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# Sediment waves: linear stability of a turbidity current boundary layer


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Turbidity current workshop, UCSB, June 2009

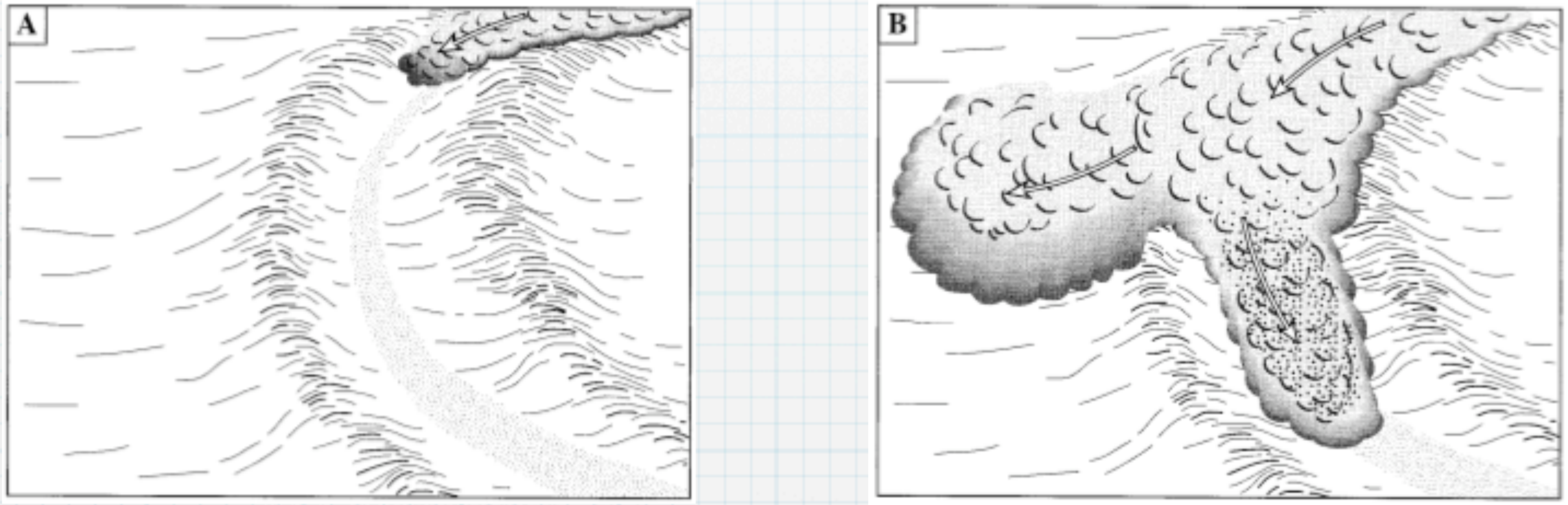
# *Outline*

1. Intro: sediment waves
2. Linear stability analysis of a turbidity current over an erodible bed:
   - full turbidity current flow profile
   - boundary layer only
3. Instability mechanism: internal waves

# *Turbidity Currents in submerged channels*

Frequent turbidity currents tend to scour out meandering channel systems.

(linear instability: Hall, Meiburg & Kneller 2008, *J. Fluid Mech*. 615)

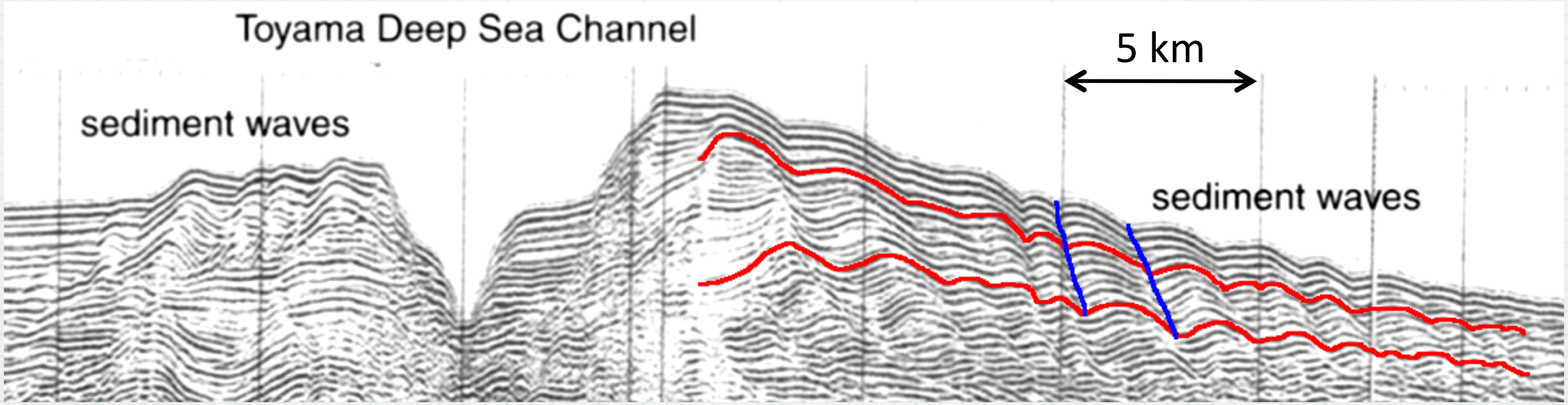



Turbidity current spilling over the levee of a submarine channel

(Peakall et al. 2000, *Journal of Sedimentary Research*, vol. 70)

# *Sediment Waves*

Wavy structures in sedimentary rock, commonly observed on the back slope of submarine channel levees




Seismic image of subsurface topography, Yamato Basin, Japan Sea
(Nakajima & Satoh 2001, *Sedimentology*, vol. 48)

- Wavelength: 1 - 10 km, Amplitude: 1 – 70 m
- typical upslope migration

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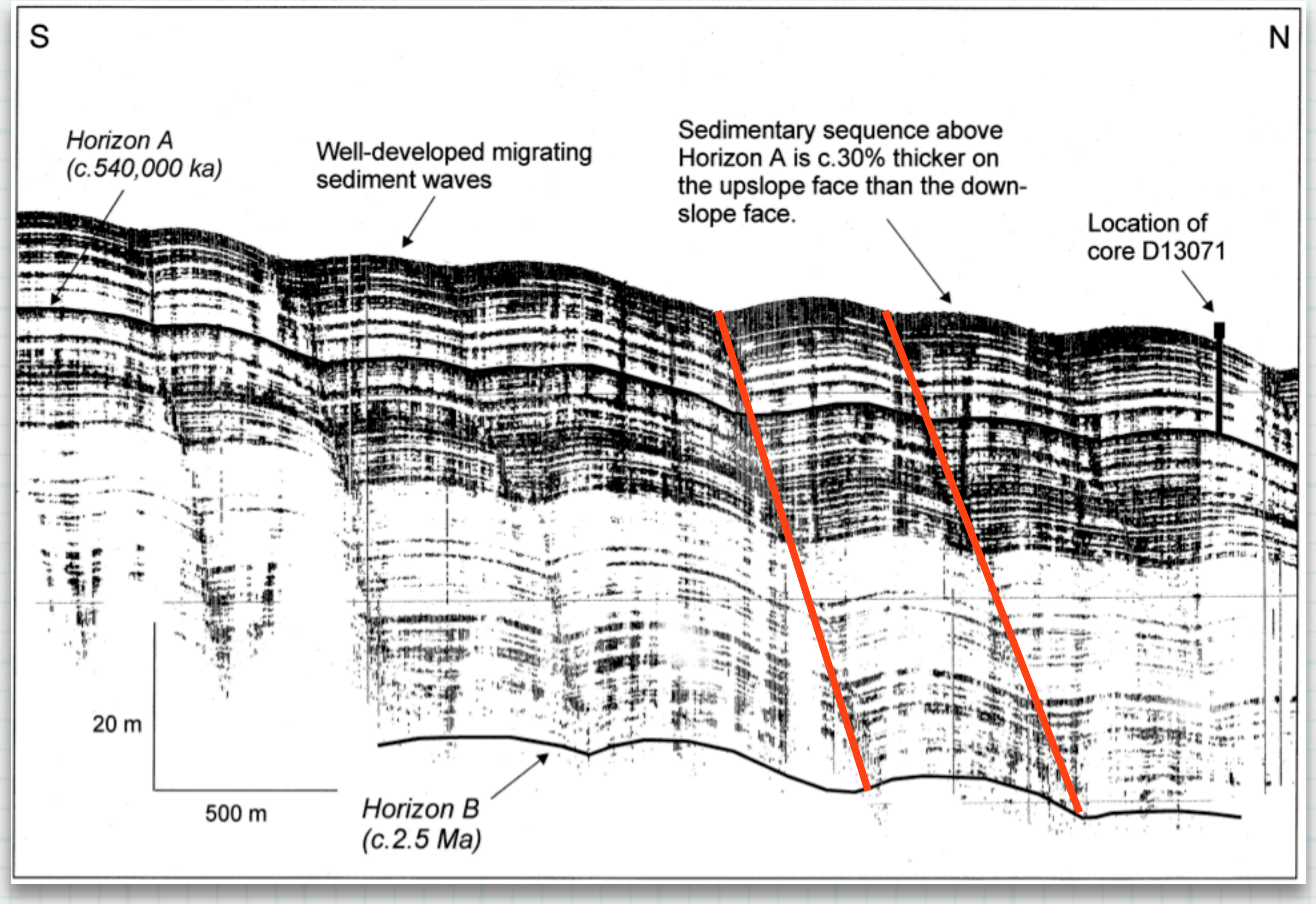



