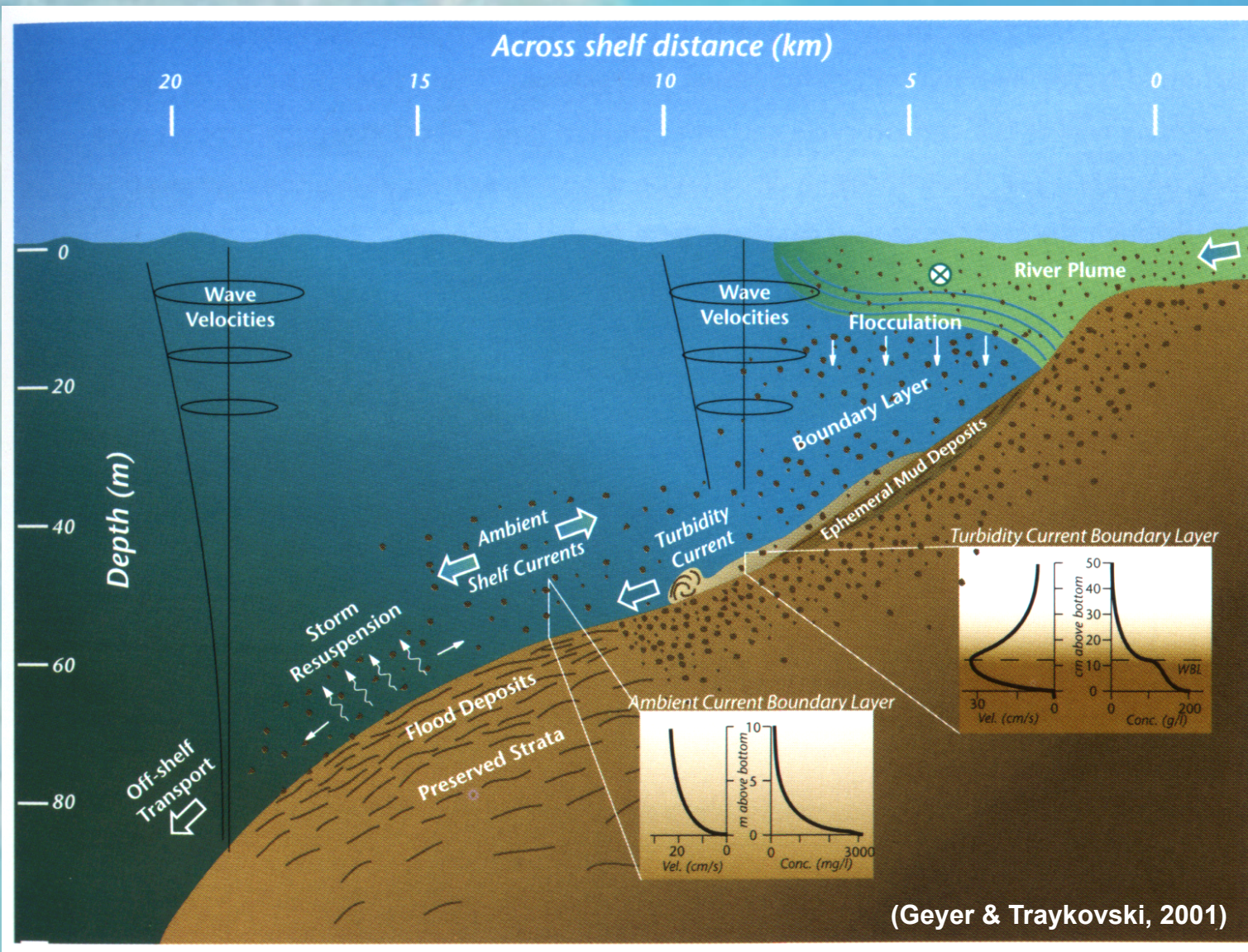


Modeling of Clinoforms Created By Wave/Current Supported Gravity Flows:

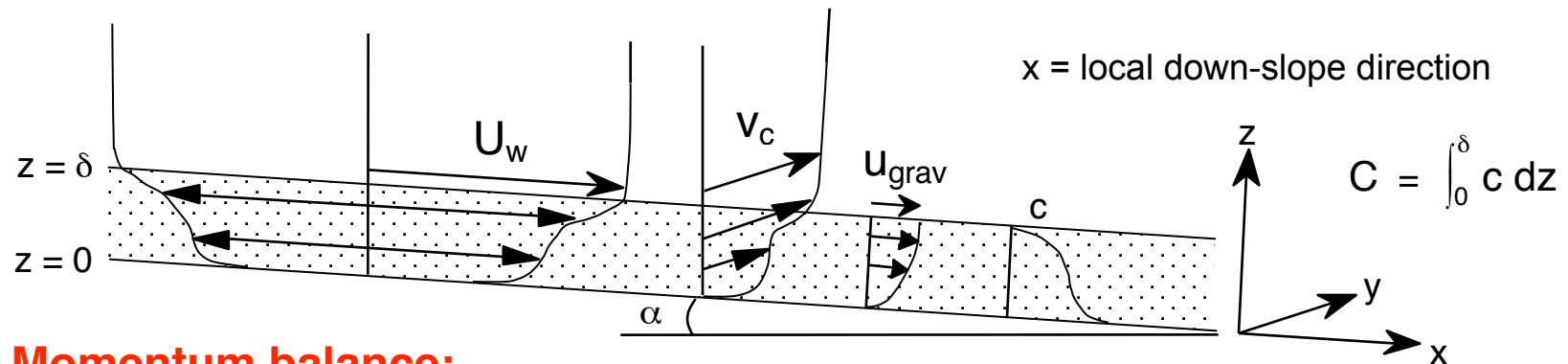
Carl Friedrichs, Virginia Institute of Marine Science, presented at CSDMS 10/19/09



Original motivation: Wave-supported sediment gravity flows are the dominant offshore transport mechanism on the Eel River shelf, CA, during floods (Ogston et al., 2000; Traykovski et al. 2000).

Wave and Current Supported Sediment Gravity Flow Analytical Model (WSGFAM):

(Friedrichs, Wright, Scully, Ma 2001 - 2009)



(i) Momentum balance:

Down-slope pressure gradient = Bottom friction

$$\alpha B = c_d U_{\max} u_{\text{grav}}$$

B = depth-integrated buoyancy anomaly
(i.e., suspended sediment)
 c_d = drag coefficient = 0.003

$$U_{\max} = (U_w^2 + v_c^2 + u_{\text{grav}}^2)^{1/2}$$

(ii) Maximum turbulent sediment load:

Richardson Number = $\frac{\text{Buoyancy}}{\text{Shear}}$ = Critical value

$$Ri = \frac{B}{(U_{\max})^2} = Ri_c = 1/4$$

(iii) Solve for u_{grav}

(iv) Deposition = $-\frac{d}{dx} (B u_{\text{grav}}) - (B u_{\text{grav}})/r$

= downslope convergence + lateral convergence

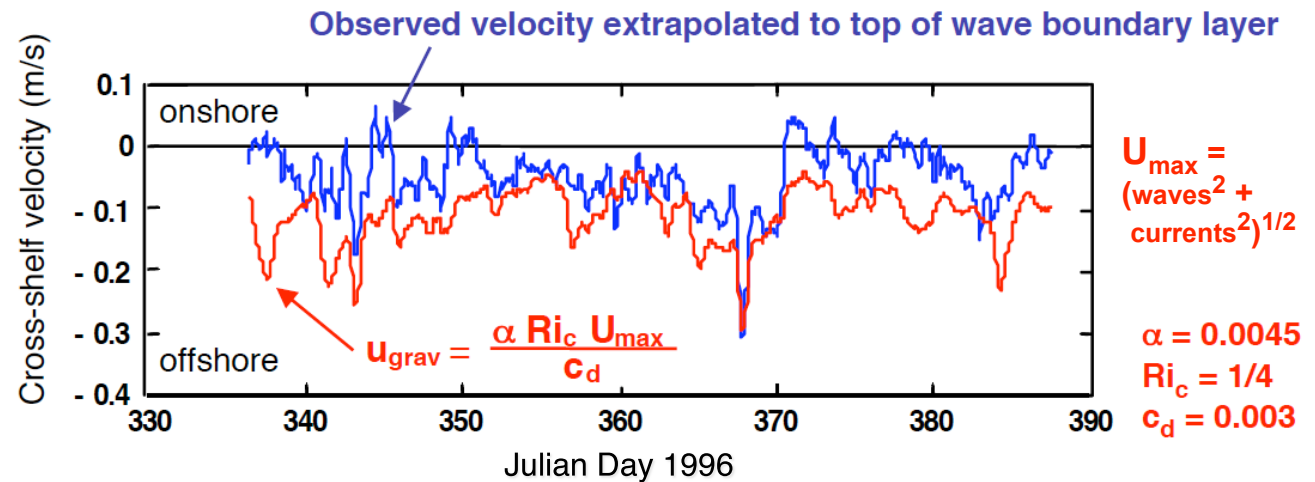
r = radius of curvature of local bathymetric contour

