Coupled turbidity current/substrate interactions

Navier-Stokes based computational studies

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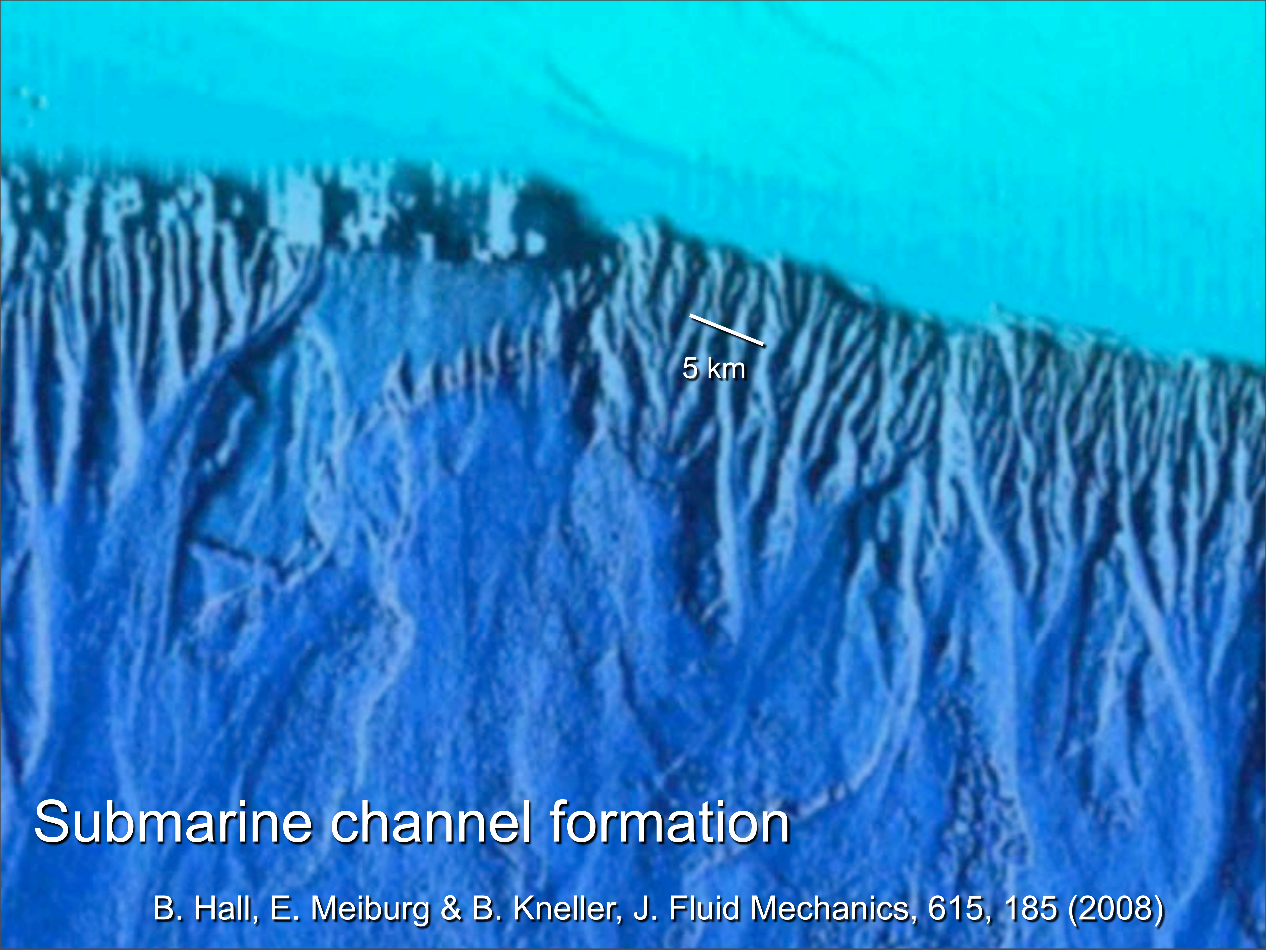
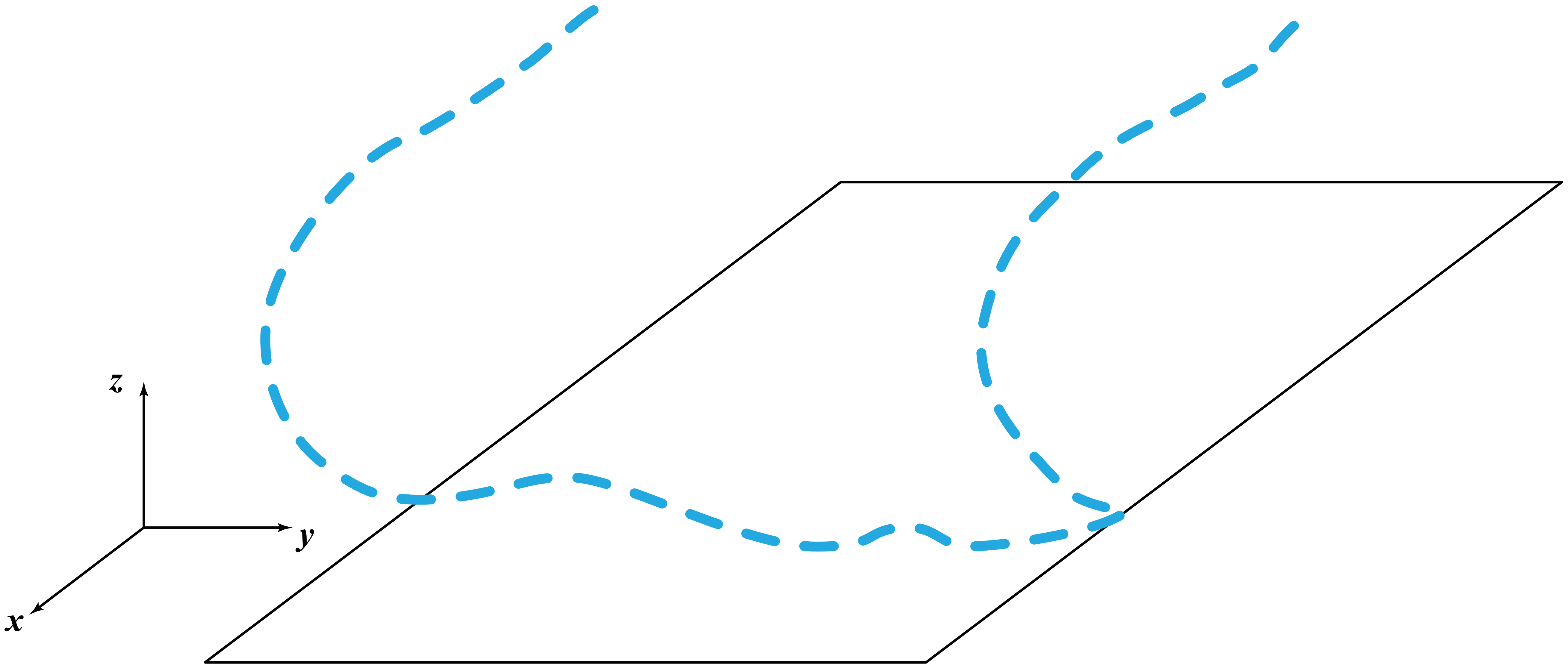
engineering

Outline

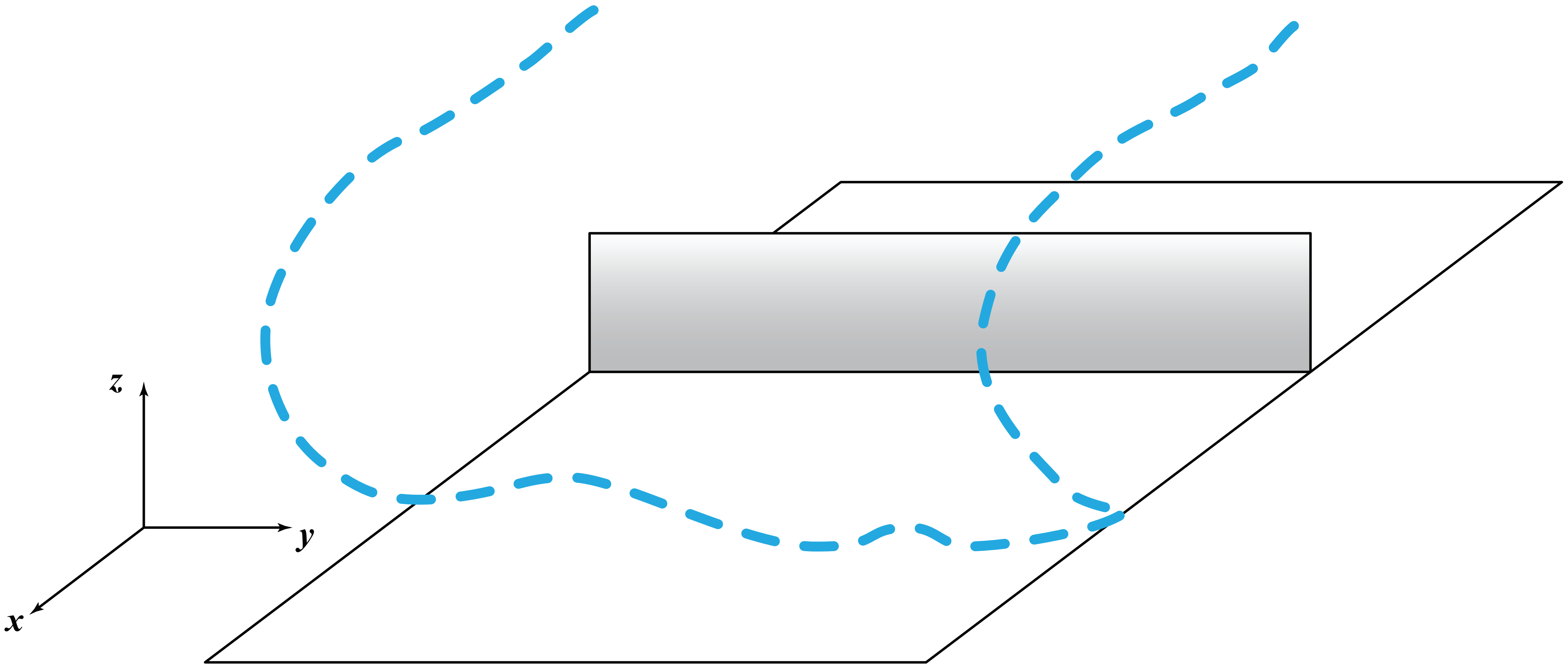
- Introduction
- Linear stability analysis
  - submarine channels
  - sediment waves
- Non-linear simulations
- Summary and conclusions


5 km
Submarine channel formation
B. Hall, E. Meiburg & B. Kneller, J. Fluid Mechanics, 615, 185 (2008)