Selvage sediment wave field, North Atlantic

(Wynn et al. 2000, *Sedimentology*, vol. 47)

# *Proposed Mechanisms*

In the literature:

- Antidunes (Normark et al. 1980)
- Lee waves (Flood 1988)
- "Cyclic steps" (Parker & Izumi 2000)

Here:

May wavy perturbations of an erodible bed grow due to linear instability mechanisms?

→ Consider the Navier-Stokes equations, coupled with an evolution equation for the bottom geometry.

## *Governing equations*

2D Navier-Stokes in Boussinesq approximation:

$$\nabla \cdot \vec{u} = 0$$

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = -\nabla p + \frac{1}{\mathrm{Re}} \nabla^2 \vec{u} - Gc\vec{e}_y$$

Advection-diffusion equation for sediment concentration:

$$\frac{\partial c}{\partial t} + \left( \vec{u} + \frac{1}{\mathrm{Pe}} \vec{e}_y \right) \cdot \nabla c = \frac{1}{\mathrm{Pe}} \nabla^2 c$$

Evolution of the bottom interface: sediment deposition and erosion

$$\frac{\partial \eta}{\partial t} = \frac{c_b}{\mathrm{Pe}} c(\eta) - N\tau_s(\eta)$$

# *Nondimensional Parameters*

Length scale: sediment diffusion vs. settling velocity $L = \dfrac{D}{v_s}$

Reynolds number: $$\mathrm{Re} = \frac{D\,U_{max}}{\nu\,v_s} = 5000$$

Peclet number: $$\mathrm{Pe} = \frac{U_{max}}{v_s} = 5000$$

Gravity parameter: $$G = \frac{\rho_p - \rho_f}{\rho_f} c_b g = 0.1$$

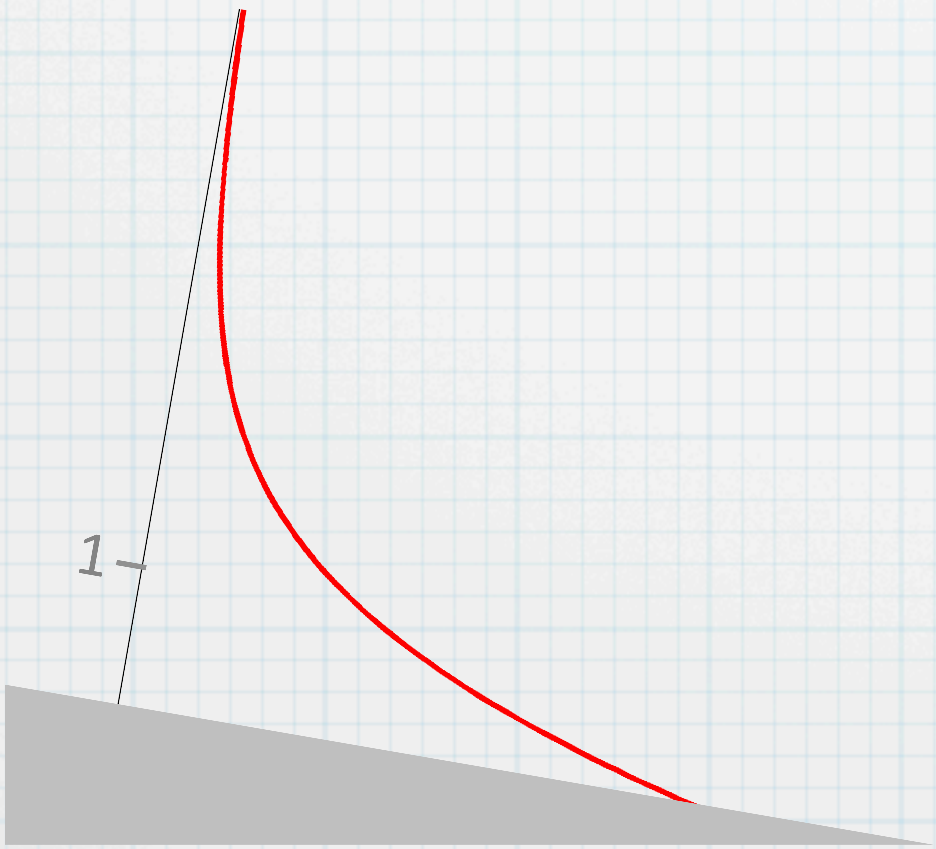
Erosion parameter: $$N = \frac{\beta \rho_f \nu v_s}{D} = 10^{-5}$$

# *Baseflow*

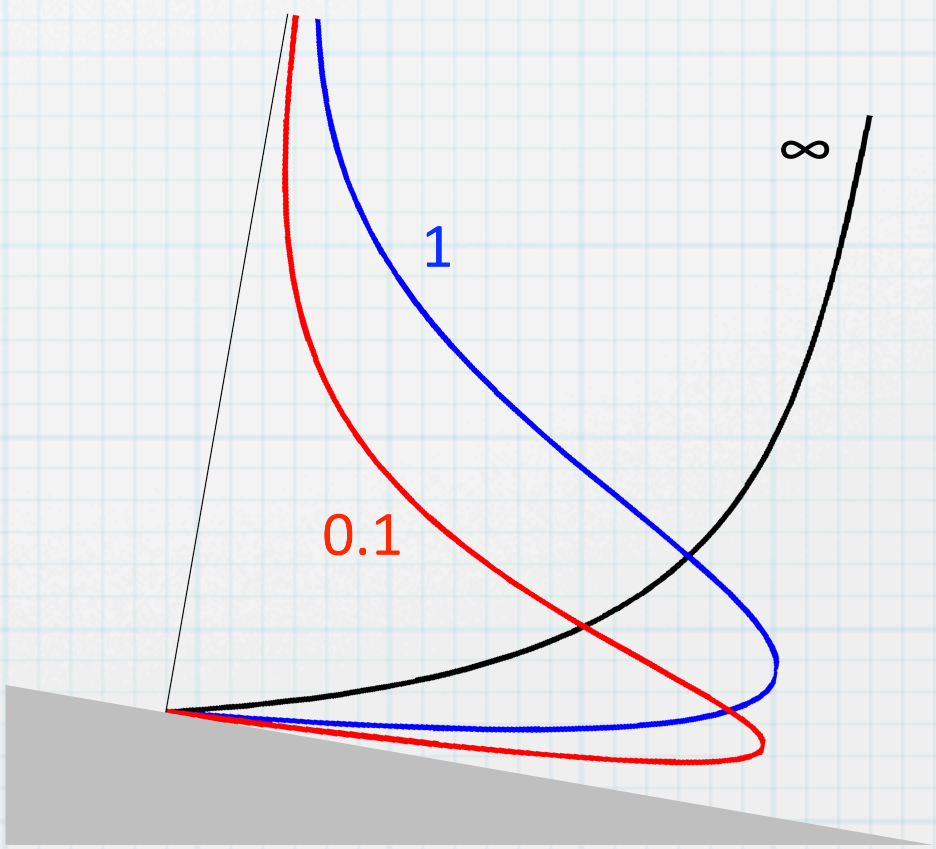
Stacey & Bowen 1988 (*J. Geophys. Res.*, vol. 93):