Application of WSGFAM to Eel River shelf, CA (Scully, Friedrichs, Wright, 2002)

(iii) Solve for u_{grav}

Comparison of analytical solution to observed across-shelf velocity:

(Observations from Ogston et al. 2000)

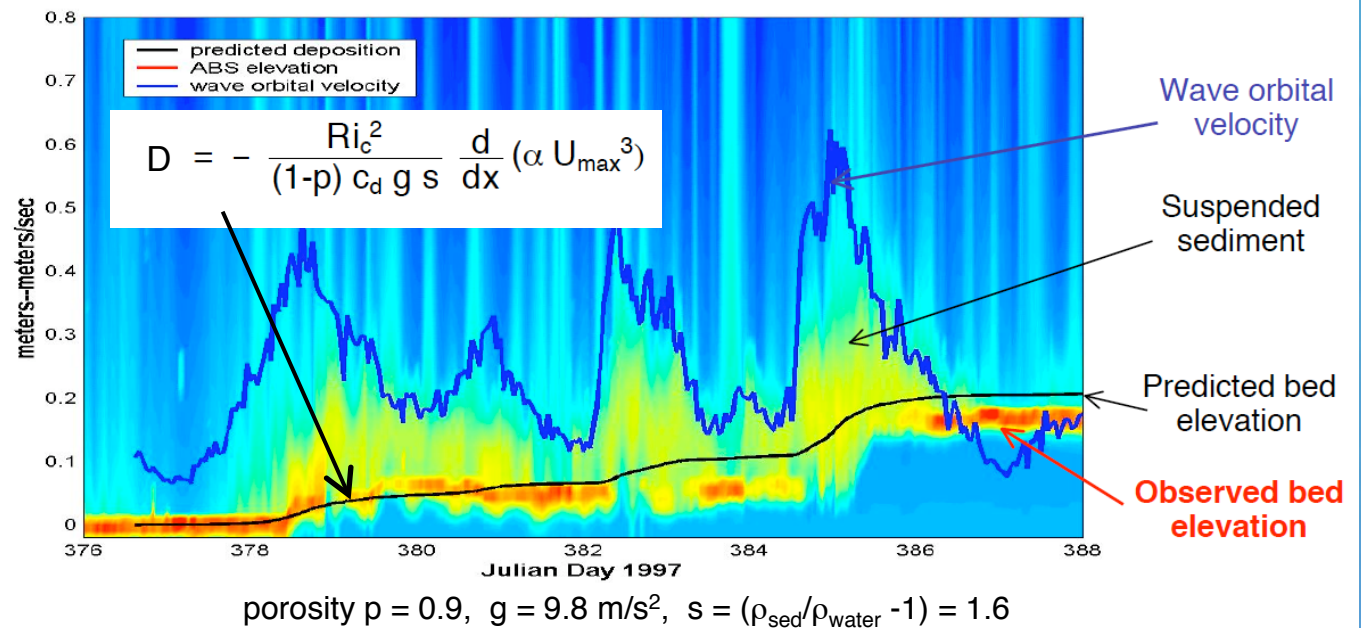


(iv) Deposition $= d/dx (B u_{\text{grav}})$

(neglecting lateral convergence in this case)

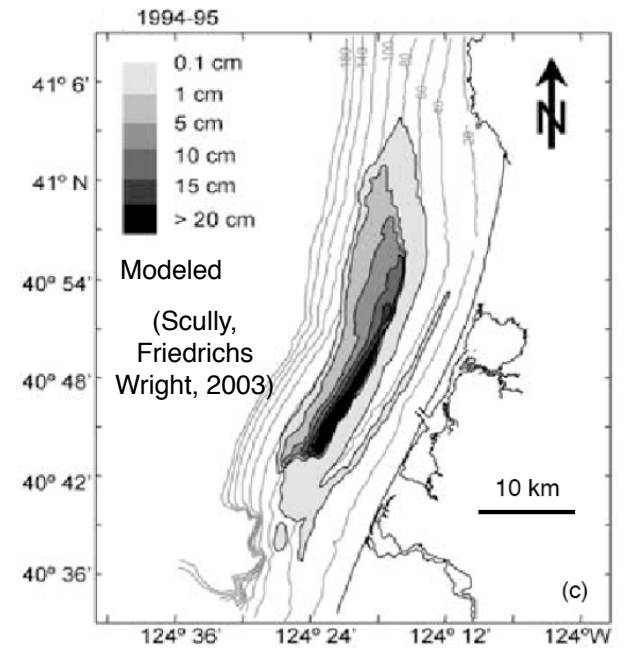
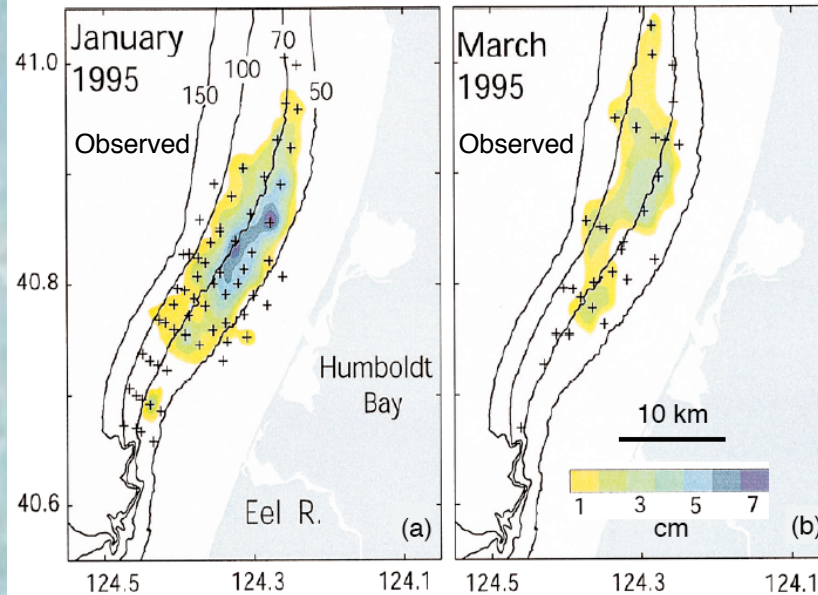
Comparison of analytical solution to observed deposition:

(Observations from Traykovski et al. 2000)

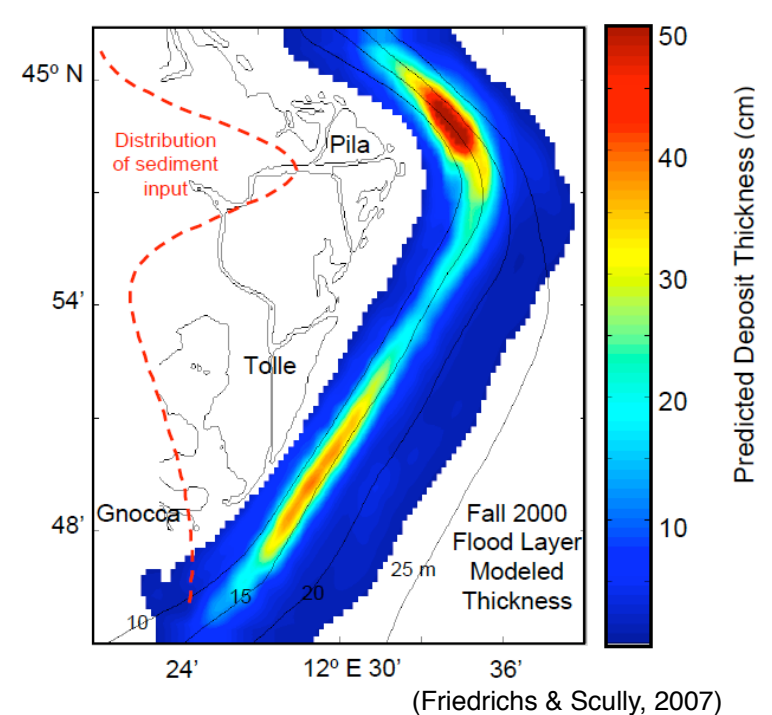
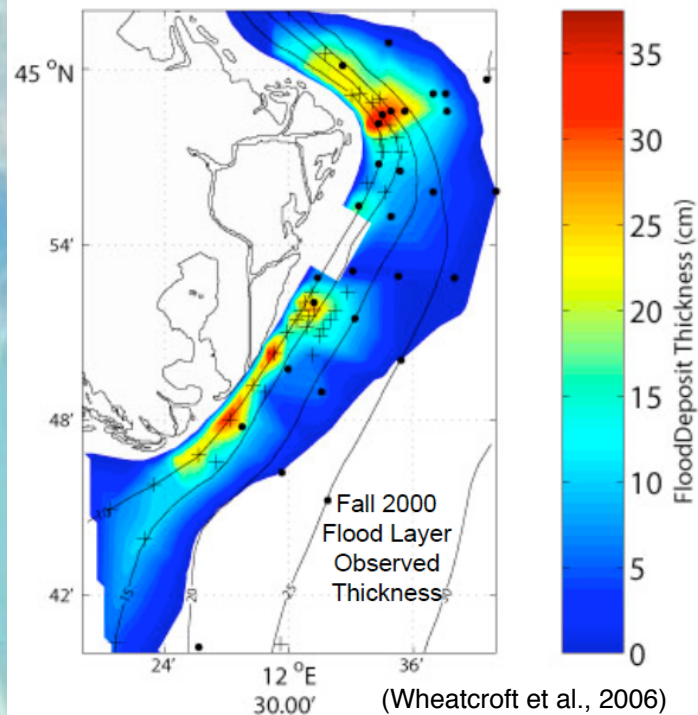