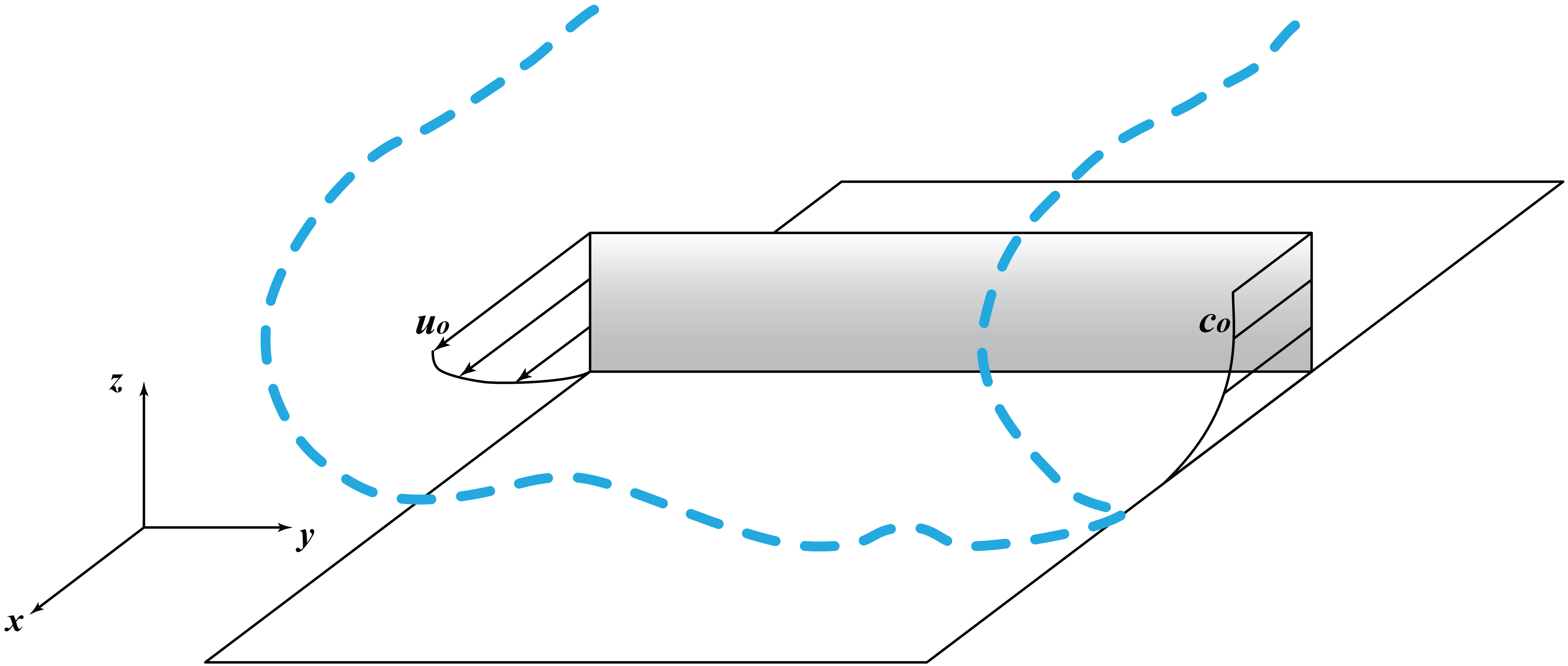
Linear stability analysis



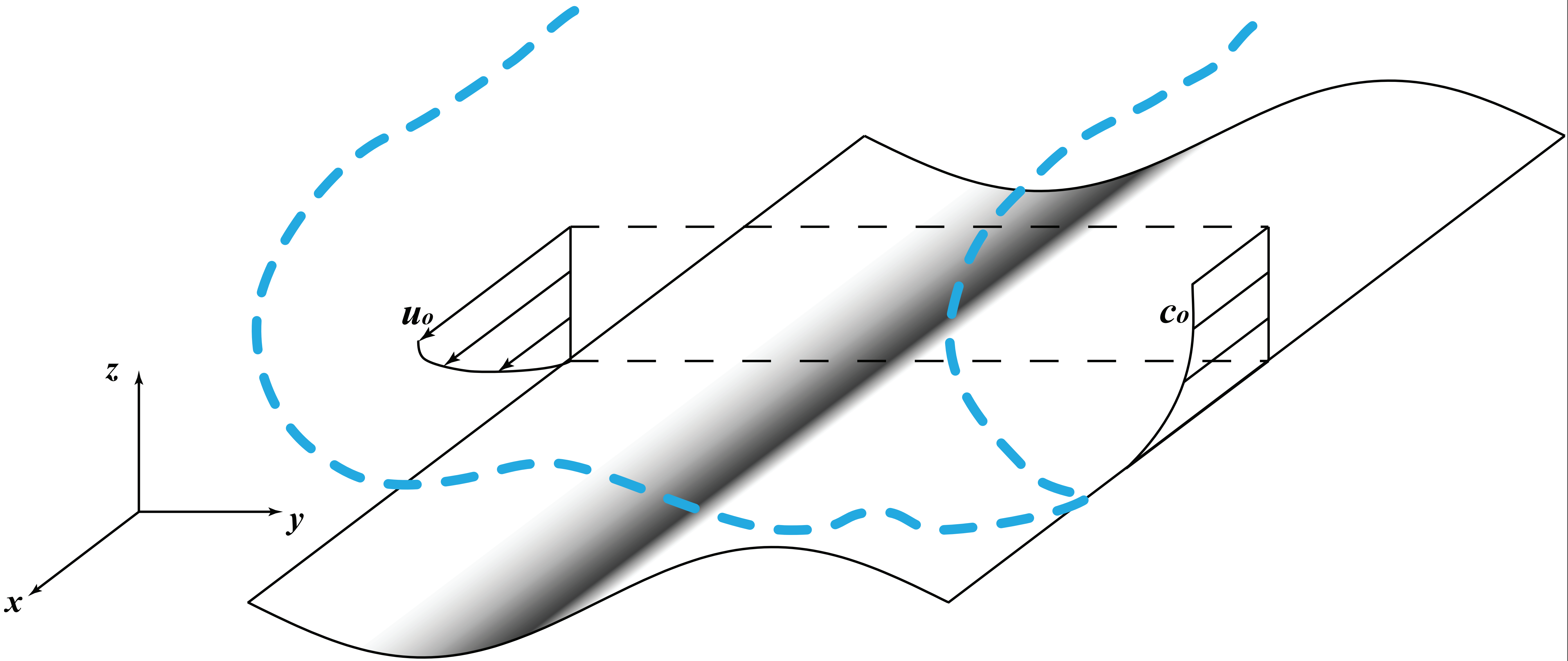
Linear stability analysis



Linear stability analysis



Linear stability analysis



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Previous work

2D depth averaged stability analysis

Smith & Bretherton (1972), Izumi & Parker (1995, 2000), Imran & Parker (2000), Izumi (2004), Izumi & Fujii (2006)

- channelization due to depositional and/or erosive flows in various environments
- predict spacing, but don’t provide detailed physical mechanism

Stability analysis of sand ridge formation by bedload transport

Colombini (1993), Colombini & Parker (1995)

- impose secondary flow through turbulence closure model
- importance of counter-rotating streamwise vortices

Navier–Stokes equations (Boussinesq)

Continuity

$$\nabla \cdot \vec{u_f} = 0$$

Momentum equations

$$\frac{\partial \vec{u_f}}{\partial t} + \left(\vec{u_f} \cdot \nabla\right) \vec{u_f} \quad = \quad -\nabla p + \frac{1}{Re} \nabla^2 \vec{u_f} + Ri \vec{e_g} c$$

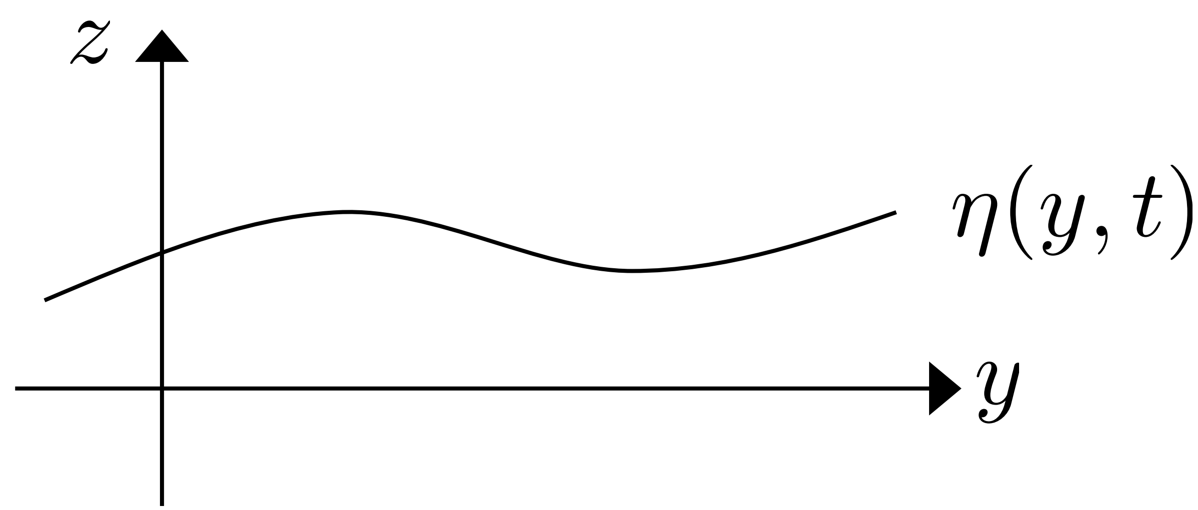
effective density

Concentration equation

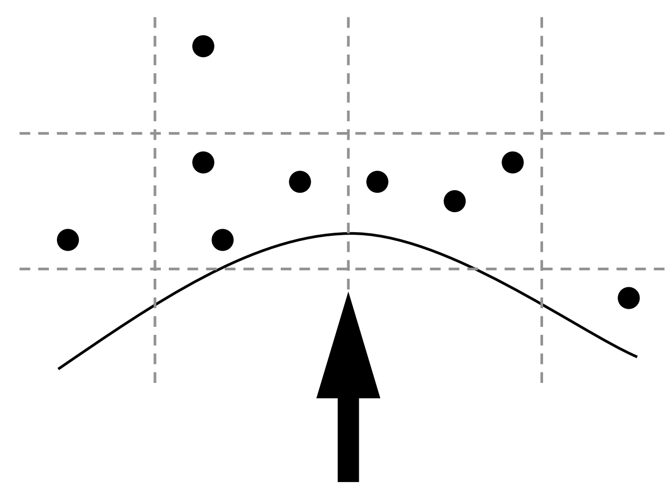
$$\frac{\partial c}{\partial t} \quad + \quad \left[\left(\vec{u_f} + W_s \vec{e_g}\right) \nabla\right] c = \frac{1}{ScRe} \nabla^2 c$$