- Steady sediment concentration profile on a slope
- Time-evolving velocity profile due to gravitational acceleration

→ use time as profile parameter τ, freeze baseflow in time




concentration




velocity at τ = 0.1, 1, ∞

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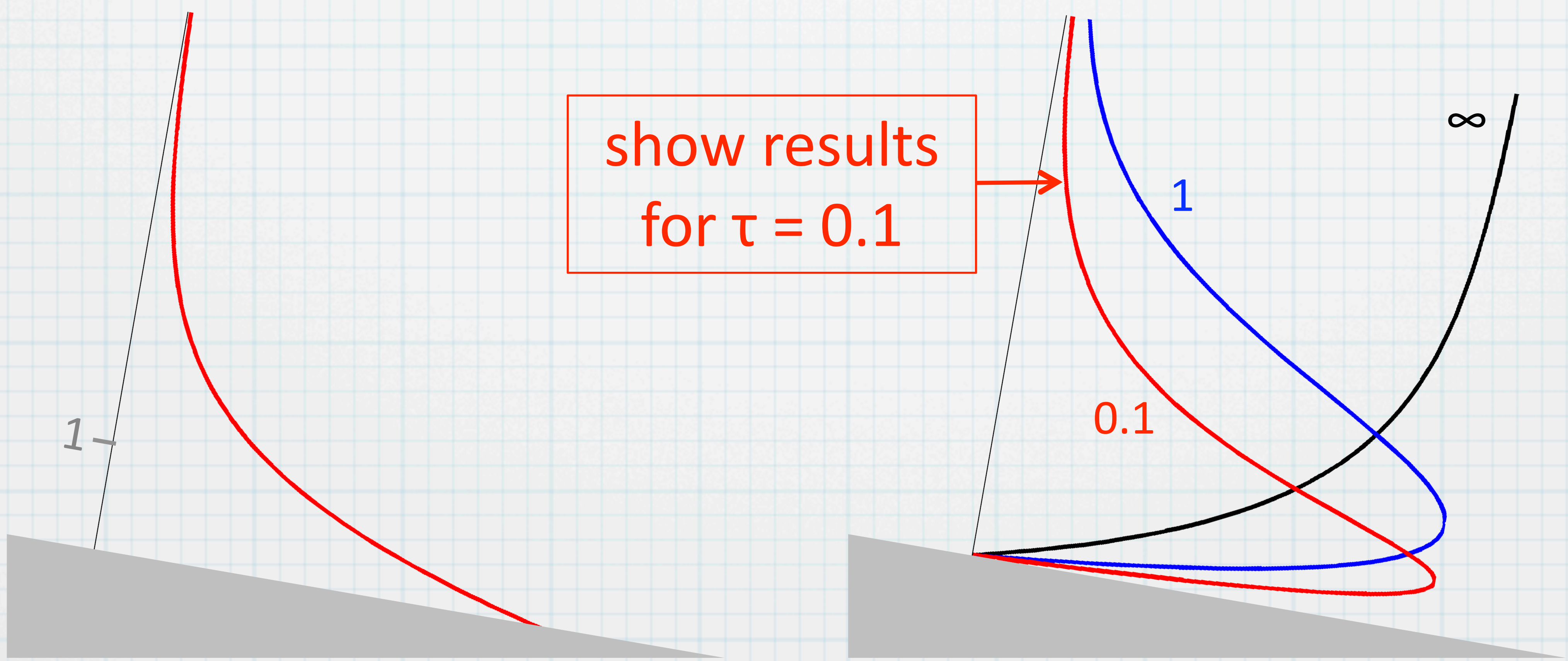
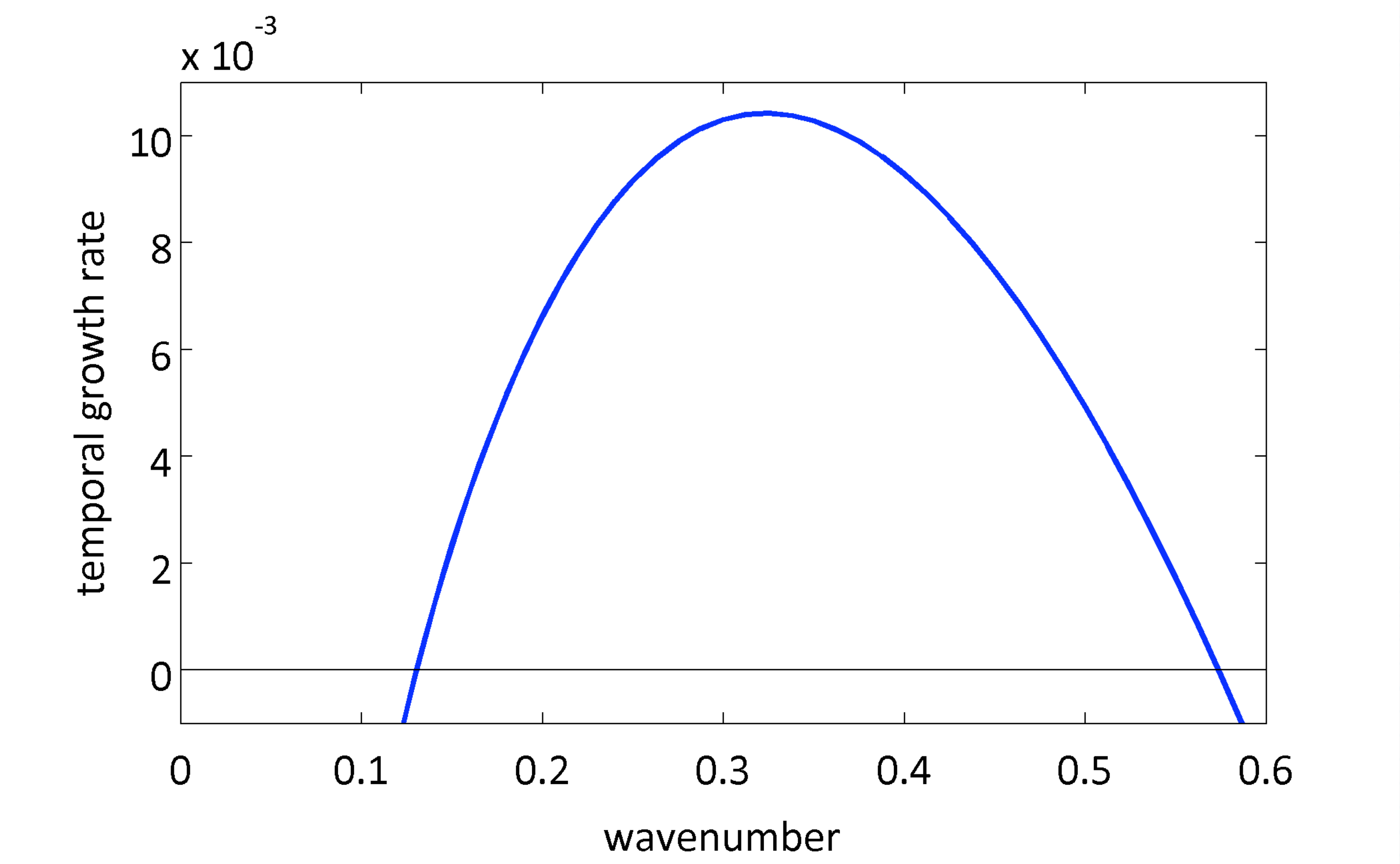