Comparison of WSGFAM results to observed seasonal deposition on Eel shelf, Northern California:

(Wheatcroft & Borgeld, 2000)

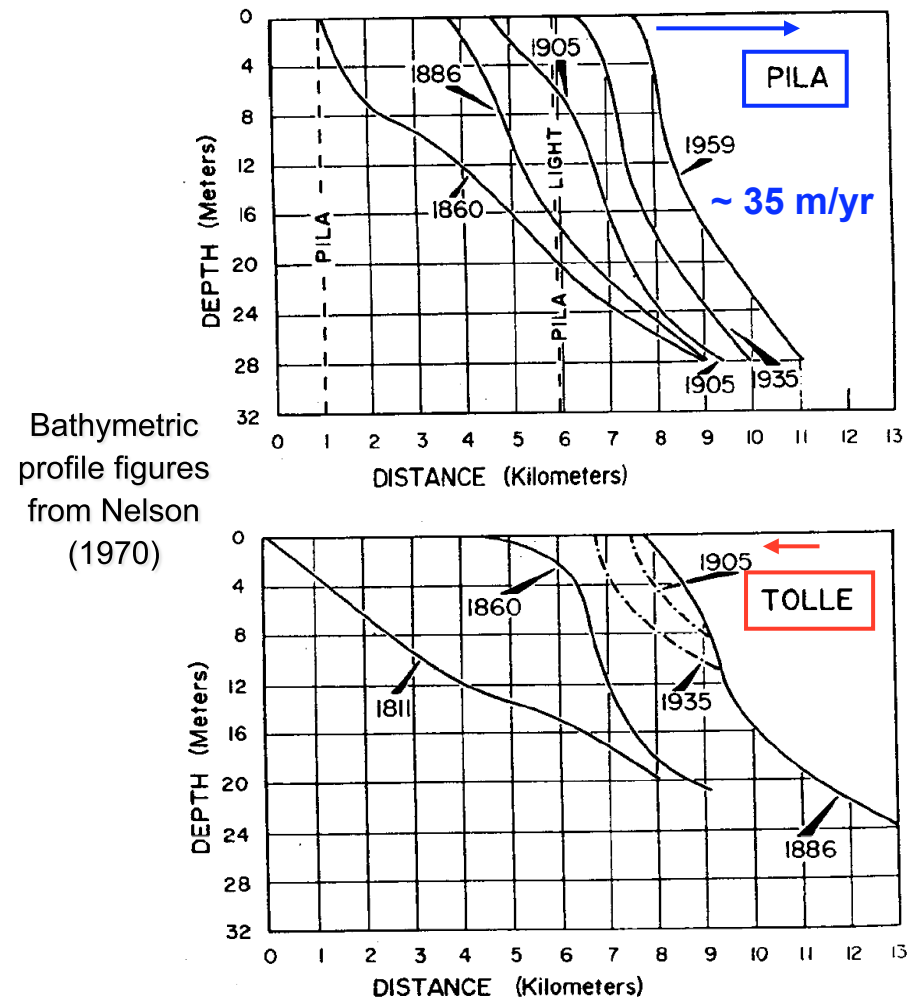
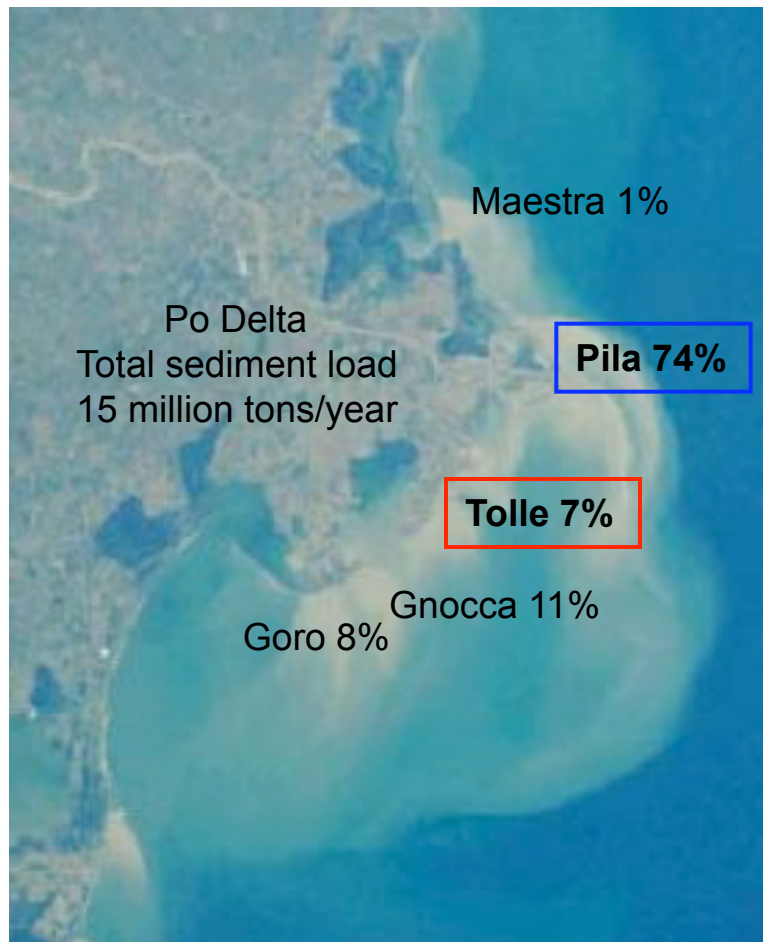


Comparison WSGFAM results to observed seasonal deposition on Po subaqueous delta, NW Adriatic Sea:

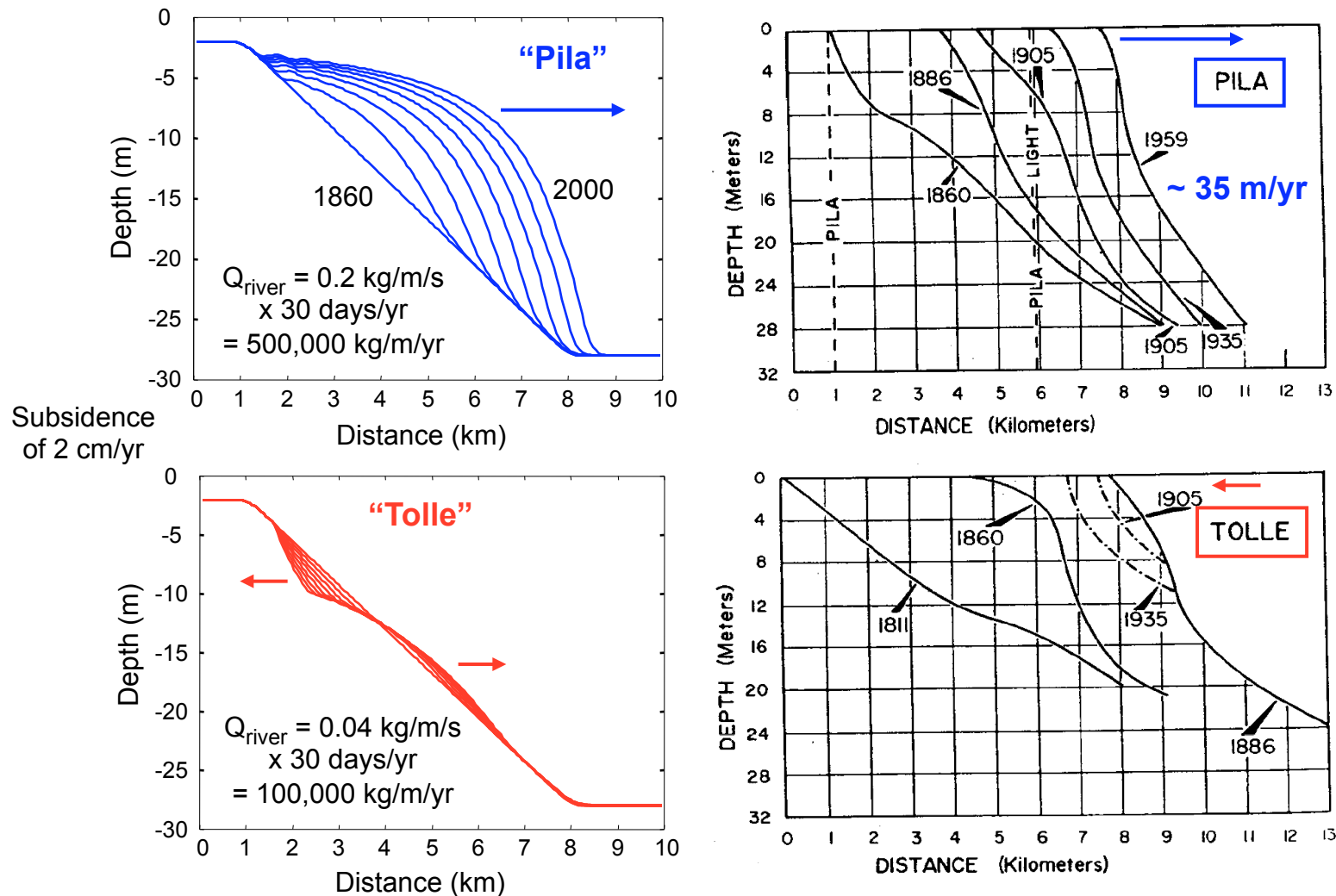


Distinct evolution of Po distributaries: Pila vs. Tolle (post 1880)

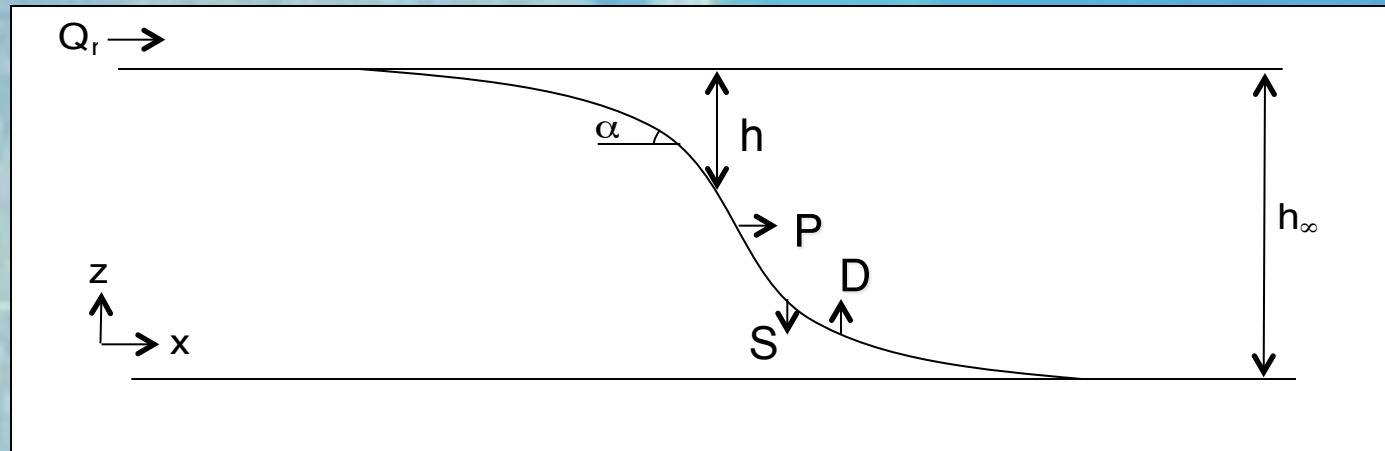
- Pila prograded as an equilibrium form, while Tolle eroded and steepened near shore
- Carminati & Martinelli (2002) suggest Po delta is subsiding by up to several cm/year



Century-scale modeling of subsiding delta (Friedrichs and Scully, 2007)



- Sediment load overwhelms subsidence off Pila, and delta evolves toward equilibrium
- Subsidence overwhelms sedimentation near shore off Tolle, steepening delta near shore



1. Momentum Balance:

$$\alpha B = c_d U_{\max} u_{\text{grav}}$$

2. Maximum Turbulent Sediment Load:

$$Ri = \frac{B}{(U_{\max})^2} = Ri_c = 1/4$$

3. Prograding Equilibrium Condition:

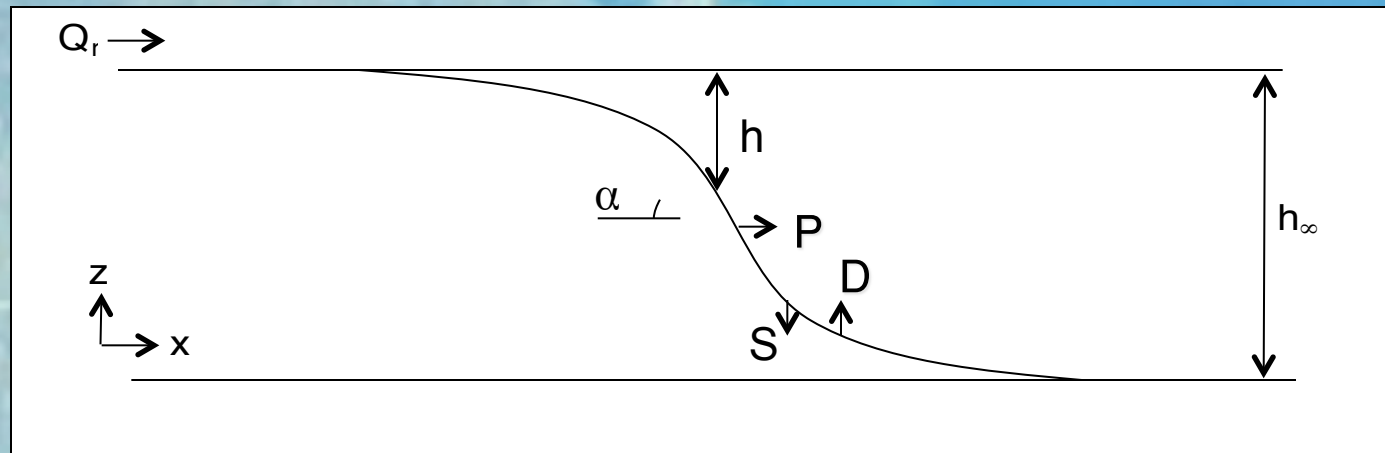
$$\text{Progradation Rate} = \frac{\text{Deposition} - \text{Subsidence}}{\text{Bed Slope}} = \text{Constant in } x, \text{ i.e., } P = \frac{D - S}{\alpha} = \text{Const.}$$

4. Conservation of Mass:

$$\text{Riverine Sediment Supply} = \text{Total Mass Deposited} + \text{Bypassing}, \text{ i.e.,}$$

$$Q_r = (1-p) \rho_s \int_0^\infty D \, dx + \text{Bypassing}$$

(Friedrichs and Wright, 2005)



Equilibrium bed slope
(α) is controlled by:

Riverine sediment
supply, Q_r

Accommodation space
from subsidence,

$$A = (1-p) \rho_s \int_0^\infty S dx$$

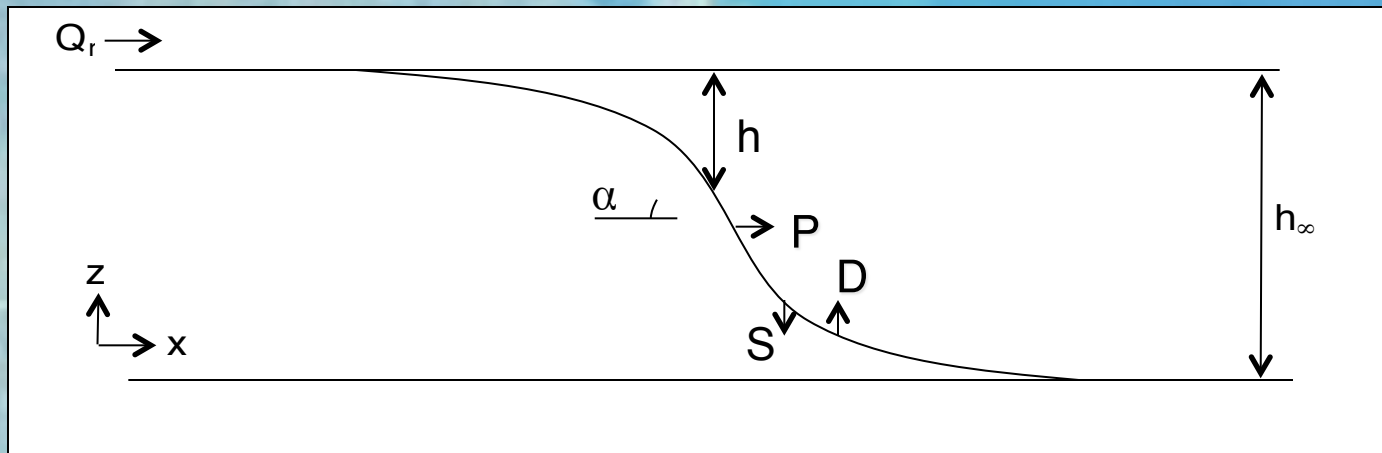
$$\frac{\alpha}{\{1 - (\alpha Ri_c / c_d)^2\}^{3/2}} = \frac{c_d sg}{Ri_c^2 \rho_s} \frac{Q_r}{u_w^3} \left(1 - \frac{h}{h_\infty}\right) \left(1 - \frac{A}{Q_r}\right)$$