settling
velocity

Interface evolution



deposition of particles



$$\frac{\partial \eta^{dep}}{\partial t} = w_s c(z=\eta)$$

erosion of particles

$$\frac{\partial \eta^{ero}}{\partial t} = -\beta \frac{\tau_n|_{z=\eta}}{n_z}$$

Reynolds number

$$Re = \frac{u_\infty D}{\nu w_s}$$

Schmidt number

$$Sc = \frac{\nu}{D}$$

Richardson number

$$Ri = g' \frac{c_\infty D}{u_\infty^2 w_s}$$

Erosion parameter

$$N = \frac{\beta \nu \rho_f w_s}{D}$$

Continuity Equation

Momentum equations x3

Concentration equation

Interface equation

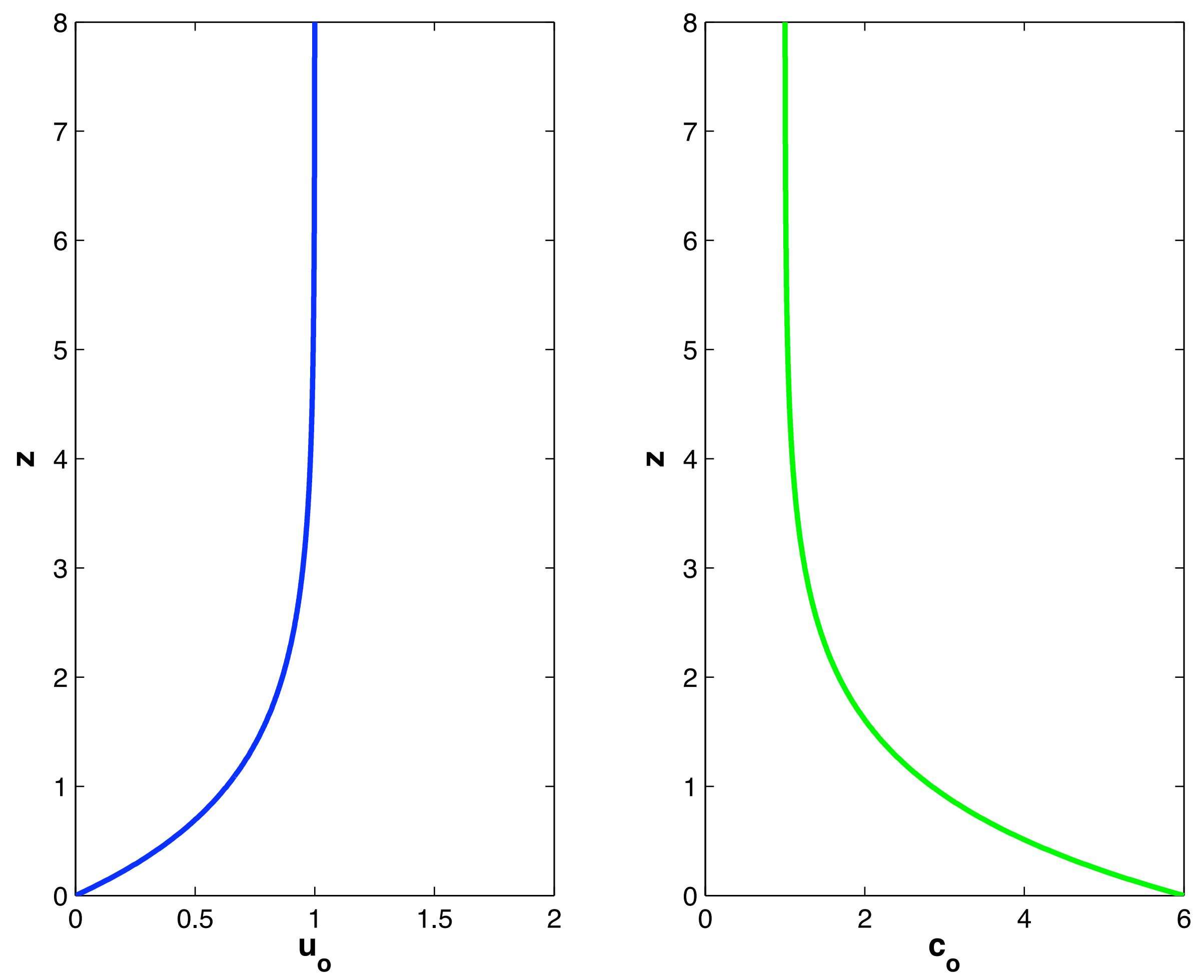
$$\left.\begin{array}{l} \\ \\ \\ \\ \end{array}\right\} \begin{pmatrix} u \\ v \\ w \\ p \\ c \\ \eta \end{pmatrix}$$

Governing parameters: Re, Sc, c_∞, Ri, N

One dimensional & quasi-steady

$$u_0(z) = 1 - e^{-z/L} \quad , \quad c_0(z) = \frac{N\,Pe}{L\,c_\infty} e^{-z} + 1$$

Important parameter: $$L = \frac{\text{length over which } u_o \text{ decays}}{\text{length over which } c_o \text{ decays}}$$



L = 1

One dimensional & quasi-steady

$$u_0(z) = 1 - e^{-z/L} \quad , \quad c_0(z) = \frac{N\,Pe}{L\,c_\infty} e^{-z} + 1$$

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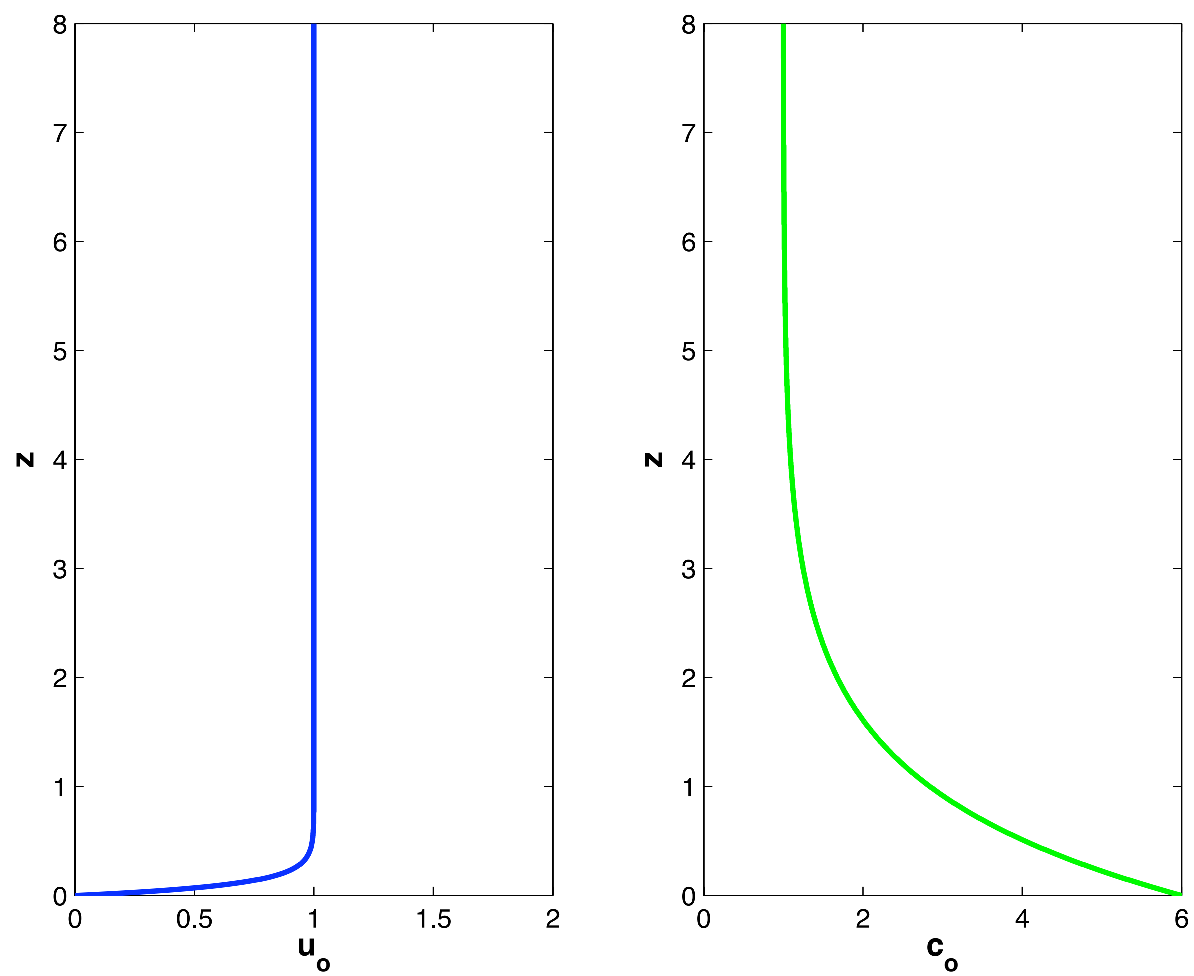
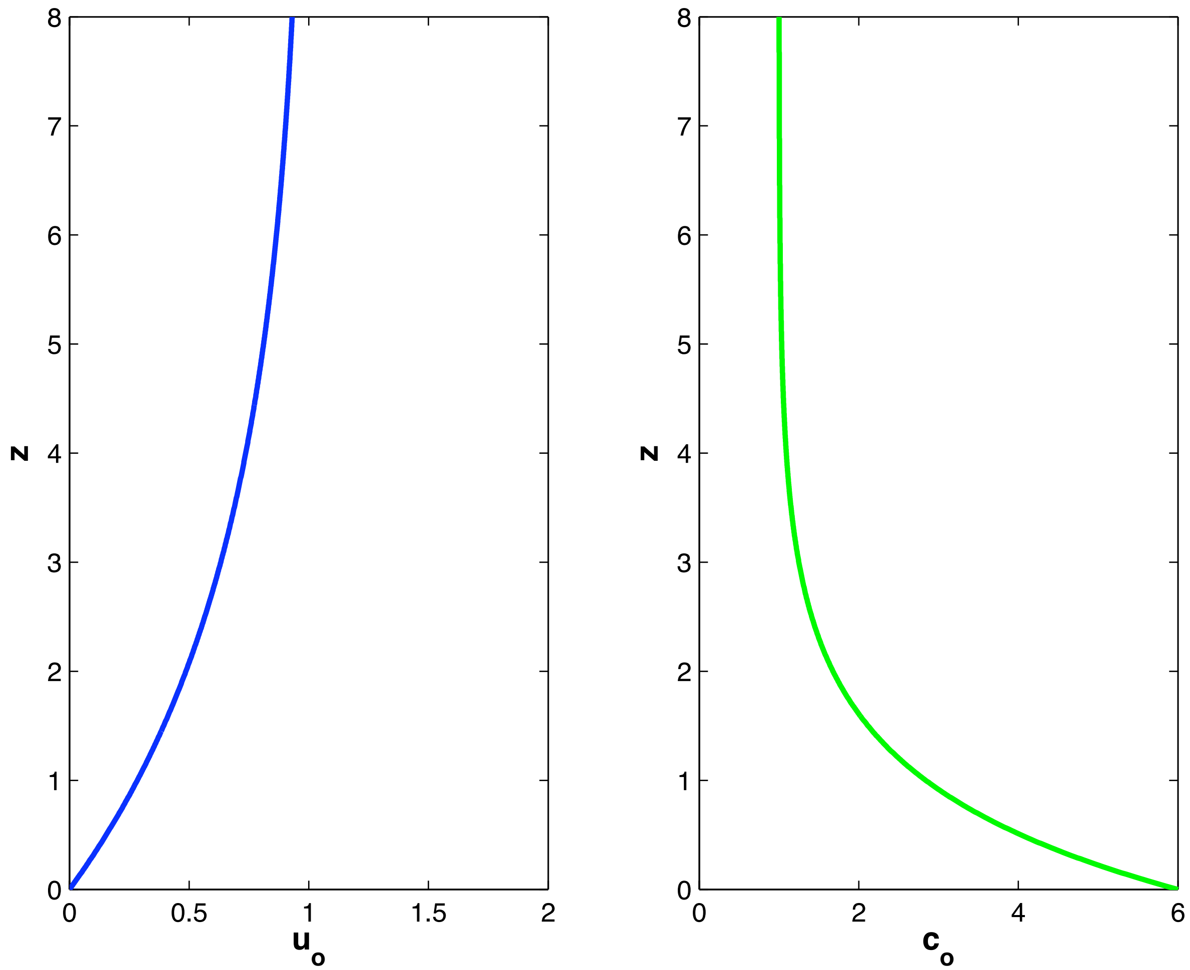


