMS.fracCFL=0.5; % distance by which a marker is allowed to move over a time step, expressed as a fraction
132 - PARAMS.R=8.314; % gas constant
133 - PARAMS.etamax=1e25; % maximum viscosity
134 - PARAMS.etamin=1e18; % minimum viscosity
135 - PARAMS.Tsolve=1; % yes (1) or no (0) solve for temperature
```

If set to 1, the temperature field will evolve by advection (in the solid flow field) and diffusion.

If set to 0, it will only be advected- the heat equation will not be solved.

Input file structure

Thermal parameters

```
136 % initial temperature profile, polynomial with depth
137 %  $T = a_0 + a_1 y + a_2 y^2 + a_3 y^3 + \text{amp} \sin(2\pi x / \text{lam})$ 
138 % (make sure it matches the BCs)
139 - PARAMS.a0=0;
140 - PARAMS.a1=0;
141 - PARAMS.a2=0;
142 - PARAMS.a3=1000/(30e3)^3;
143 - PARAMS.amp=0; % amplitude of sinusoidal perturbation
144 - PARAMS.lam=1; % wavelength of sinusoidal perturbation
145 - PARAMS.ynTreset=1; % if ==1, reset  $T=T_0$  where  $im==1$  (sticky layer)
146 - PARAMS.T0=0;
147 % reference values for the constant diffusivity thermal solver
148 % ( $\kappa = \kappa_{\text{ref}} / (\rho_{\text{ref}} c_{\text{pref}})$ )
149 - PARAMS.rhoref=MAT(2).rho0;
150 - PARAMS.kref=3;
151 - PARAMS.cpref=1000;
```

The initial temperature structure follows this functional form:

$$T = a_0 + a_1 y + a_2 y^2 + a_3 y^3 + \text{amp} \sin(2\pi x / \text{lam})$$

See **SiStER_Initialize.m** – Make sure it is not incompatible with the imposed boundary conditions! And do not forget that $y=0$ is the top of the domain, not of the rock layer (it includes the sticky layer)

Input file structure

Thermal parameters

```
136 % initial temperature profile, polynomial with depth
137 %  $T = a_0 + a_1 y + a_2 y^2 + a_3 y^3 + \text{amp} \sin(2\pi X / \text{lam})$ 
138 % (make sure it matches the BCs)
139 - PARAMS.a0=0;
140 - PARAMS.a1=0;
141 - PARAMS.a2=0;
142 - PARAMS.a3=1000/(30e3)^3;
143 - PARAMS.amp=0; % amplitude of sinusoidal perturbation
144 - PARAMS.lam=1; % wavelength of sinusoidal perturbation
145 - PARAMS.ynTreset=1; % if ==1, reset  $T=T_0$  where  $im==1$  (sticky layer)
146 - PARAMS.T0=0;
147 % reference values for the constant diffusivity thermal solver
148 % ( $\kappa = k_{\text{ref}} / (\rho_{\text{ref}} c_{\text{pref}})$ )
149 - PARAMS.rhoref=MAT(2).rho0;
150 - PARAMS.kref=3;
151 - PARAMS.cpref=1000;
```

With `PARAMS.ynTreset = 1`, the temperature field will be automatically set to `PARAMS.T0` throughout the sticky layer. This is a way to enforce T_0 at the air-rock interface.

Input file structure

Thermal parameters

```
136 % initial temperature profile, polynomial with depth
137 %  $T = a_0 + a_1*y + a_2*y^2 + a_3*y^3 + \text{amp}*\sin(2*\pi*X/\text{lam})$ 
138 % (make sure it matches the BCs)
139 - PARAMS.a0=0;
140 - PARAMS.a1=0;
141 - PARAMS.a2=0;
142 - PARAMS.a3=1000/(30e3)^3;
143 - PARAMS.amp=0; % amplitude of sinusoidal perturbation
144 - PARAMS.lam=1; % wavelength of sinusoidal perturbation
145 - PARAMS.ynTreset=1; % if ==1, reset  $T=T_0$  where  $im==1$  (sticky layer)
146 - PARAMS.T0=0;
147 % reference values for the constant diffusivity thermal solver
148 % ( $\kappa = k_{\text{ref}} / (\rho_{\text{ref}}*c_{\text{pref}})$ )
149 - PARAMS.rhoref=MAT(2).rho0;
150 - PARAMS.kref=3;
151 - PARAMS.cpref=1000;
```

The present version of the thermal solver uses a constant diffusivity that can be set here as $k_{\text{ref}} / (\rho_{\text{ref}}*c_{\text{pref}})$

This only gets used if `PARAMS.Tsolve=1`

Input file structure

Picard iterations parameters