Wave orbital velocity, u_w

Underlying shelf
depth, h_∞

General Equilibrium
Solution

(Friedrichs and Wright, 2005)



Equilibrium bed slope
(α) is controlled by:

Riverine sediment
supply, Q_r

Accommodation space
from subsidence,

$$A = (1-p) \rho_s \int_0^\infty S \, dx$$

$$\alpha = \frac{c_d \, s g}{Ri_c^2 \, \rho_s} \frac{Q_r}{u_w^3} \left(1 - \frac{h}{h_\infty}\right) \left(1 - \frac{A}{Q_r}\right)$$

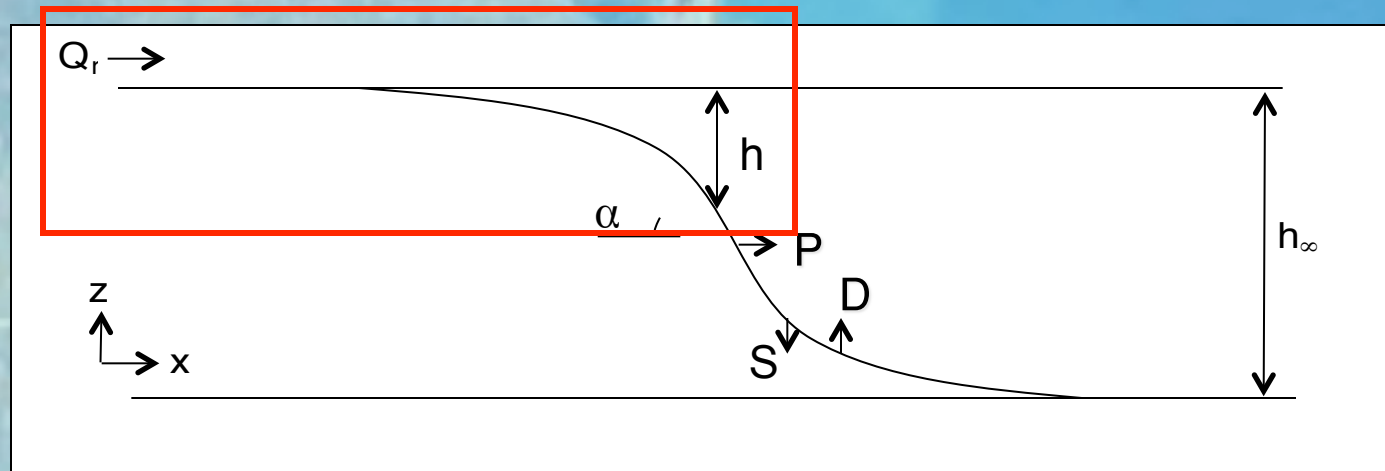
Wave orbital velocity, u_w

Underlying shelf
depth, h_∞

General Equilibrium
Solution

Depositional slope cannot be larger than $\alpha = c_d/Ri_c \approx 0.012 - 0.016$.
For $\alpha > 0.012 - 0.016$, turbidity current will be auto-suspending and erosional.

(Friedrichs and Wright, 2005)



Equilibrium bed slope
(α) is controlled by:

Riverine sediment
supply, Q_r

Accommodation space
from subsidence,

$$\frac{\alpha}{\{1 - (\alpha Ri_c / c_d)^2\}^{3/2}} = \frac{c_d sg}{Ri_c^2 \rho_s} \frac{Q_r}{u_w^3} \left(1 - \frac{h}{h_\infty}\right) \left(1 - \frac{A}{Q_r}\right)$$

Wave orbital velocity, u_w

Underlying shelf
depth, h_∞

$$A = (1-p) \rho_s \int_0^\infty S dx$$

General Equilibrium
Solution

First consider waves on landward, bypassing portion of profile with no subsidence.

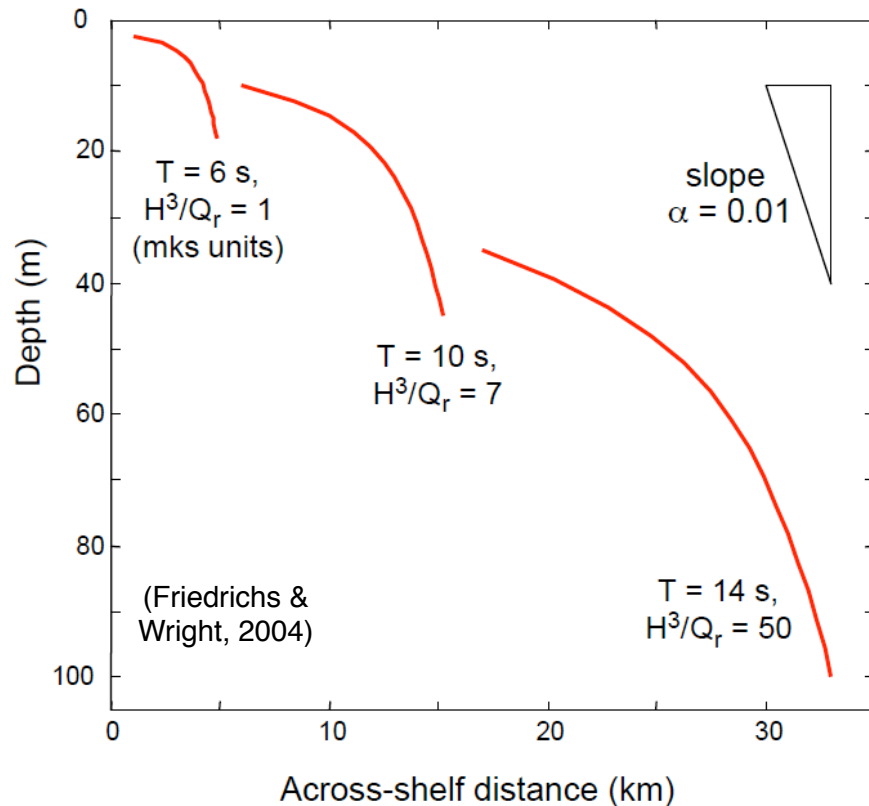
For wind waves, substitute in $u_w = \omega (H/2) (\sinh kh)^{-1}$, where H is wave height, ω is wave radian frequency and k is wave number.

(Friedrichs and Wright, 2005)

Equilibrium bypassing solution:

$$\frac{\alpha}{\{1 - (\alpha Ri_c / c_d)^2\}^{3/2}} = \frac{8 (\sinh kh)^3 c_d sg}{(H^3/Q_r) Ri_c^2 \rho_s \omega^3}$$