L < 1

One dimensional & quasi-steady

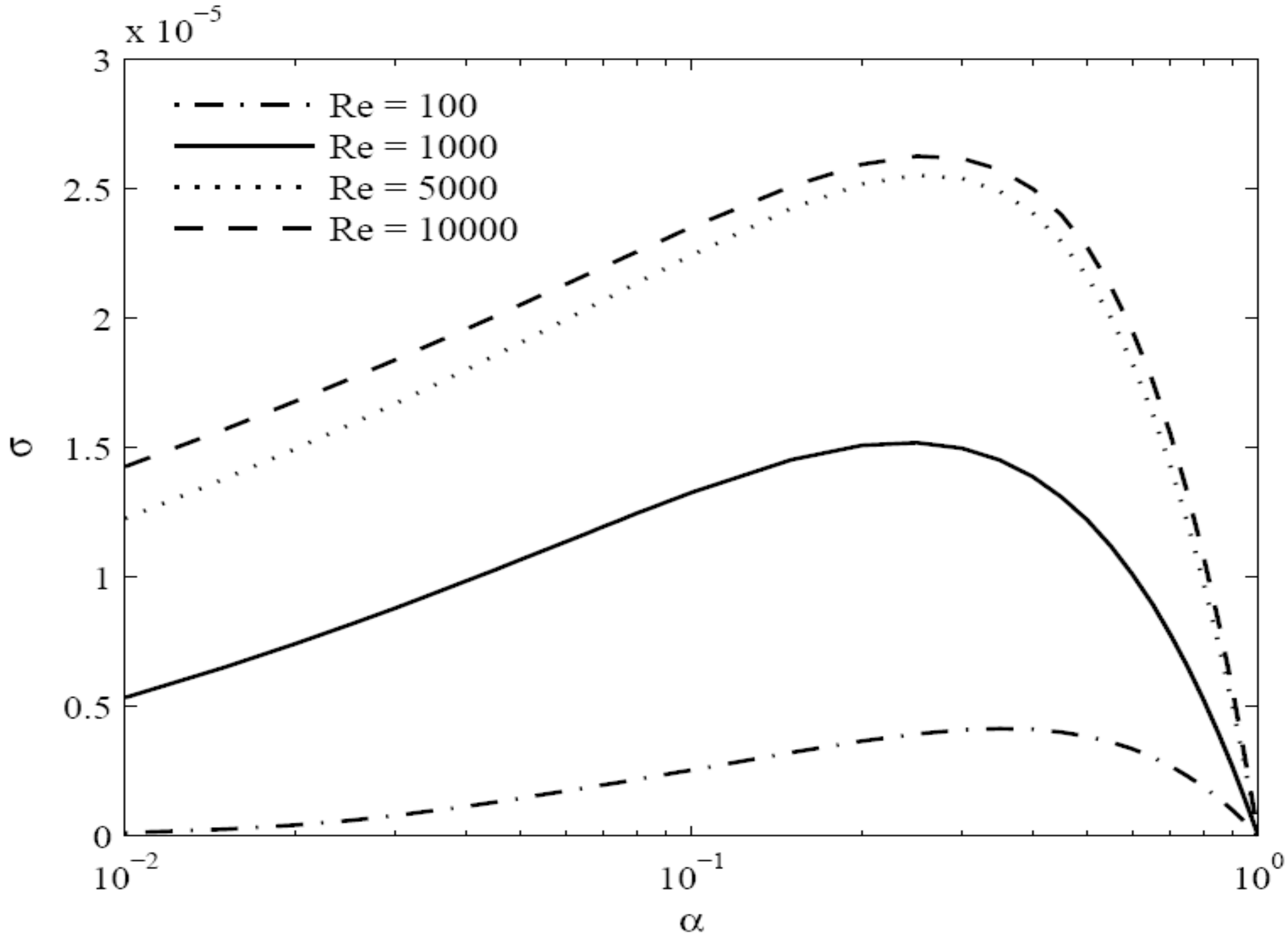
$$u_0(z) = 1 - e^{-z/L} \quad , \quad c_0(z) = \frac{N\,Pe}{L\,c_\infty} e^{-z} + 1$$

Important parameter: $L = \dfrac{\text{length over which } u_o \text{ decays}}{\text{length over which } c_o \text{ decays}}$



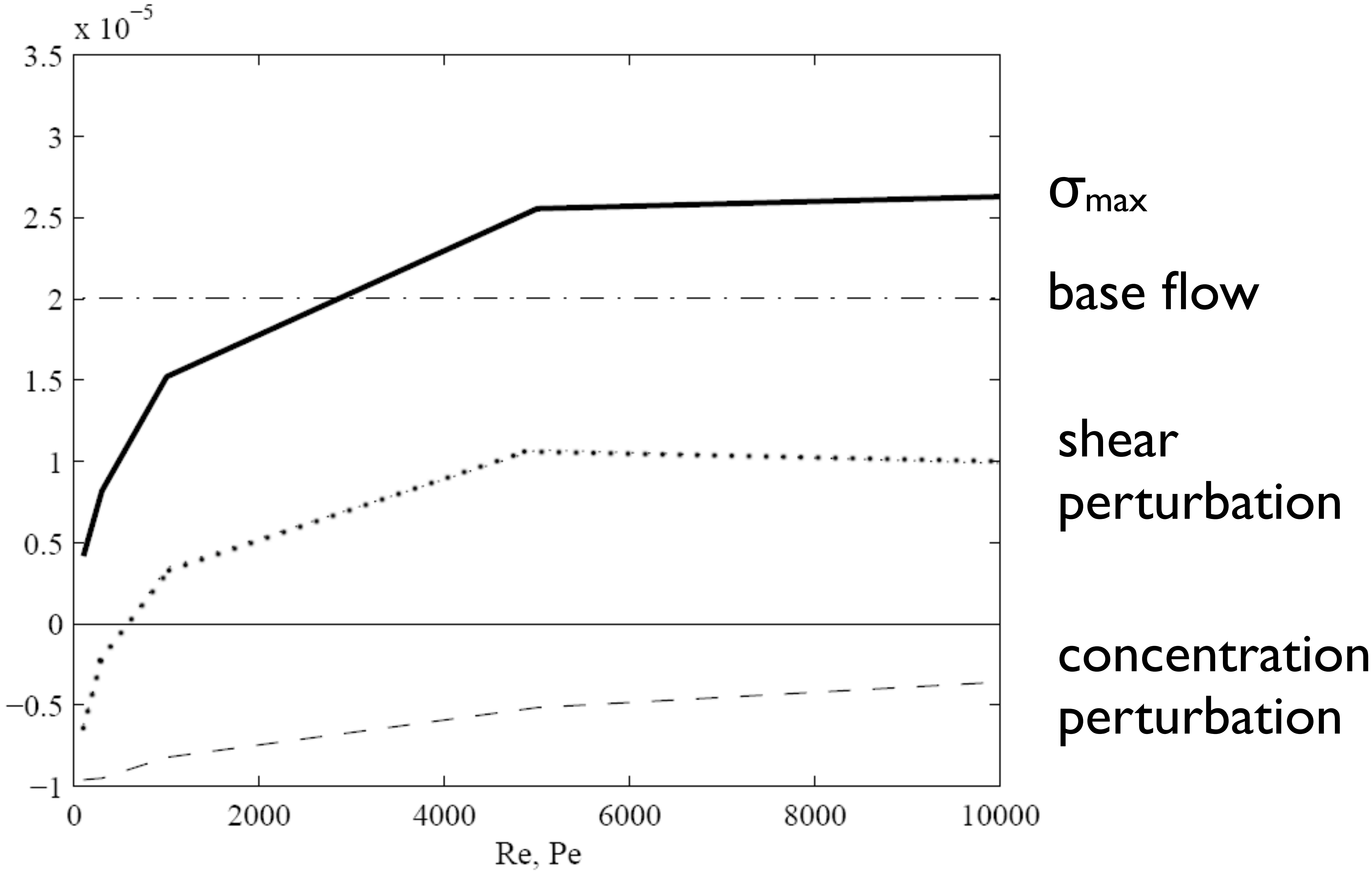
L > 1

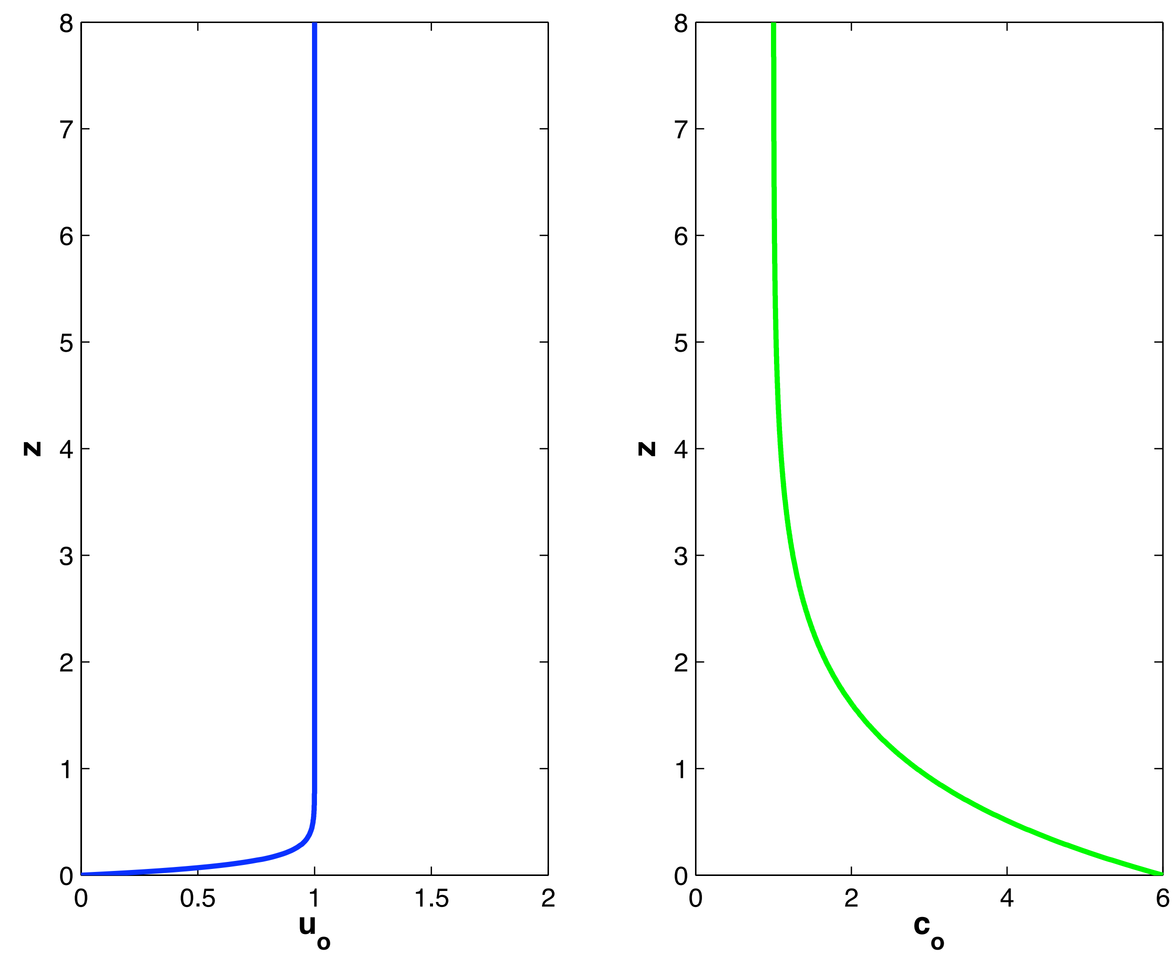
$$L = 0.5$$
$$G = 0.1$$
$$c_\infty = 10^{-2}$$
$$N = 10^{-5}$$