concentration

velocity at $\tau$ = 0.1, 1, $\infty$

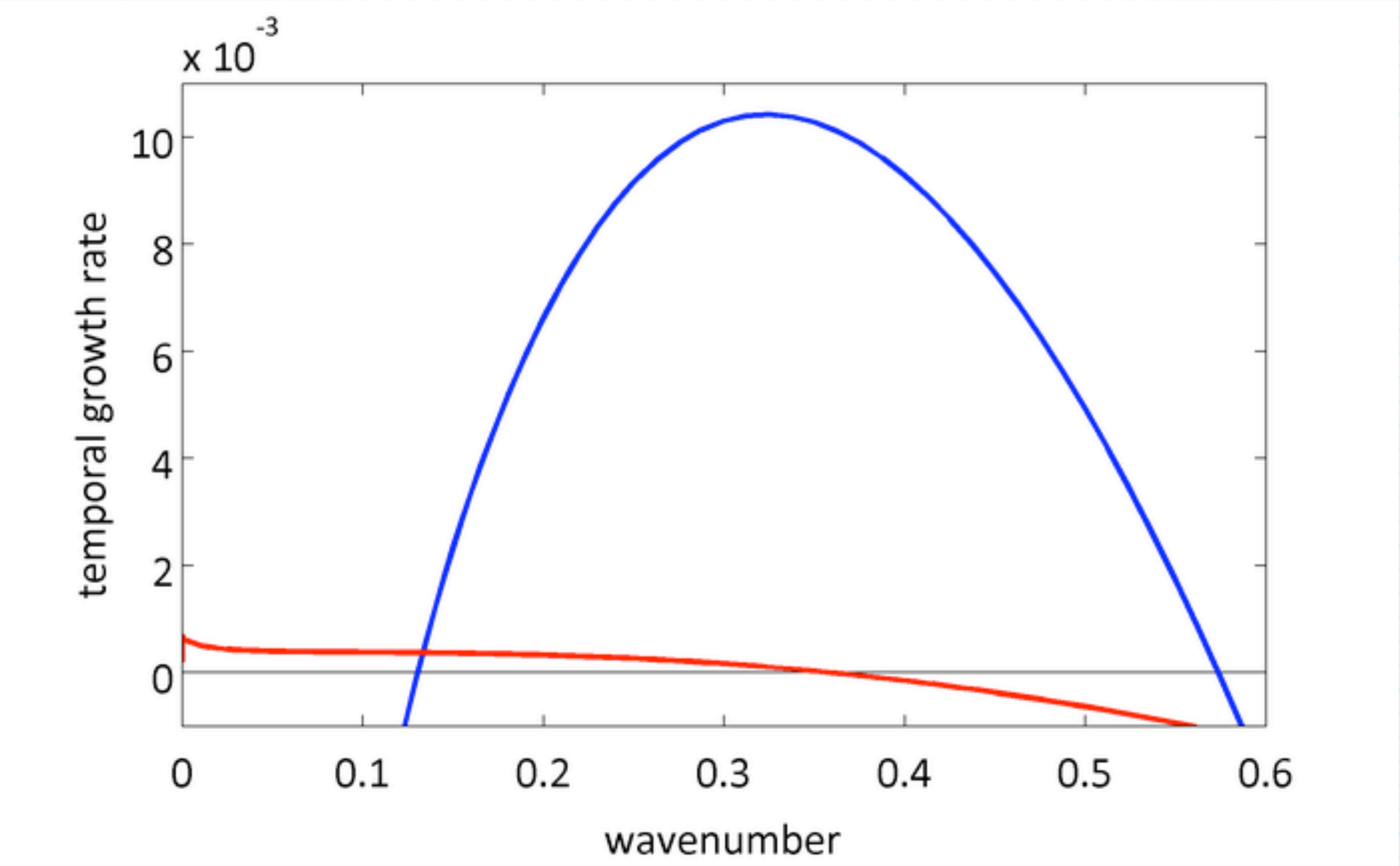
# *Temporal modes with unchanging flat bottom*

Only Kelvin-Helmholtz mode in outer shear layer is unstable.



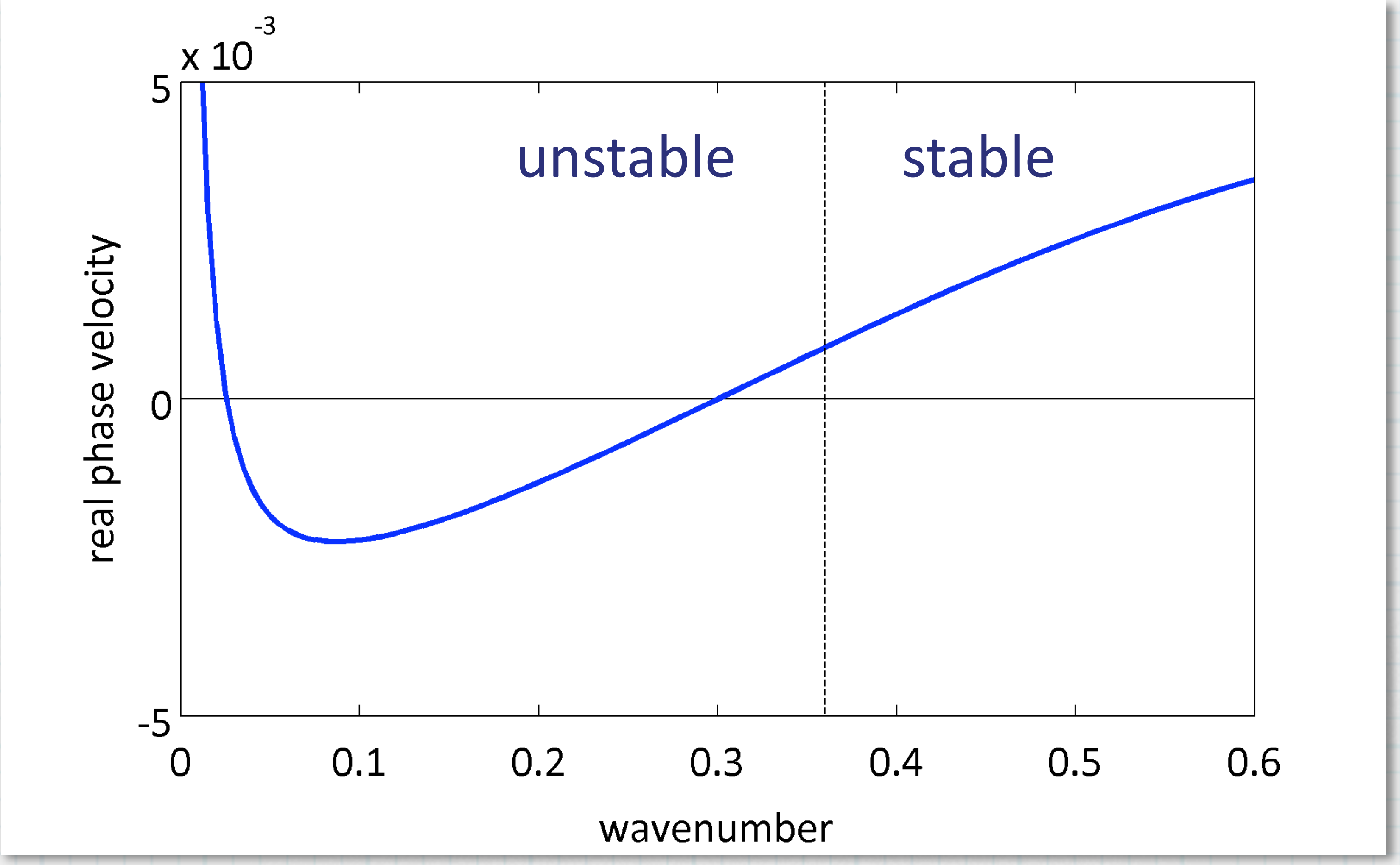
# *Temporal modes with erodible bed*

A new unstable mode emerges at low wavenumbers!