```
152 % Picard iterations
153 - PARAMS.Npicard_min=3; % minimum number of Picard iterations per time step
154 - PARAMS.Npicard_max=50; % maximum number of Picard iterations per time step
155 - PARAMS.conv_crit1=0.01;
156 - PARAMS.conv_crit2=0.03;
157 % convergence is assumed if a fraction of the domain smaller
158 % than PARAMS.conv_crit2 * domain size has more than relative
159 % change of PARAMS.conv_crit1 in strain rate and velocity field
160
```

This run will always use at least 3 Picard iterations, but will stop after 50, regardless of the convergence criterion.

Input file structure

Picard iterations parameters

```
152 % Picard iterations
153 - PARAMS.Npicard_min=3; % minimum number of Picard iterations per time step
154 - PARAMS.Npicard_max=50; % maximum number of Picard iterations per time step
155 - PARAMS.conv_crit1=0.01;
156 - PARAMS.conv_crit2=0.03;
157 % convergence is assumed if a fraction of the domain smaller
158 % than PARAMS.conv_crit2 * domain size has more than relative
159 % change of PARAMS.conv_crit1 in strain rate and velocity field
160
```

Convergence is assumed when the strain rate and velocity field change by more than 1% in only 3% of the domain or less. This formulation helps when iterating over the formation of narrow shear bands.

See **SiStER_run_Picard_iterations**

Input file structure

Boundary conditions (pressure)

```
161  
162 % BOUNDARY CONDITIONS %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  
163  
164 % pressure  
165 - PARAMS.p0cell=0; % pressure in the top-left corner of the domain (anchor point)  
166
```

Pressure is set to 0 in the top-left corner of the domain.
This is done to anchor the solution to a specific pressure
and stabilize the inversion.

Input file structure

```
167
168 % flow      Boundary conditions (flow)
169
170 % boundary conditions
171 % entries in BC correspond to
172 % 1/ rollers? (1=yes, 0=no)
173 % 2/ type of velocity normal to boundary (0=constant)
174 % 3/ value of normal velocity
175
176 - BC.top=[1 0 1.0563e-10];
177 - BC.bot=[1 0 -1.0563e-10];
178 - BC.left=[1 0 -3.1688e-10];
179 - BC.right=[1 0 3.1688e-10];
180 - PARAMS.BalanceStickyLayer=1; % if set to 1, the code will reset the inflow
181 % / outflow BCs to balance the inflow / outflow of sticky layer material,
182 % and rock separately, based on the position of the sticky layer / air
183 % interface
184
```

BCs are specified in this BC structure for the top, bottom, left and right edges of the domain. The first entry is 1 for free slip ($d v_{\text{tangential}} / d \text{normal} = 0$), 0 for no-slip ($v_{\text{tangential}} = 0$)

Input file structure

```
167
168 % flow      Boundary conditions (flow)
169
170 % boundary conditions
171 % entries in BC correspond to
172 % 1/ rollers? (1=yes, 0=no)
173 % 2/ type of velocity normal to boundary (0=constant)
174 % 3/ value of normal velocity
175
176 - BC.top=[1 0 1.0563e-10];
177 - BC.bot=[1 0 -1.0563e-10];
178 - BC.left=[1 0 -3.1688e-10];
179 - BC.right=[1 0 3.1688e-10];
180 - PARAMS.BalanceStickyLayer=1; % if set to 1, the code will reset the inflow
181 % / outflow BCs to balance the inflow / outflow of sticky layer material,
182 % and rock separately, based on the position of the sticky layer / air
183 % interface
184
```

The second entry specifies the type of normal velocity BC – it can only be 0 in the current version, which enforces a constant normal velocity (m/s), specified in the third entry (careful with sign conventions, and make sure inflow matched outflow!)

Input file structure

```
167
168 % flow      Boundary conditions (flow)
169
170 % boundary conditions
171 % entries in BC correspond to
172 % 1/ rollers? (1=yes, 0=no)
173 % 2/ type of velocity normal to boundary (0=constant)
174 % 3/ value of normal velocity
175
176 - BC.top=[1 0 1.0563e-10];
177 - BC.bot=[1 0 -1.0563e-10];
178 - BC.left=[1 0 -3.1688e-10];
179 - BC.right=[1 0 3.1688e-10];
180 - PARAMS.BalanceStickyLayer=1; % if set to 1, the code will reset the inflow
181 % / outflow BCs to balance the inflow / outflow of sticky layer material,
182 % and rock separately, based on the position of the sticky layer / air
183 % interface
184
```

In this example, a velocity of 1 cm/yr is applied towards the right on the right edge, and – 1 cm/yr on the left edge.

The top and bottom velocities are set to balance this outflow.

Input file structure