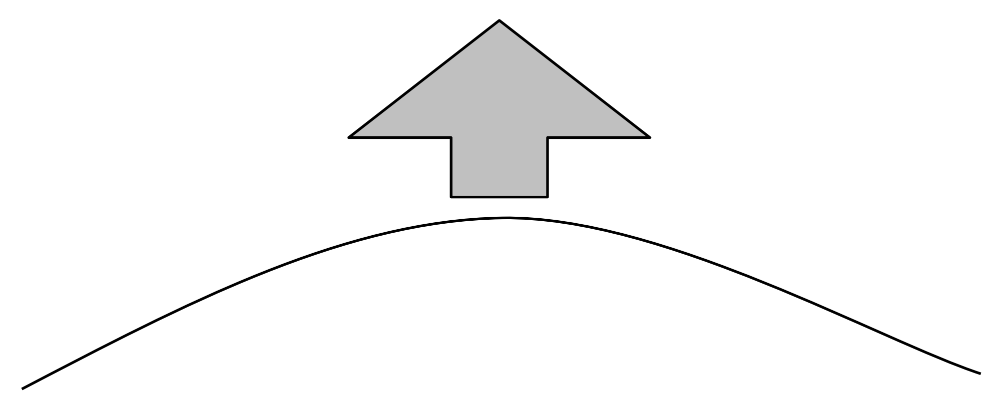
Most amplified wave number α~0.25

Instability mechanism

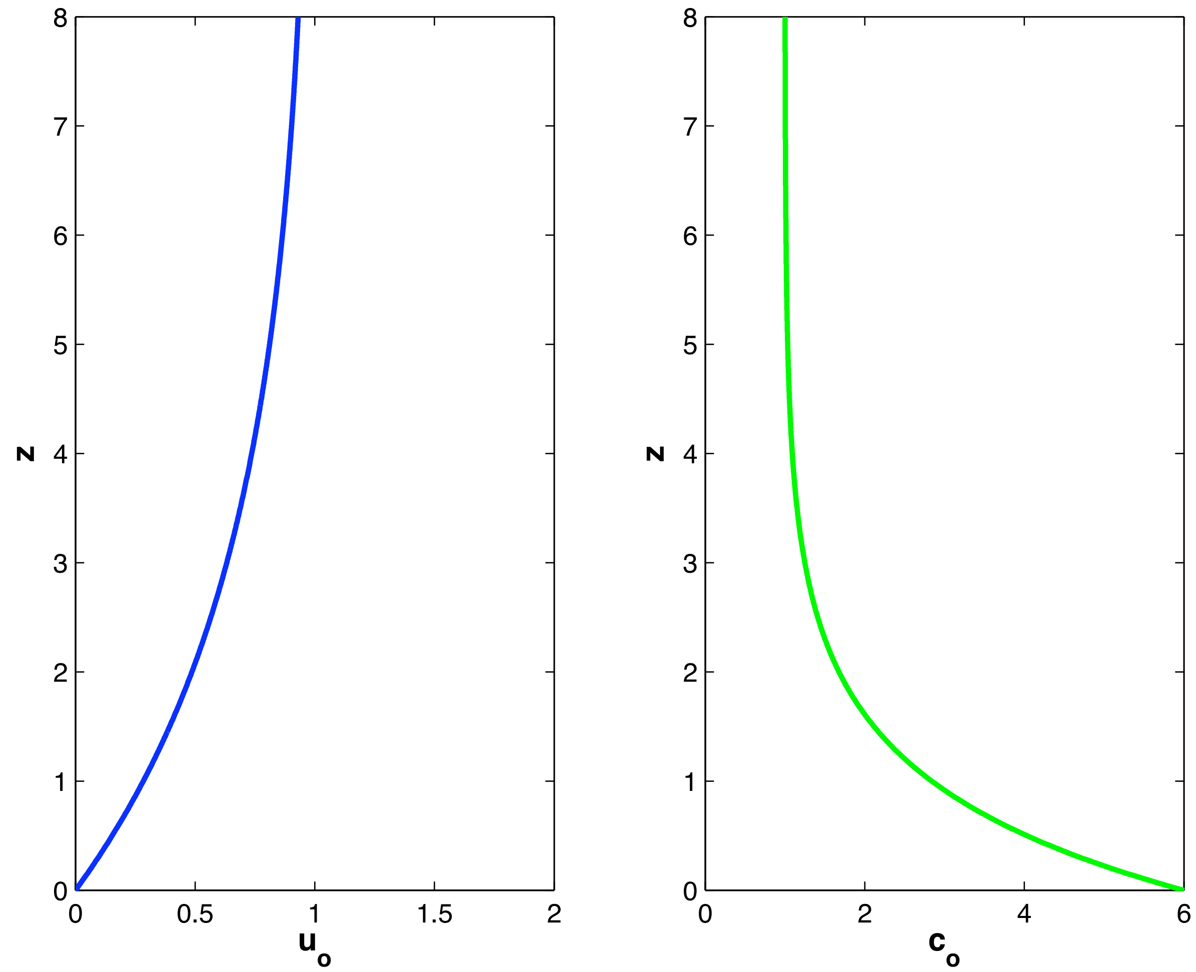




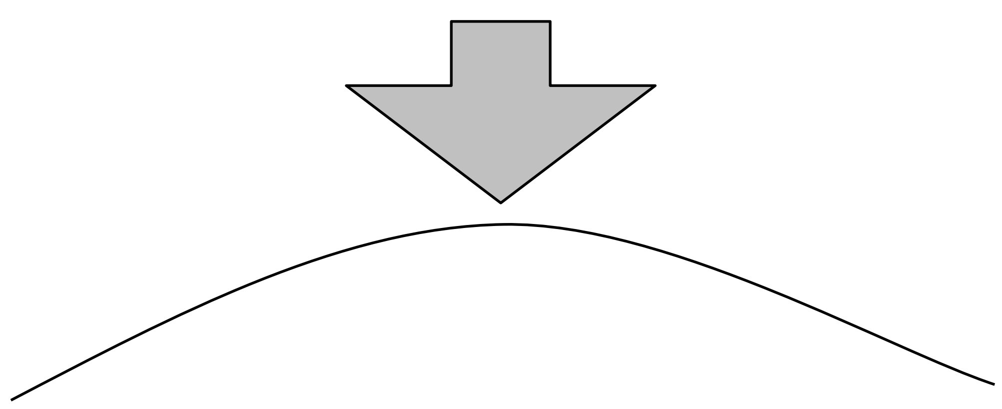
$$L < 1 : \quad \frac{\partial u_o}{\partial z} \downarrow \quad c_o \downarrow$$



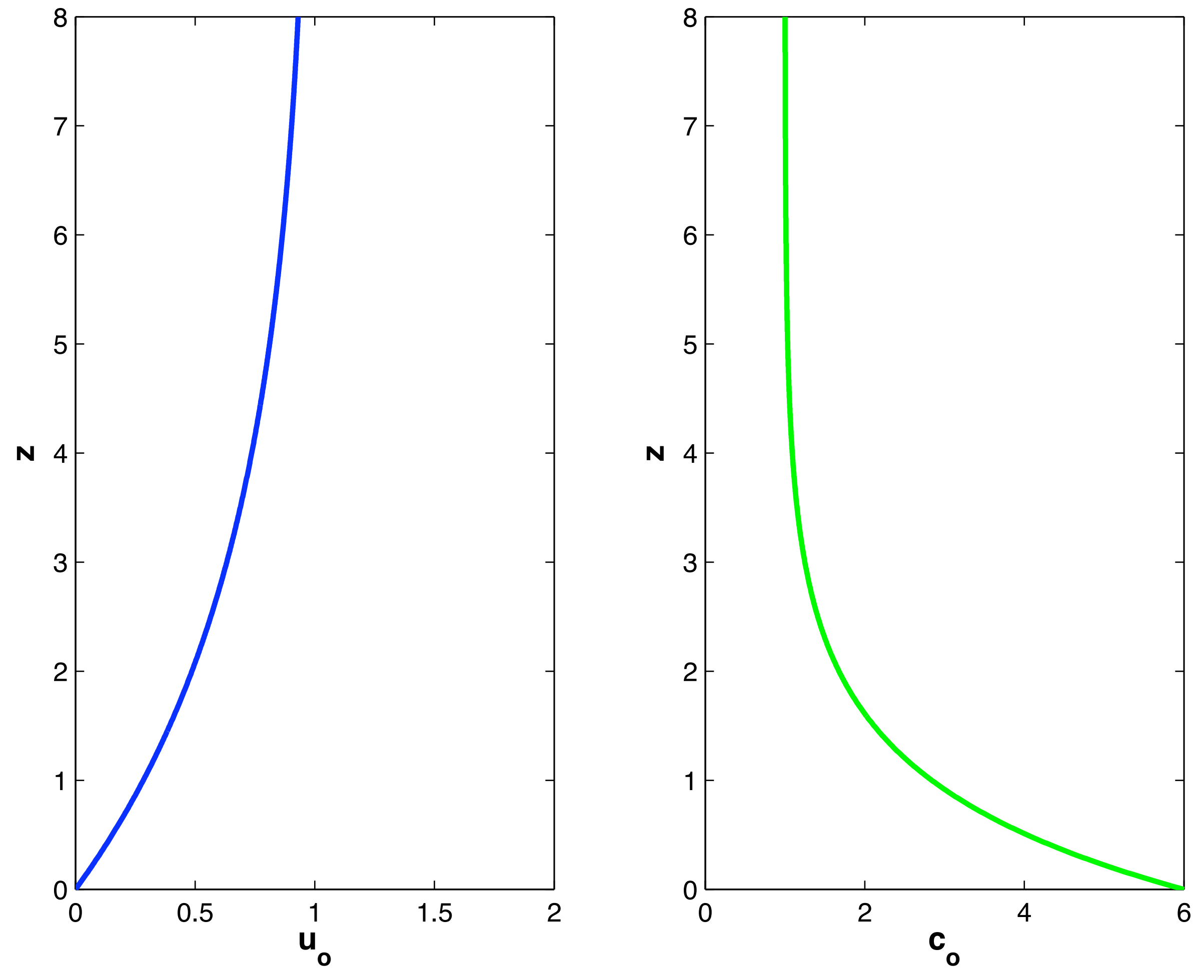
Base flow interaction



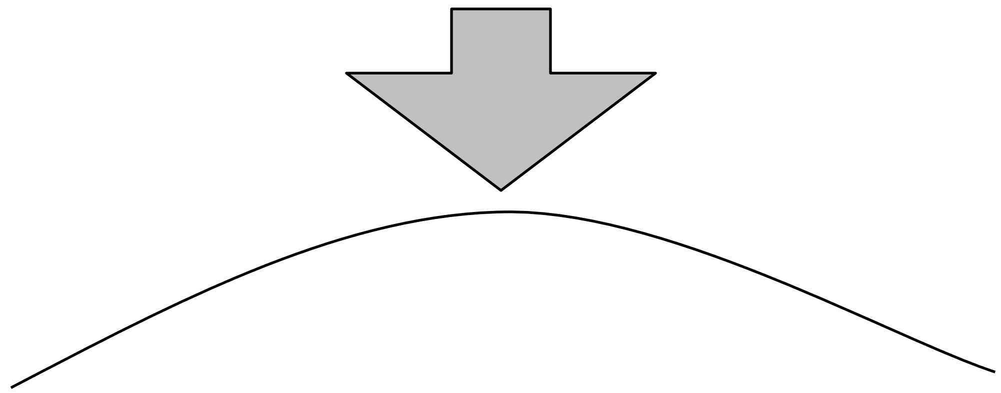
$$L > 1 : \quad \frac{\partial u_o}{\partial z} \quad c_o$$