# *Temporal modes with erodible bottom*

Phase velocity of the bed interaction mode

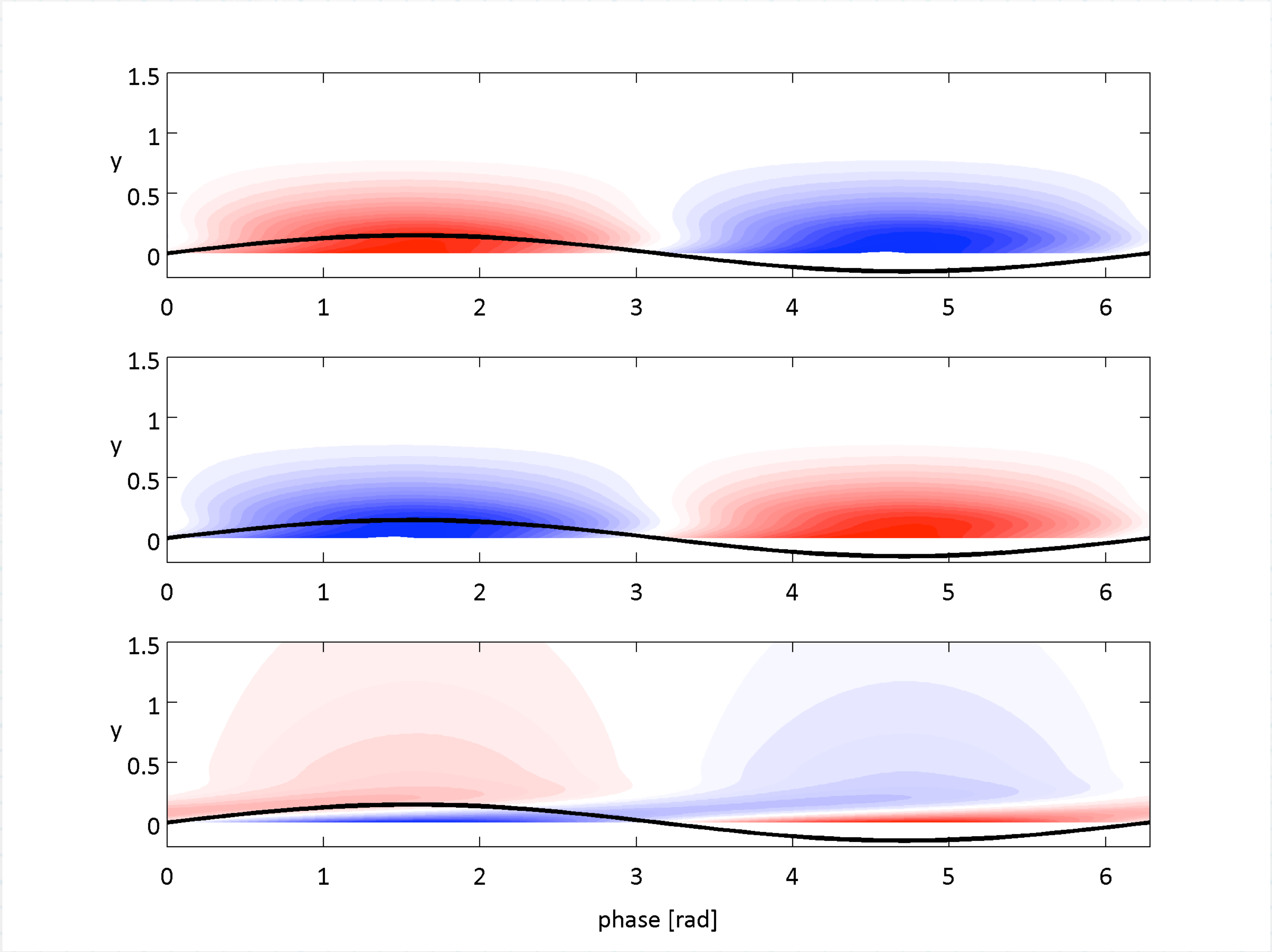



Negative phase velocity in the unstable range

→ upstream migration!

# *Instability mechanism*

Eigenfunctions




shear

vorticity

concentration

All perturbations are confined in the boundary layer:

Shear layer probably unimportant for instability.

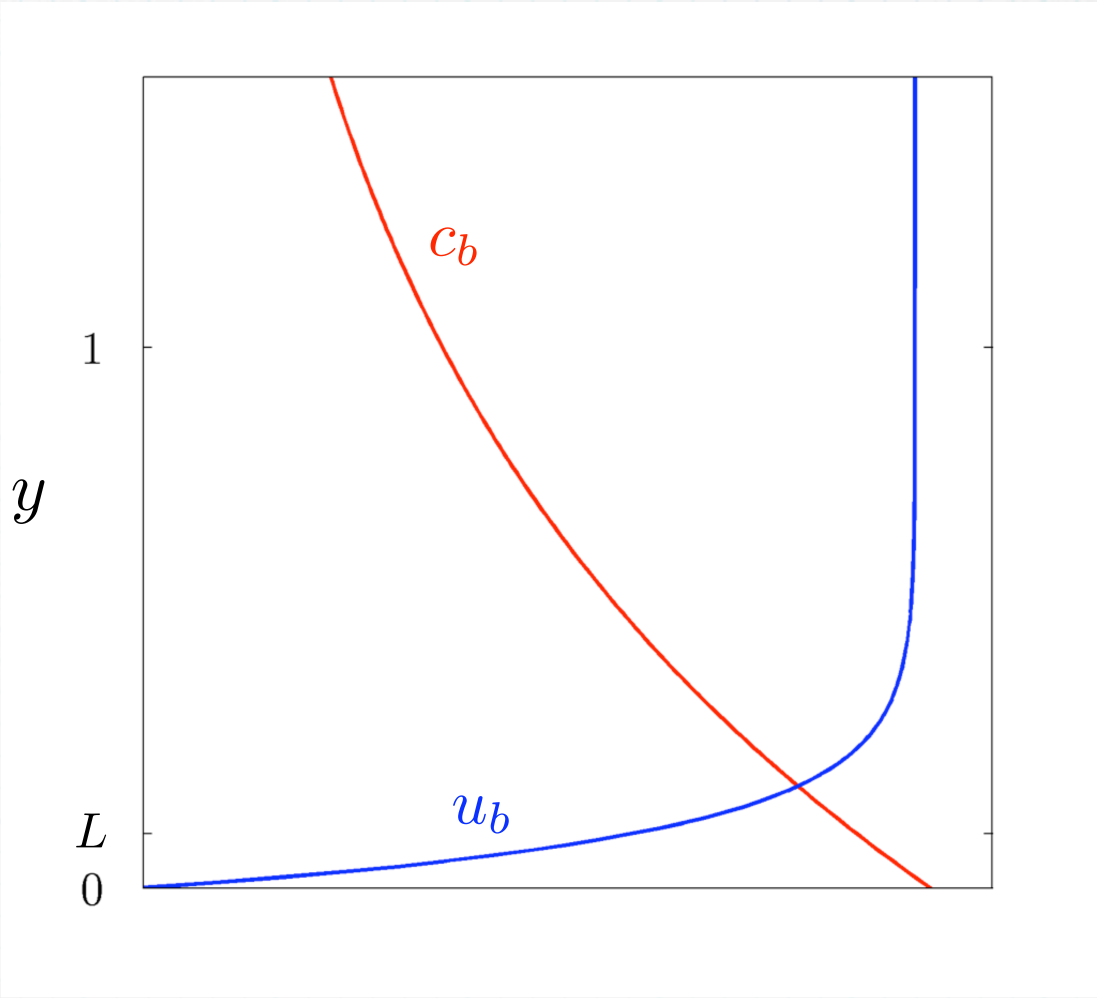
# *Boundary layer baseflow*

Consider pure boundary layer flow for further analysis:
same baseflow as in Hall, Meiburg & Kneller 2008 (JFM vol. 615)