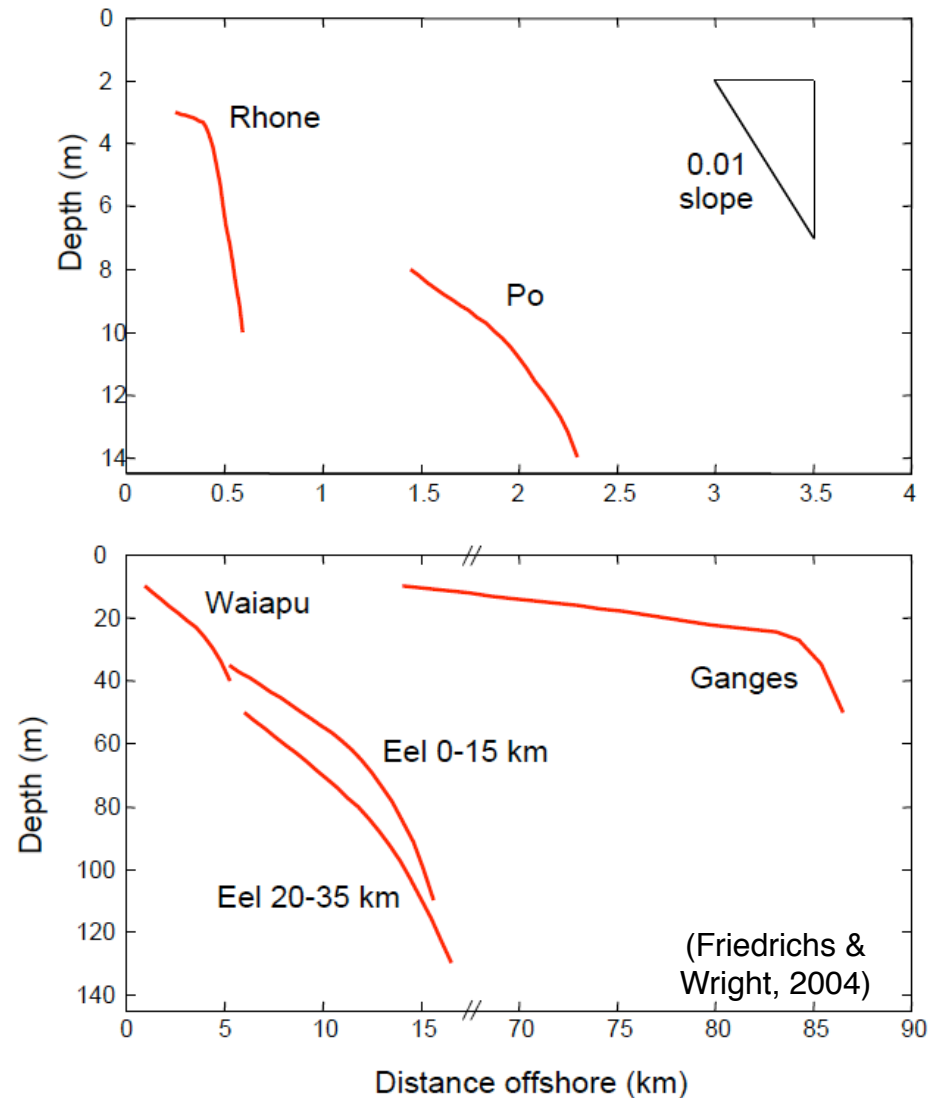
Model trends for bathymetry:

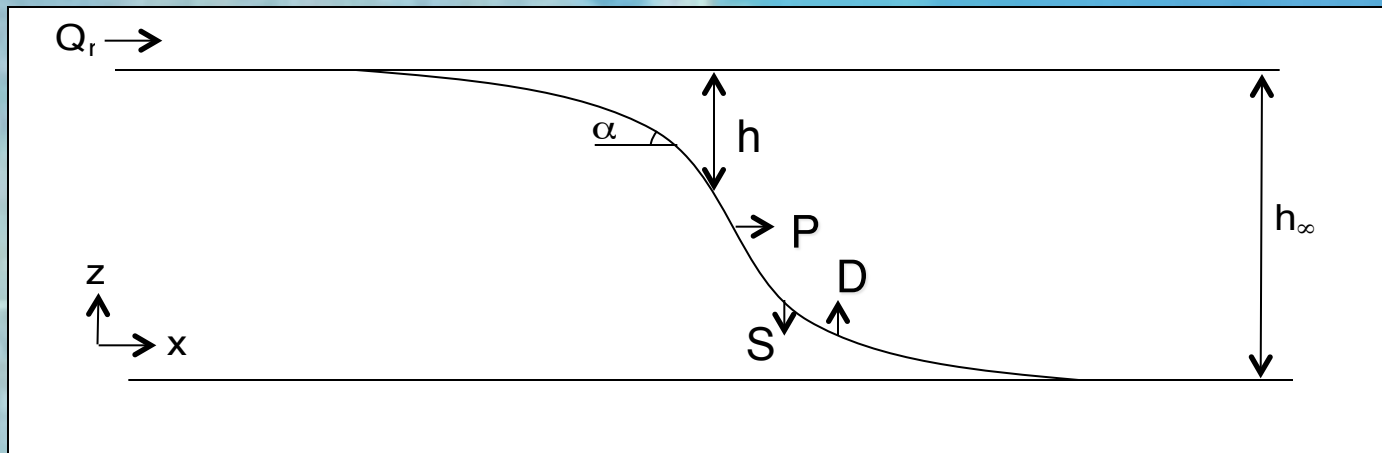


-- Only variable quantities are wave height, river discharge (H^3/Q_r) and wave period (T).

-- H and T tend to be positively correlated, so H^3/Q_r is the dominant environmental parameter.

Observed bathymetries:





Equilibrium bed slope
(α) is controlled by:

Riverine sediment
supply, Q_r

Accommodation space
from subsidence,

$$A = (1-p) \rho_s \int_0^\infty S dx$$

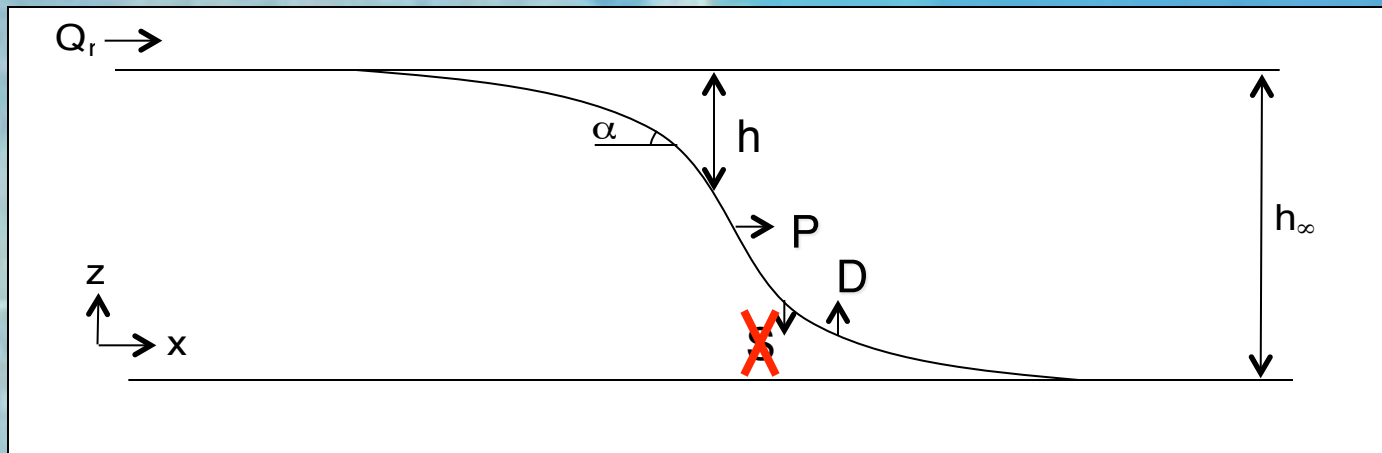
$$\frac{\alpha}{\{1 - (\alpha Ri_c / c_d)^2\}^{3/2}} = \frac{c_d sg}{Ri_c^2 \rho_s} \frac{Q_r}{u_w^3} \left(1 - \frac{h}{h_\infty}\right) \left(1 - \frac{A}{Q_r}\right)$$

Wave orbital velocity, u_w

Underlying shelf
depth, h_∞

**General Equilibrium
Solution**

(Friedrichs and Wright, 2005)



Equilibrium bed slope
(α) is controlled by:

Riverine sediment
supply, Q_r

Accommodation space
from subsidence,

$$\frac{\alpha}{\{1 - (\alpha Ri_c / c_d)^2\}^{3/2}} = \frac{c_d sg}{Ri_c^2 \rho_s} \frac{Q_r}{u_w^3} \left(1 - \frac{h}{h_\infty}\right) \left(1 - \frac{A}{Q_r}\right)$$