Base flow interaction

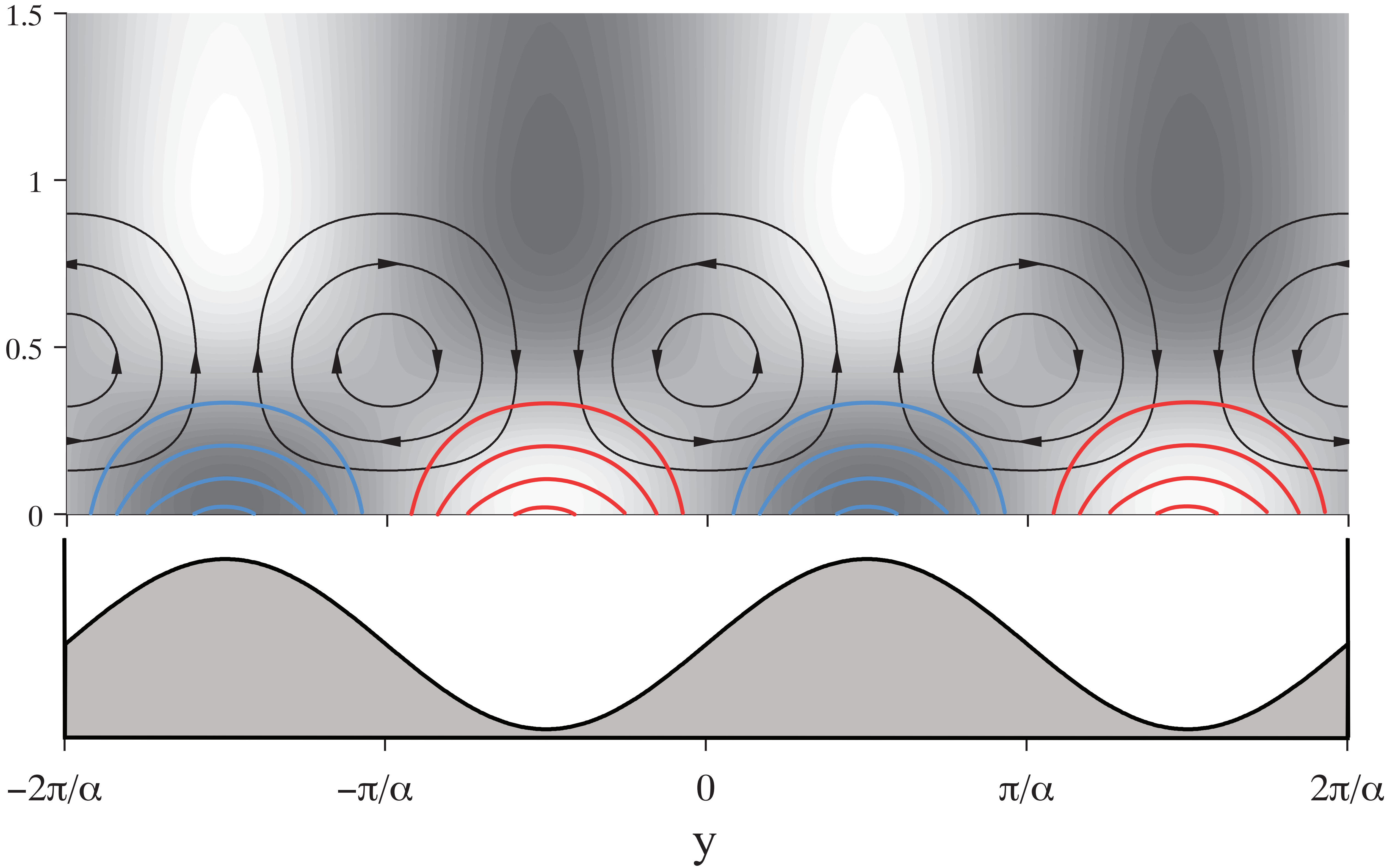


$$L > 1 : \quad \frac{\partial u_o}{\partial z} \downarrow \quad c_o \Downarrow$$