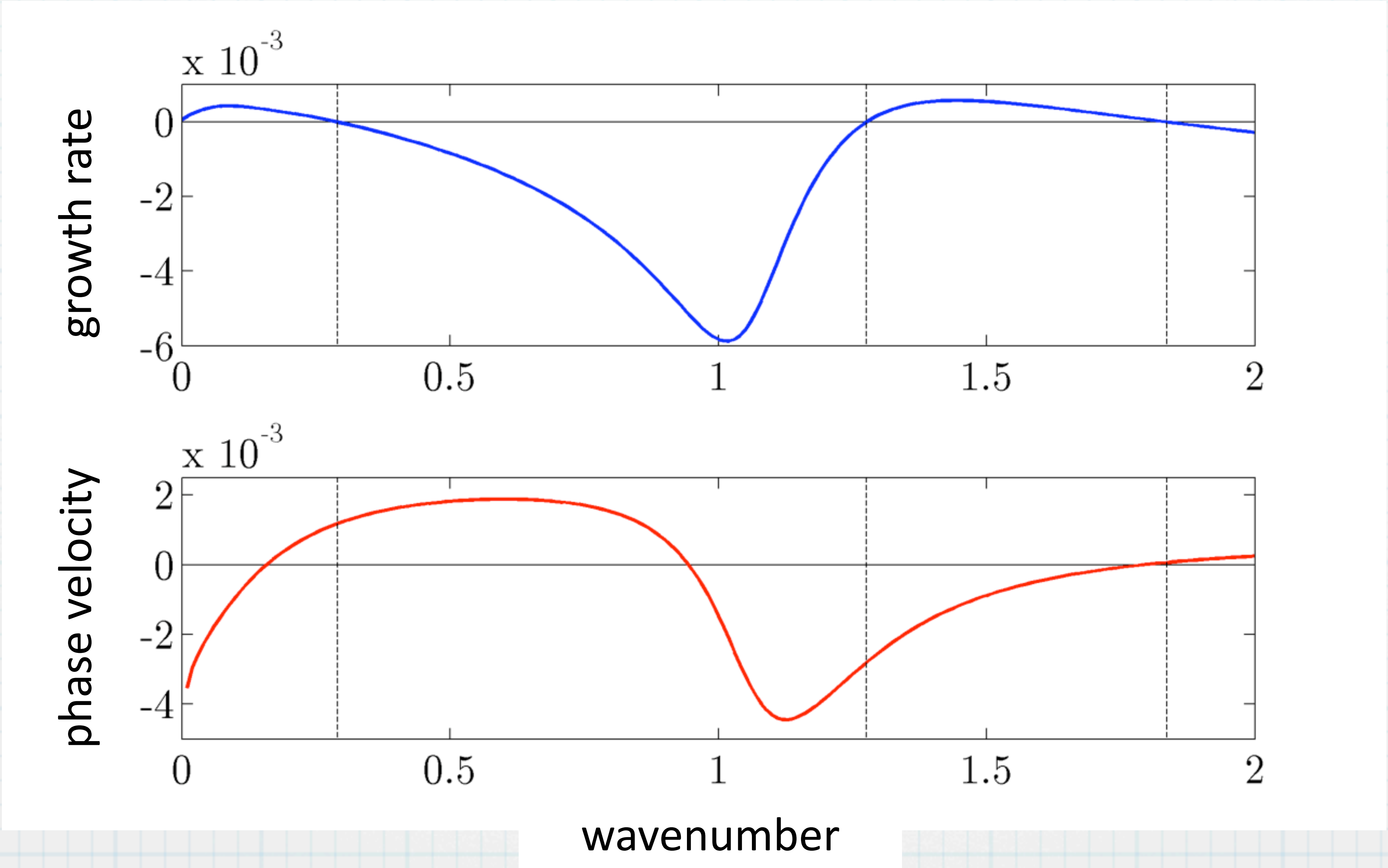
$$
\begin{aligned}
u_b(y) &= 1 - \exp(-z/L) \\
c_b(y) &= \frac{N\,Pe}{L\,c_\infty}\exp(-z) + 1
\end{aligned}
$$

with thickness ratio $L = 0.1$ :

steady solution of the
governing equations



# *Temporal modes in stratified boundary layer*



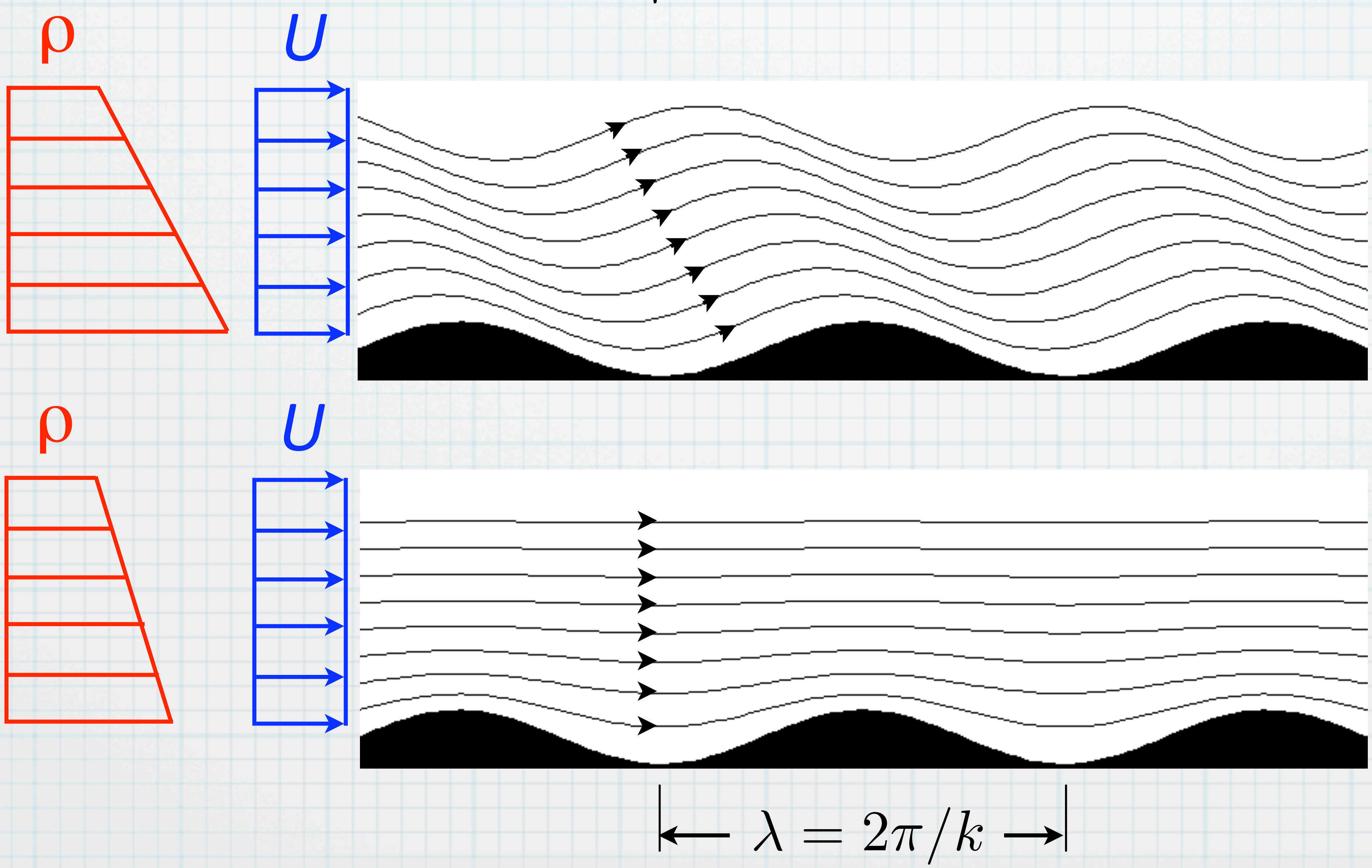

Two unstable wavenumber regions;

mainly negative phase velocity

# *Mechanism: internal waves ("lee waves")*

Radiation condition for internal wave excitation in a constant stratified medium:

$$kU \leq \Omega_b = \sqrt{-G\frac{\mathrm{d}\rho}{\mathrm{d}y}}$$ (buoyancy frequency)