```
167
168 % flow      Boundary conditions (flow)
169
170 % boundary conditions
171 % entries in BC correspond to
172 % 1/ rollers? (1=yes, 0=no)
173 % 2/ type of velocity normal to boundary (0=constant)
174 % 3/ value of normal velocity
175
176 - BC.top=[1 0 1.0563e-10];
177 - BC.bot=[1 0 -1.0563e-10];
178 - BC.left=[1 0 -3.1688e-10];
179 - BC.right=[1 0 3.1688e-10];
180 - PARAMS.BalanceStickyLayer=1; % if set to 1, the code will reset the inflow
181 % / outflow BCs to balance the inflow / outflow of sticky layer material,
182 % and rock separately, based on the position of the sticky layer / air
183 % interface
184
```

Setting this parameter to 1 will balance the inflow of sticky air and rock separately, based on the position of the air-rock interface at any given time (See **SiStER_flow_solve**)

Input file structure

Boundary conditions (temperature)

```
186      % thermal
187
188      % entries in BCtherm correspond to
189      % 1/ type? (1=Dirichlet, 0=Neumann)
190      % 2/ value
191 -    BCtherm.top=[1 0];
192 -    BCtherm.bot=[1 1000];
193 -    BCtherm.left=[2 0];
194 -    BCtherm.right=[2 0];
```

Those will be used if PARAMS.Tsolve is set to 1.

If the first entry is 1 (Dirichlet), the temperature is set equal to the second entry. If the first entry is 2 (Neumann), the gradient of temperature is set equal to the second entry.

This example has $T = 0$ on the top, $T = 1000^\circ\text{C}$ on the bottom, and no heat flux through the sides.

Visualizing the output

Output files contain (among other things):

time (in seconds)

Nodal arrays:

X, Y (coordinates of shear nodes)

vx, vy (velocities) and p (pressure)

Marker arrays:

xm, ym (marker coordinates)

im (phase) etam (viscosity) rhom (density) Tm (temperature)

ep (accumulated plastic strain) sxxm, sxym (deviatoric stresses)

epsllm (strain rate)

Topography:

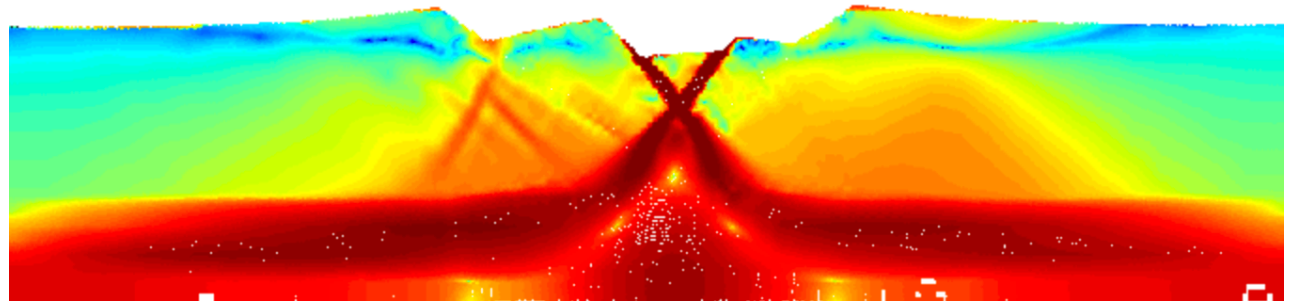
topo_x, topo_y (coordinates of a dense chain of markers tracking the sticky layer – rock interface)

More variables can be output- see l. 96 of **SiStER_MAIN**

Visualizing the output

To visualize marker arrays (here the strain rate at iteration 100, excluding the sticky air layer):

```
>> fastscatter(xm(im>1)/1e3,ym(im>1) /1e3,log10(epsIIm(im>1)),'markersize',1);  
>> axis ij  
>> axis equal  
>> grid off  
>> colorbar  
>> caxis([-18 -13])
```



And the topography:

```
>> plot(topo_x,topo_y,'k.')  
>> set(gca,'YDir','reverse')
```