Main criterion for instability: $L < 1$

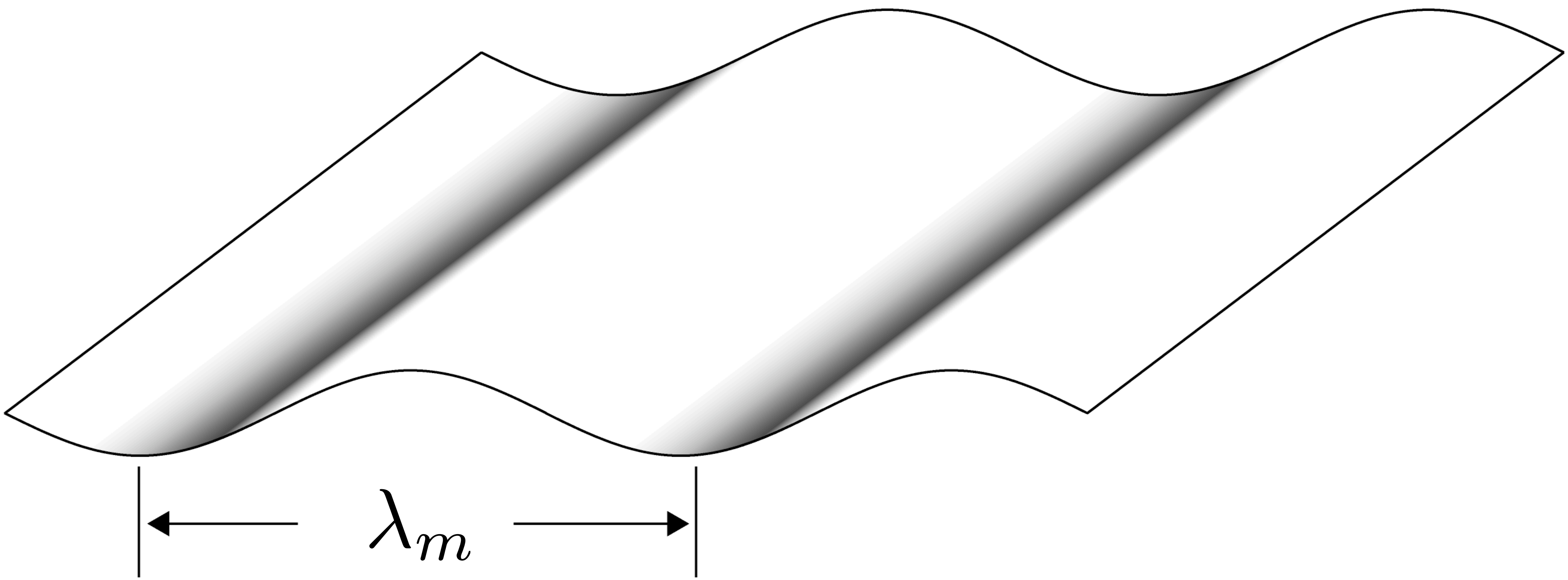
Concentration & shear perturbations



Channel spacing

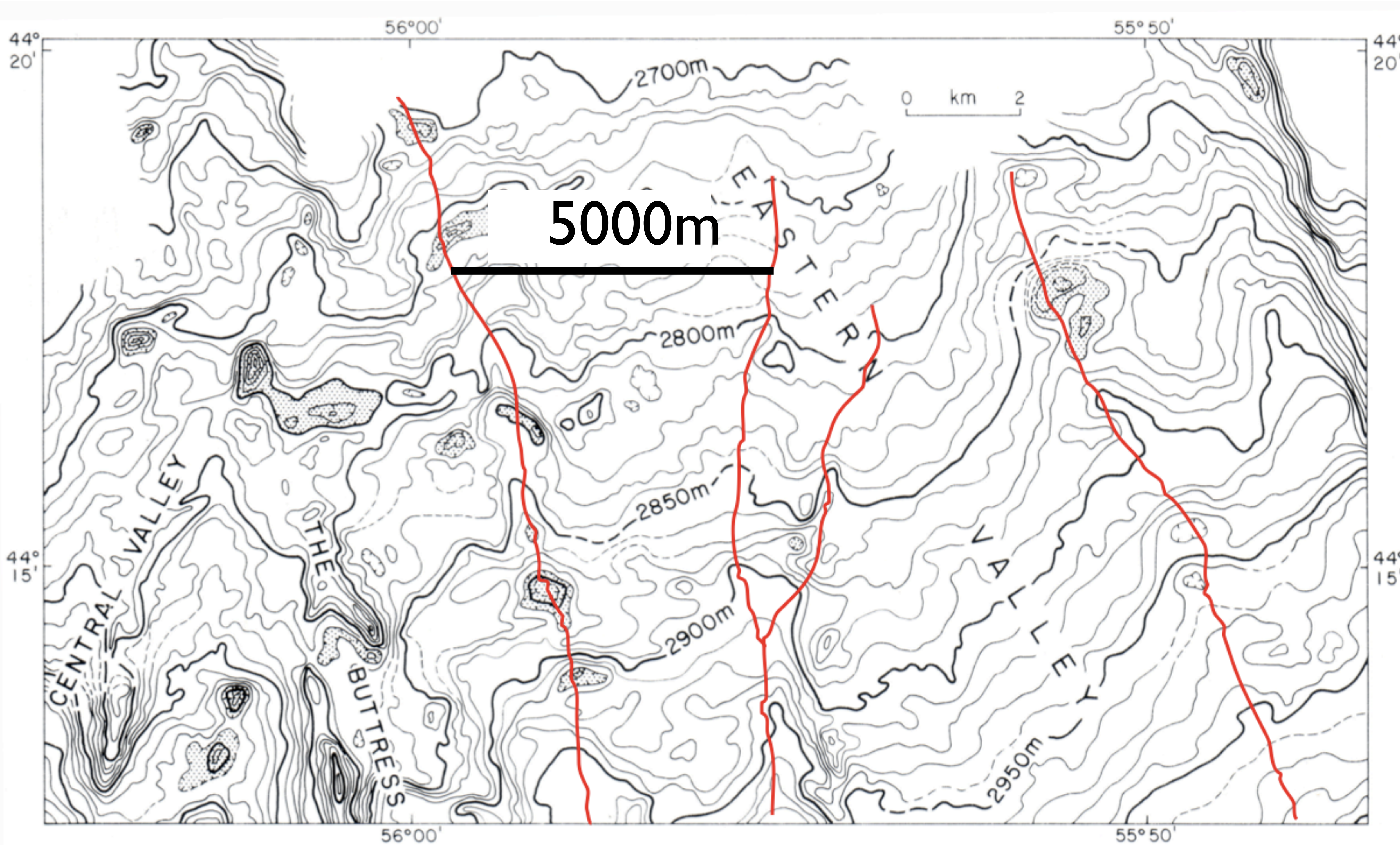
$$\alpha_m \approx 0.25$$

$$\lambda_m \approx 25\times \text{ current height}$$



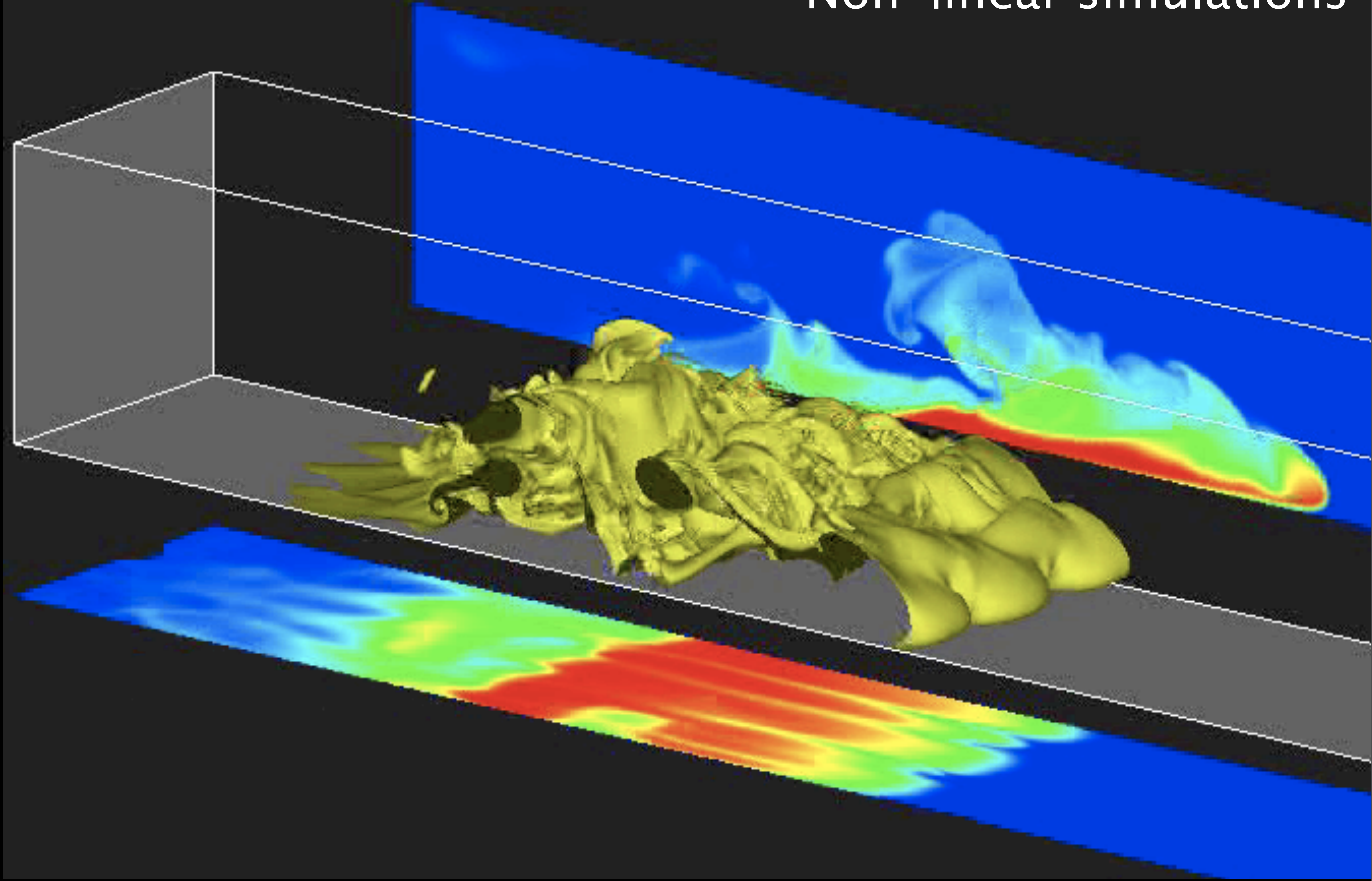
Laurentian Fan

Grand Banks event: estimated height 200-300 m



Shor et al 1990

Non-linear simulations



$$\nabla \cdot \vec{u}_f = 0$$

Momentum

$$\frac{\partial \vec{u}_f}{\partial t} + (\vec{u}_f \cdot \nabla)\,\vec{u}_f = -\nabla p + \frac{1}{Re}\nabla^2 \vec{u}_f + c_{tot}\vec{e}_g$$

Species transport

$$\frac{\partial c_i}{\partial t} + \left[\left(\vec{u}_f + U_i^s \vec{e}_g\right)\nabla\right] c_i = \frac{1}{Pe}\nabla^2 c_i$$

- incompressible, Boussinesq NS equations
- multiple concentration fields
- constant particle settling speed (Dietrich, 82)
- threshold entrainment relationship (Garcia & Parker, 93)

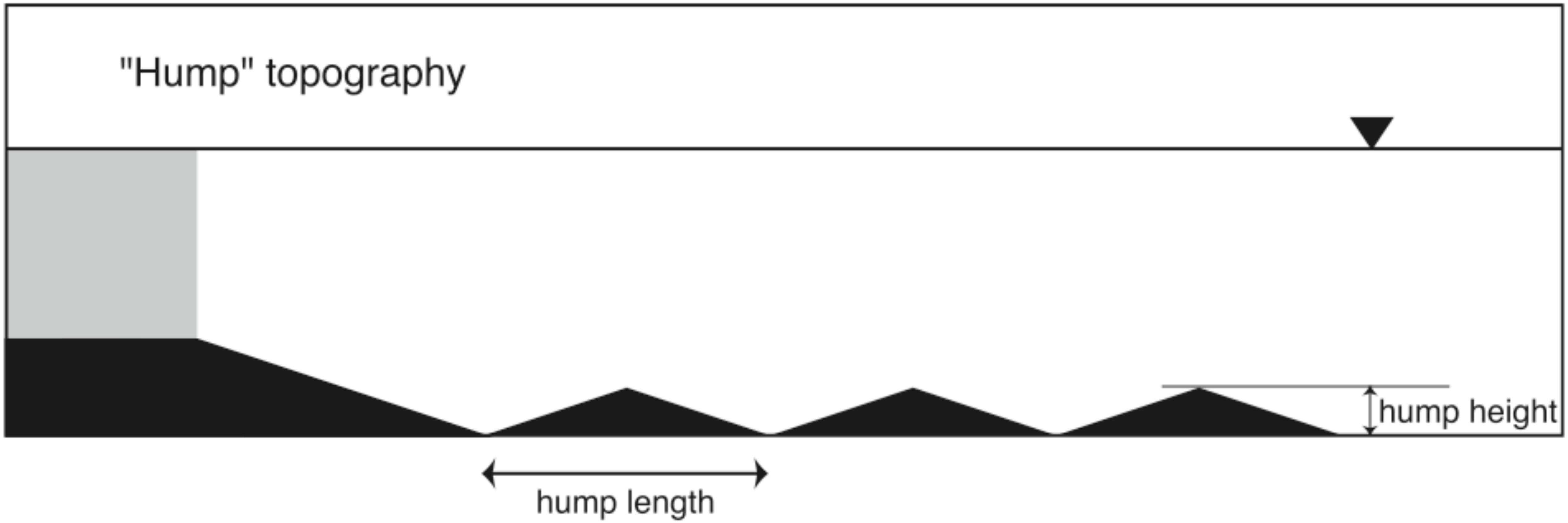
Pressure projection method

- staggered grid
- viscous terms: implicitly with 2nd order differences
- convective terms: explicit with 3rd order ENO
- Poisson equation for pressure (PCG, multigrid)
- 3rd order RK time-stepping

Masking the grid

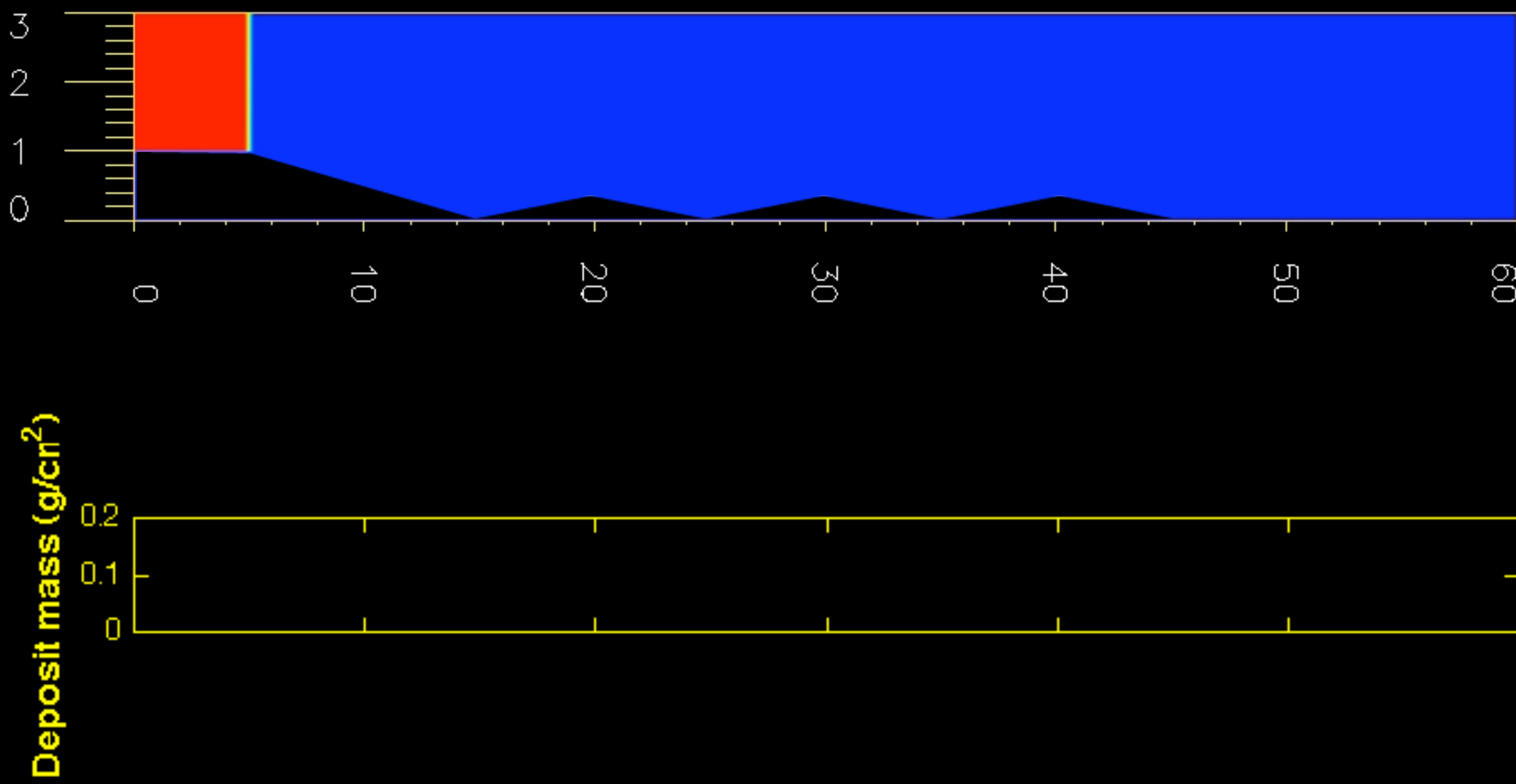
Direct numerical simulation of sediment waves