$kU < \Omega_b$

→ radiation

$kU > \Omega_b$

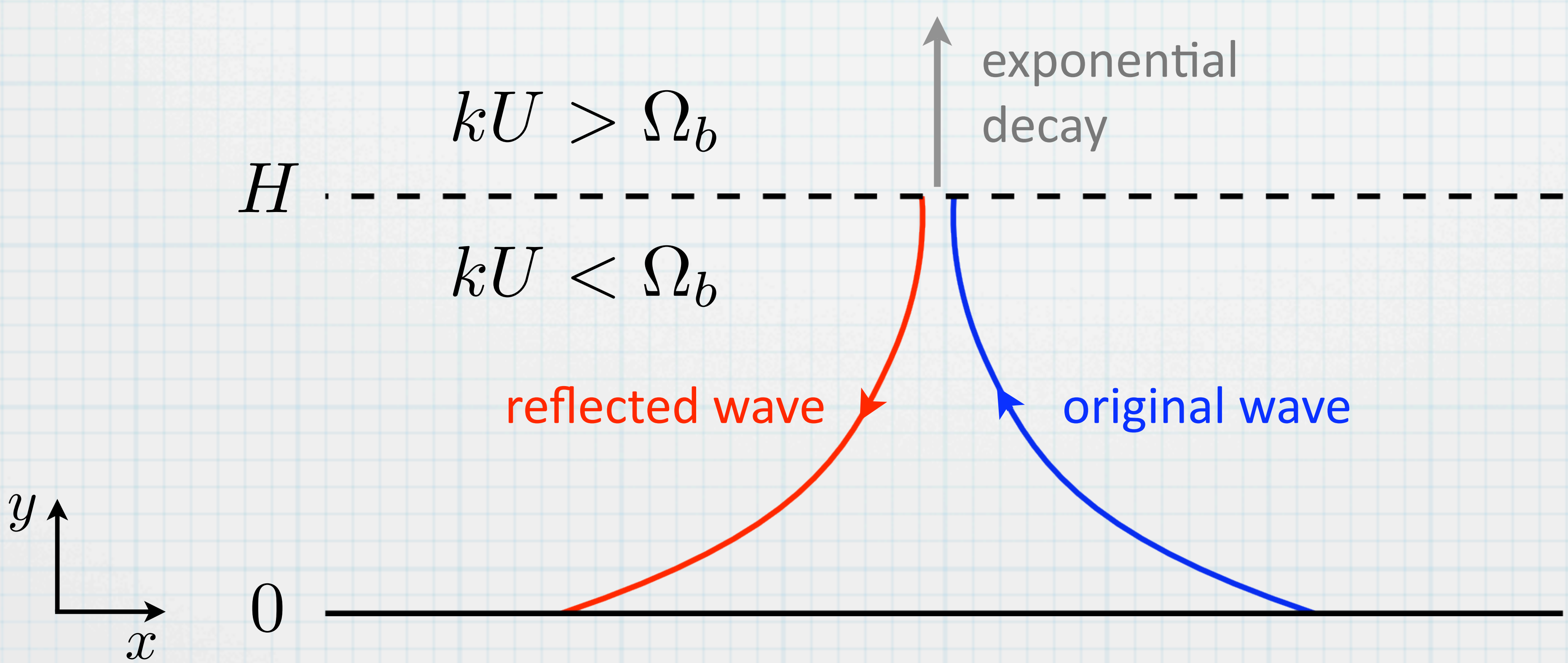
→ evanescent perturbation

# *Internal waves in the boundary layer*

Boundary layer: both $U$ and $\dfrac{\mathrm{d}\rho}{\mathrm{d}y}$ vary with height.

For *each* $k$ there exists a height $y = H$ where $kU(H) = \Omega_b(H)$.

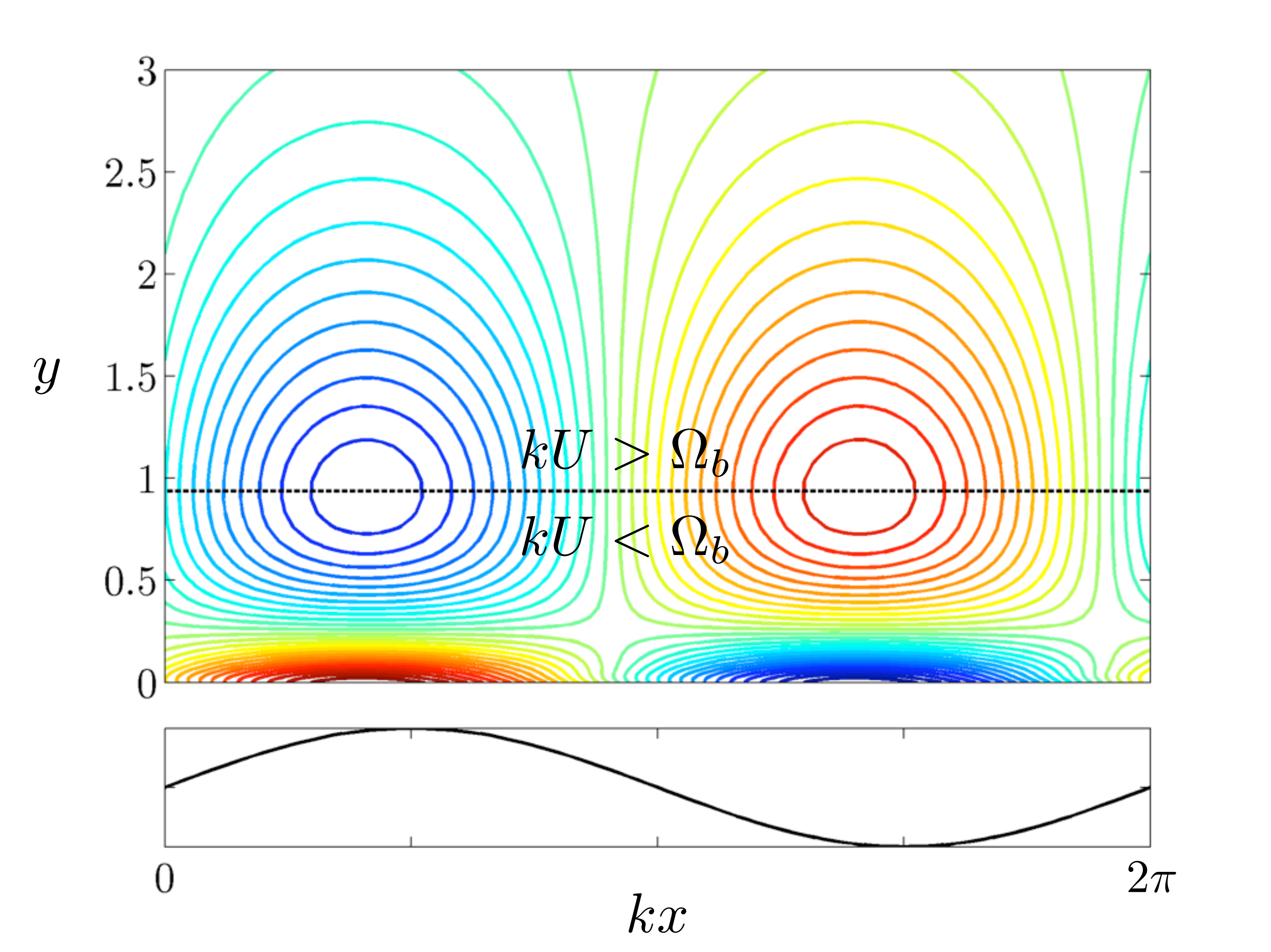
➛ internal wave radiation within a layer $0 \leq y \leq H$, exponential decay above $H$.