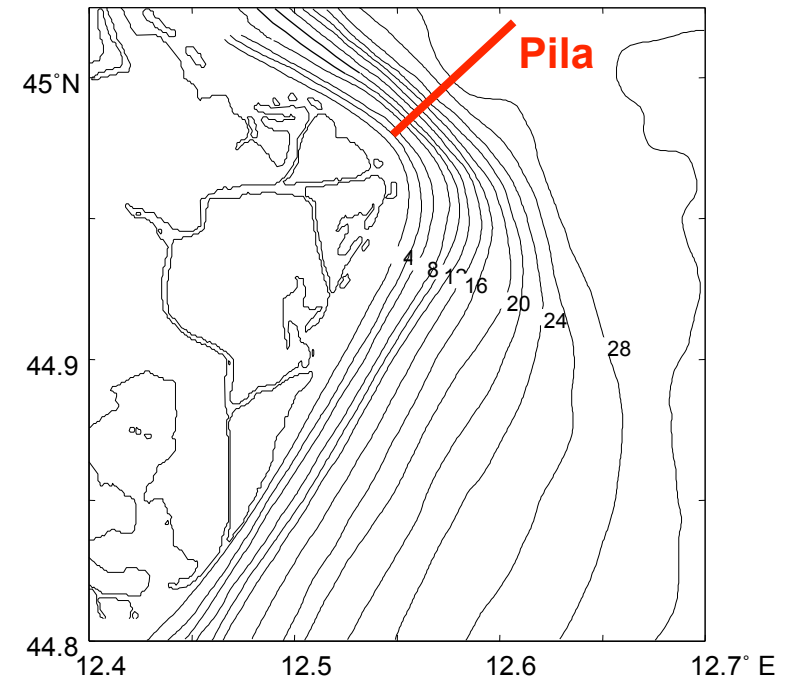
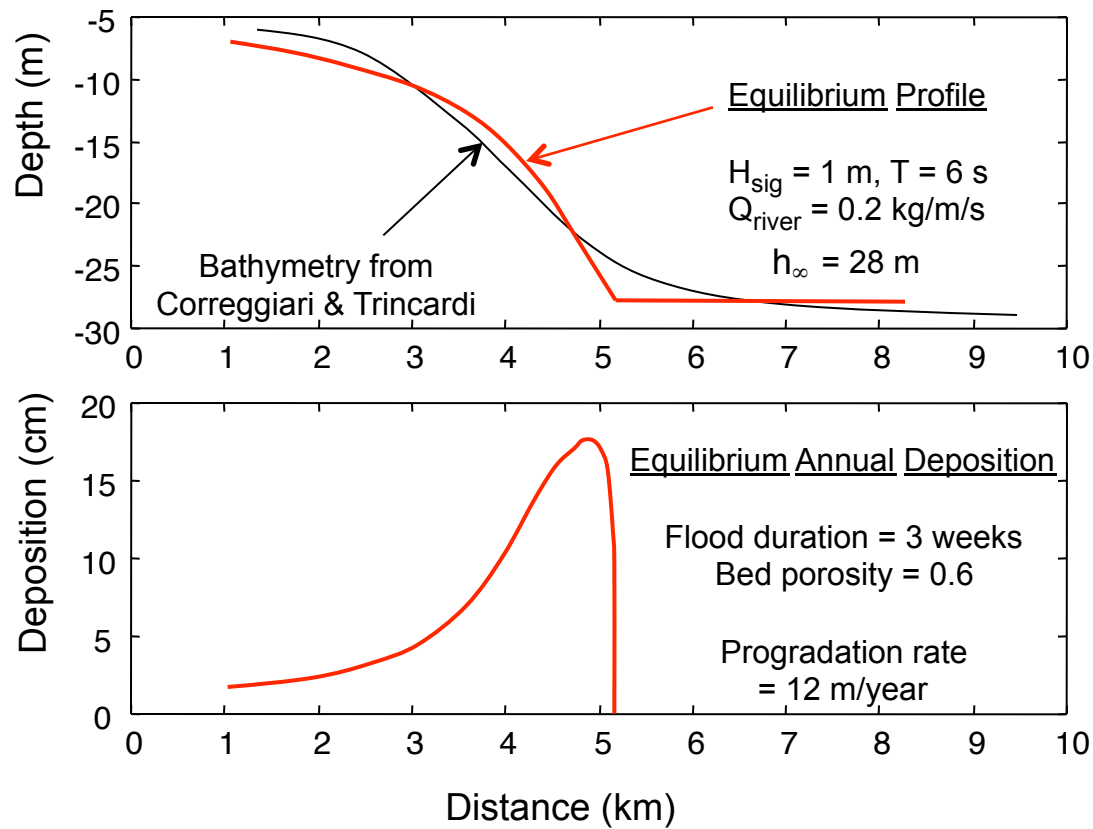
Wave orbital velocity, u_w

Underlying shelf
depth, h_∞

$$A = (1-p) \rho_s \int_0^\infty S dx$$

Next consider a delta profile prograding across a flat shelf with no subsidence.

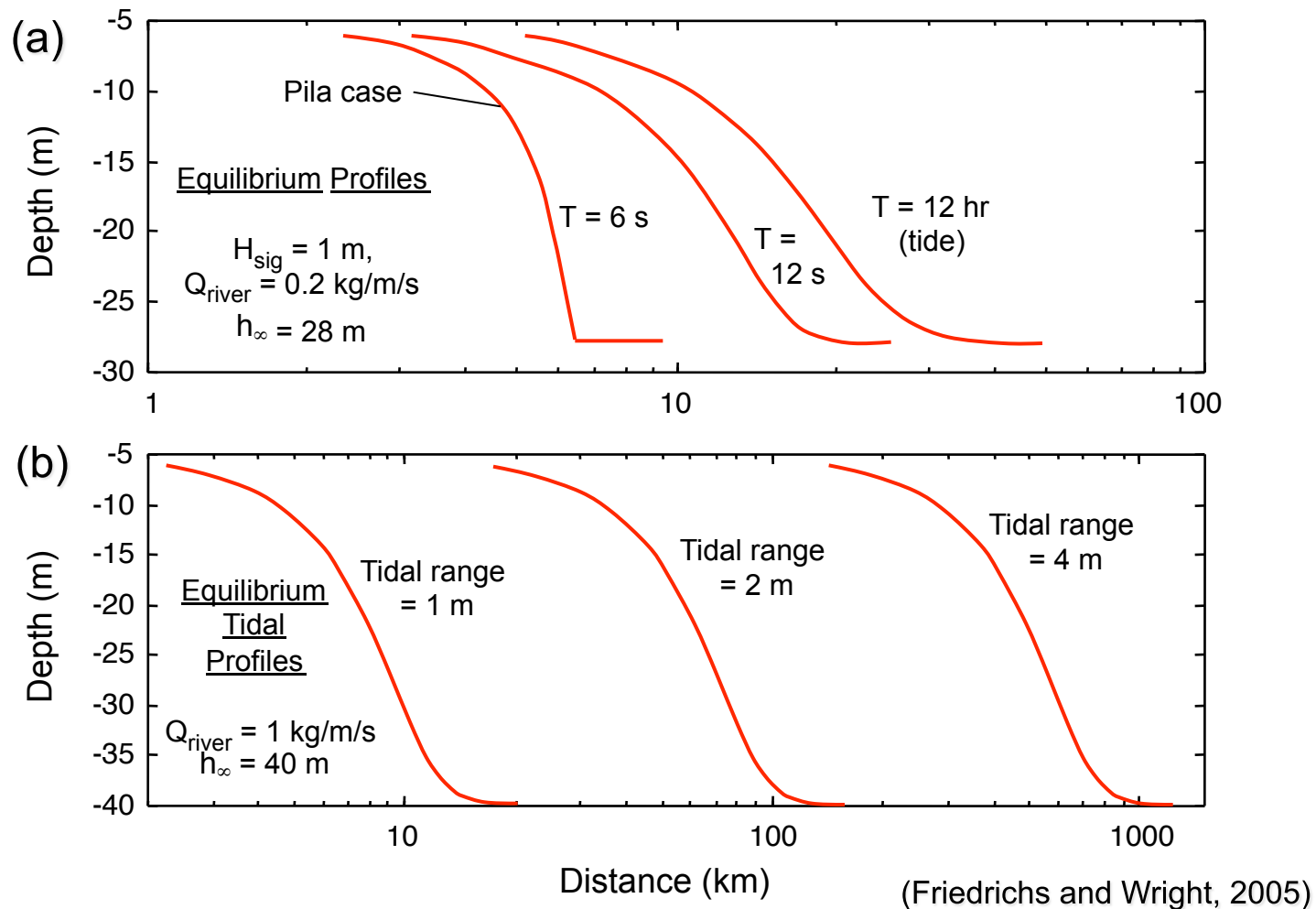
Pila mouth of Po: Present day slope and theoretical equilibrium



Pila subaqueous delta is close to equilibrium solution for reasonable model parameters

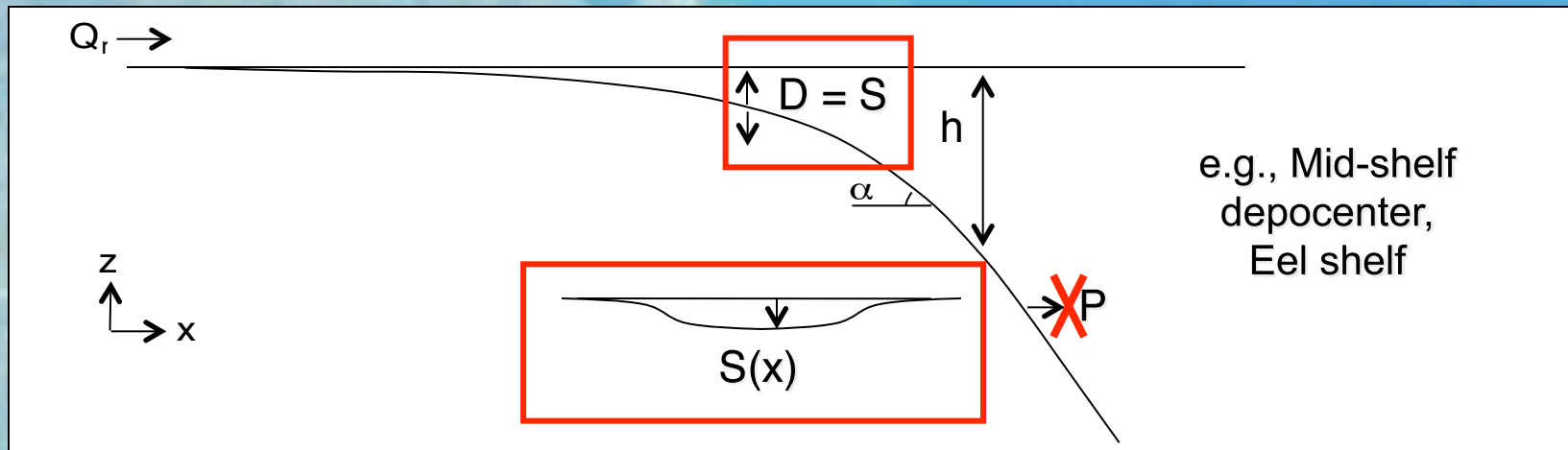
(Friedrichs and Wright, 2005)