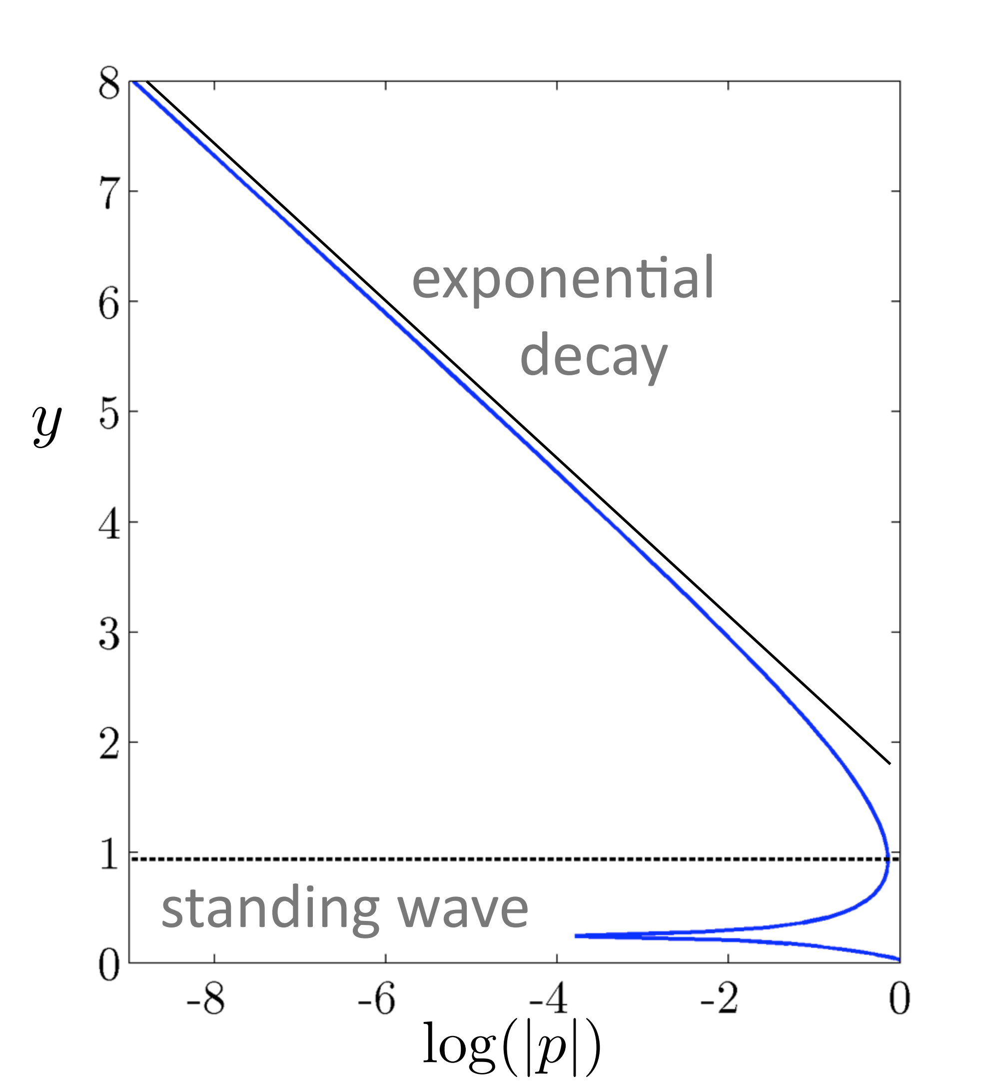


# *Sediment wave eigenmodes*

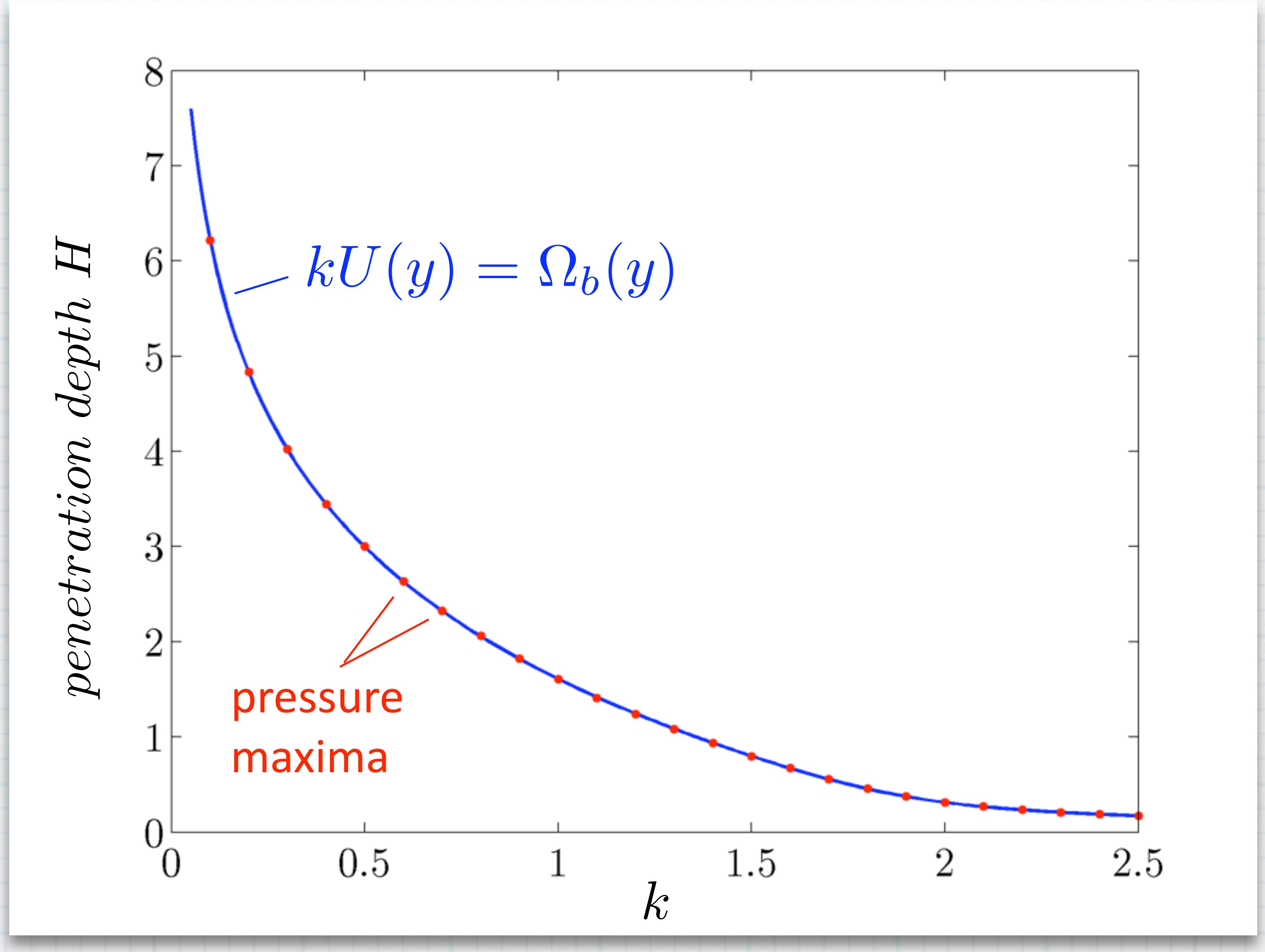
Example: mode with maximum growth rate, $k = 1.4$,

pressure perturbations



# *Penetration depth*




For all $k$ : Outer pressure maximum coincides exactly with theoretical penetration depth ➛ *observed instabilities are internal ("lee") waves.*

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- A *new unstable mode* arises from the interaction of a turbidity current with a sediment bed.

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# *Conclusions*

- A *new unstable mode* arises from the interaction of a turbidity current with a sediment bed.
- This mode is associated with internal waves (“lee waves”) within the concentration boundary layer.
- Negative phase velocity over large range of unstable wavenumbers.
  ➢ May explain the upstream migration of sediment waves.
- The new mode is unstable at low wavenumbers. With present parameters: unstable at $k < 0.29$

Compare to typical dimensions in turbidity currents:

Sediment wavelength: $\lambda \sim 1 - 10$ km

Current height: $L \sim 10 - 100$ m

➢ Wavenumber: $k = O(0.001 — 0.1)$