Effects of (a) longer wave period (waves to tides) and (b) increased tidal range on equilibrium bathymetric profiles:



Width of equilibrium subaqueous delta increases with wave period or tidal range

(c.f. Walsh et al. 2003; Vakarelov et al. 2008)



Equilibrium bed slope (α) is controlled by:

Riverine sediment supply, Q_r

Accommodation space from subsidence,

$$\frac{\alpha}{\{1 - (\alpha Ri_c / c_d)^2\}^{3/2}} = \frac{c_d sg}{Ri_c^2 \rho_s} \frac{Q_r}{u_w^3} \left(1 - \frac{h}{h_\infty}\right) \left(1 - \frac{A}{Q_r}\right)$$

Wave orbital velocity, u_w

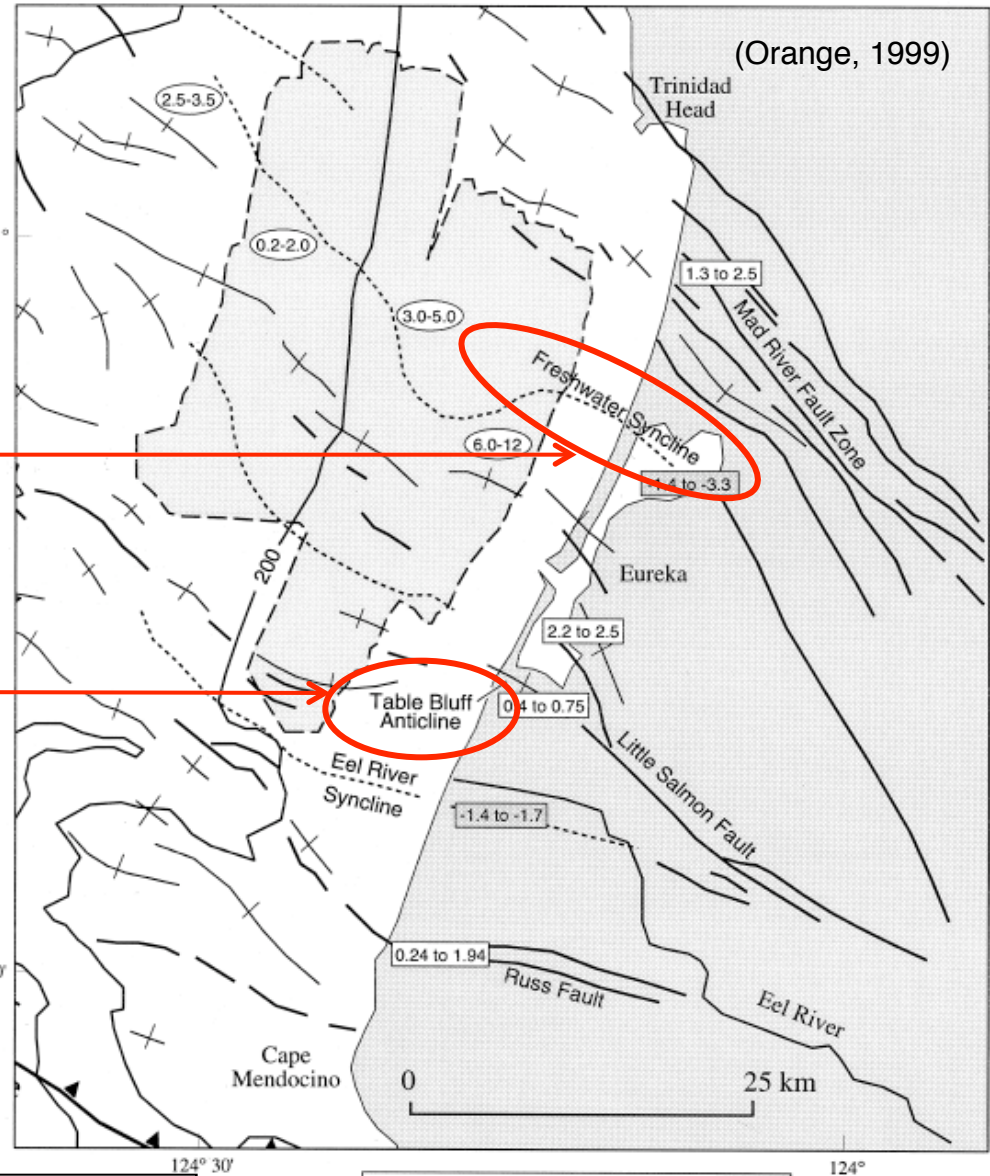
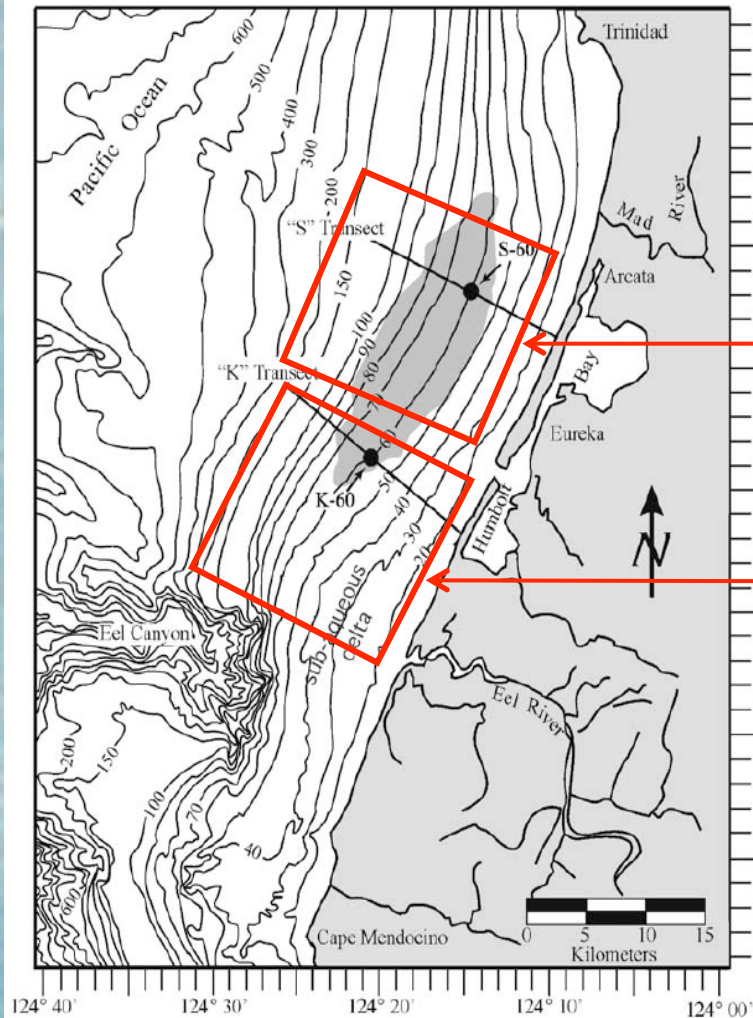
Underlying shelf depth, h_∞

$$A = (1-p) \rho_s \int_0^\infty S dx$$

- Deposition is determined by subsidence (i.e., shape of accommodation space).
- Subsidence reduces profile slope but equilibrium slope is independent subsidence details.

Eel River shelf, CA:

(Scully et al., 2003, showing 95-97 depocenter)



(Orange, 1999)

- Deposition lines up with syncline (subsidence) and shallower mid-shelf slope
- Bypassing lines up with anticline (uplift) and steeper mid-shelf slope

6-12	sediment accumulation rates (mm/yr)
-1.4 to -1.7	tectonic subsidence rates (mm/yr)
2.2 to 2.5	tectonic uplift rates (mm/yr)