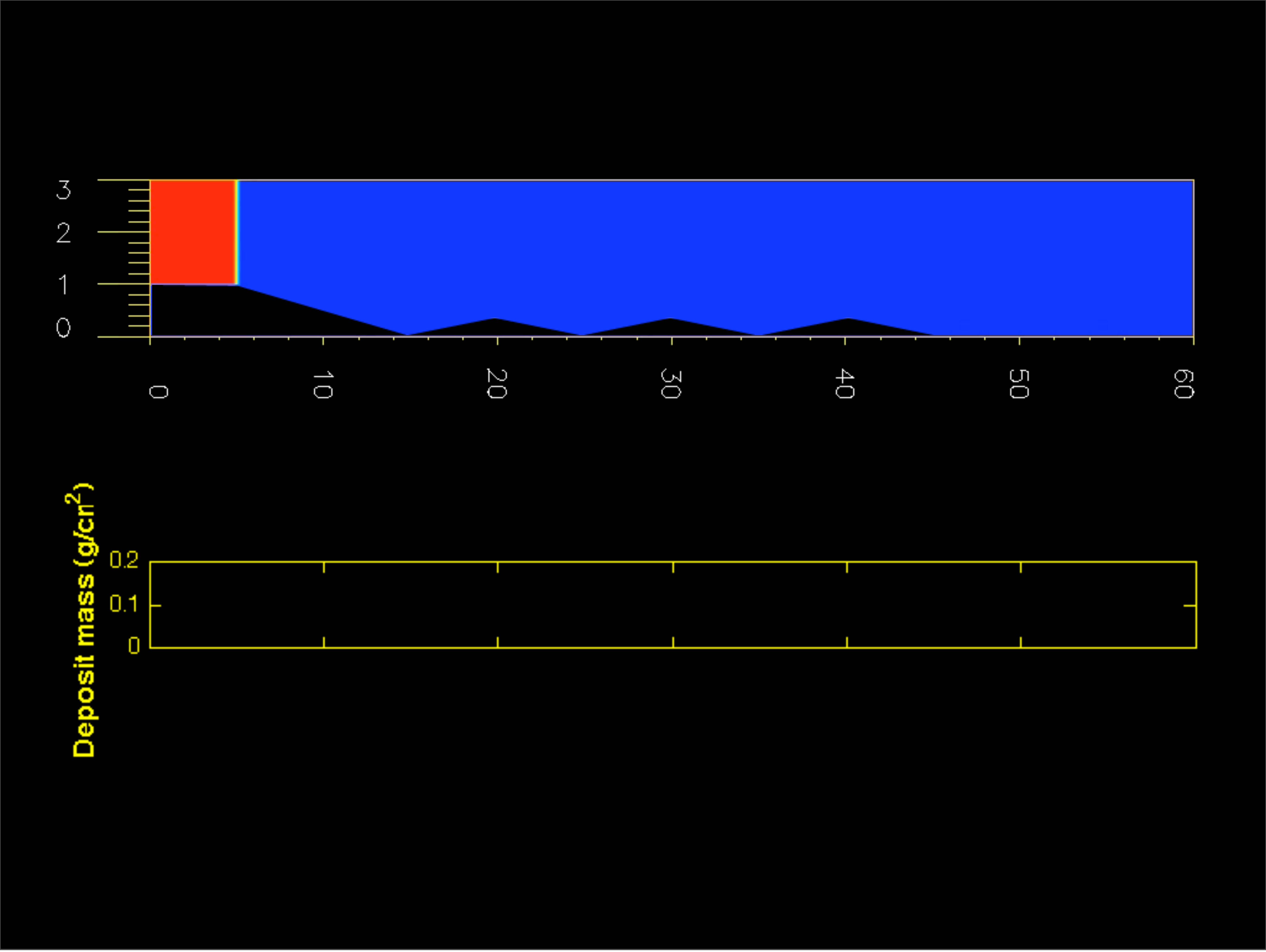
Experiments/Simulations by Kubo, 2004



- flume experiments
- depth averaged simulations
- multiple grain sizes (125 - 30 microns)

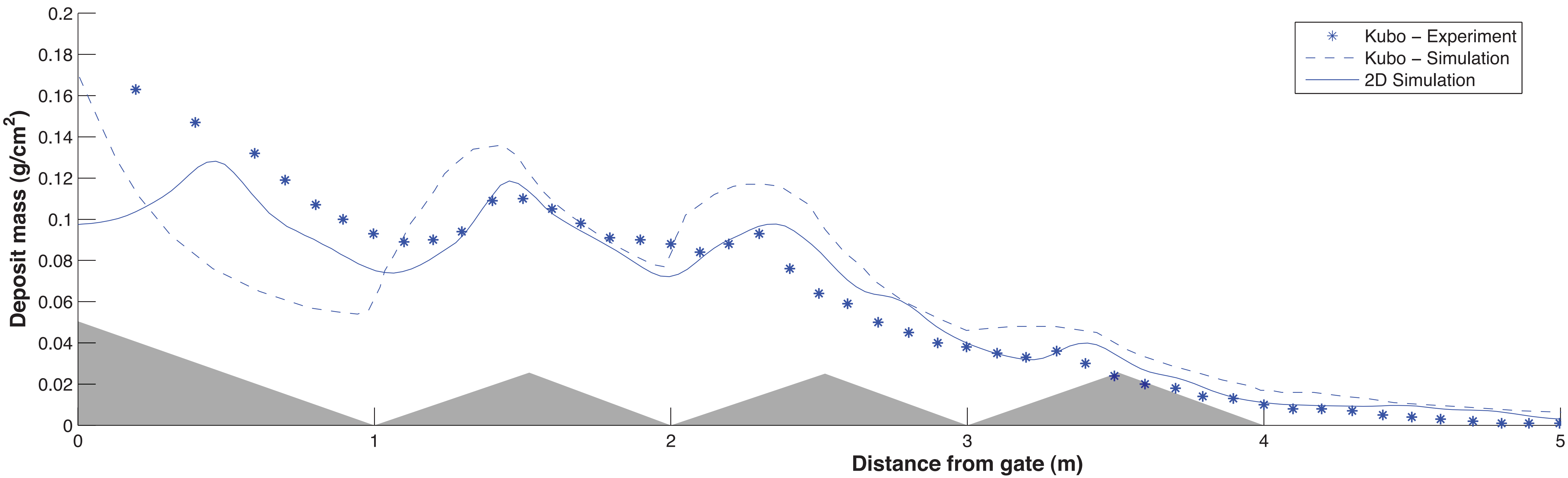
Y. Kubo, Sedimentary Geology 164 (2004)

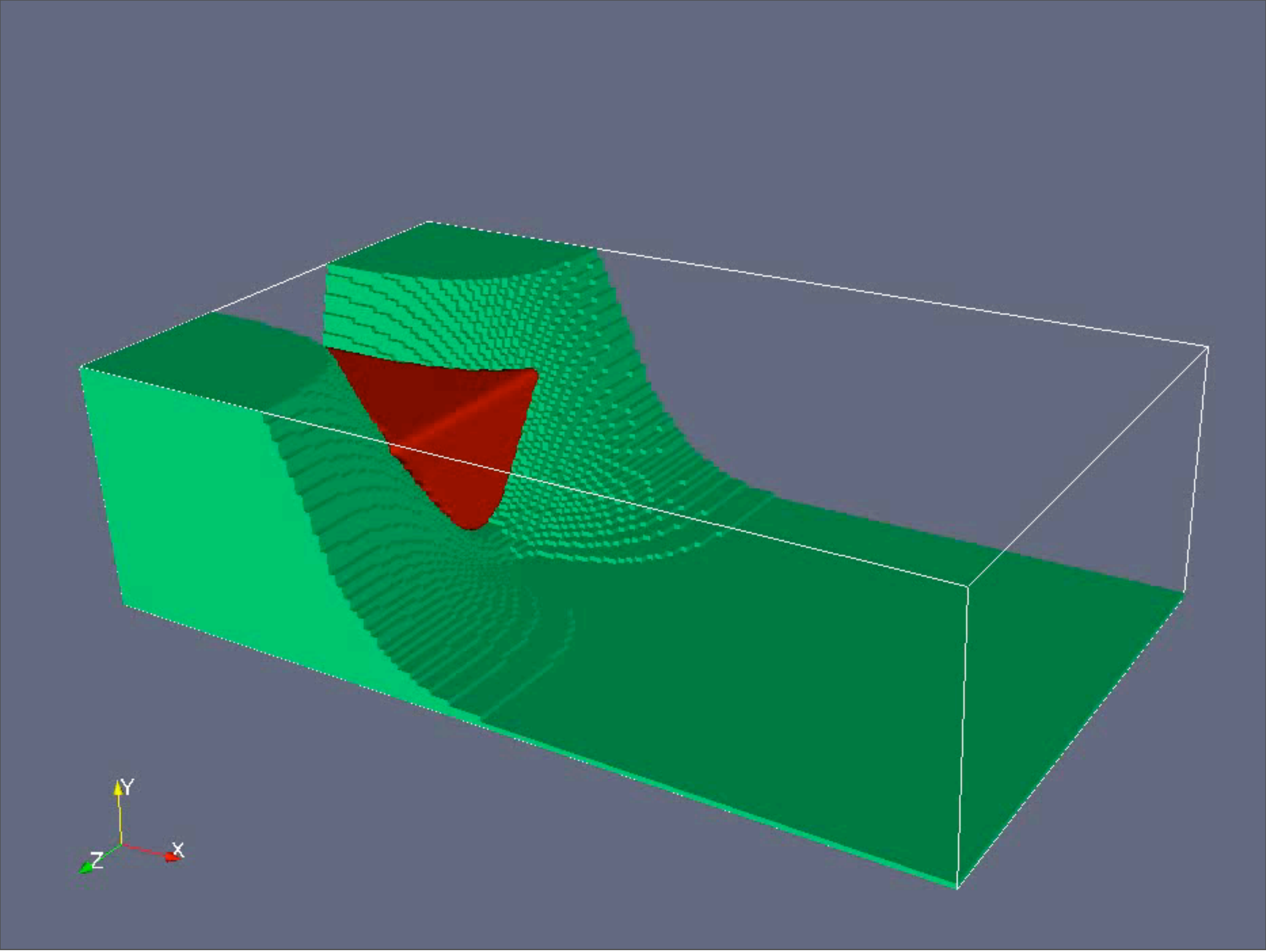

3
2
1
0
0
10
20
30
40
50
60
Deposit mass (g/cm^2)
0.2
0.1
0


3
2
1
0
0
10
20
30
40
50
60
Deposit mass (g/cm2)
0.2
0.1
0

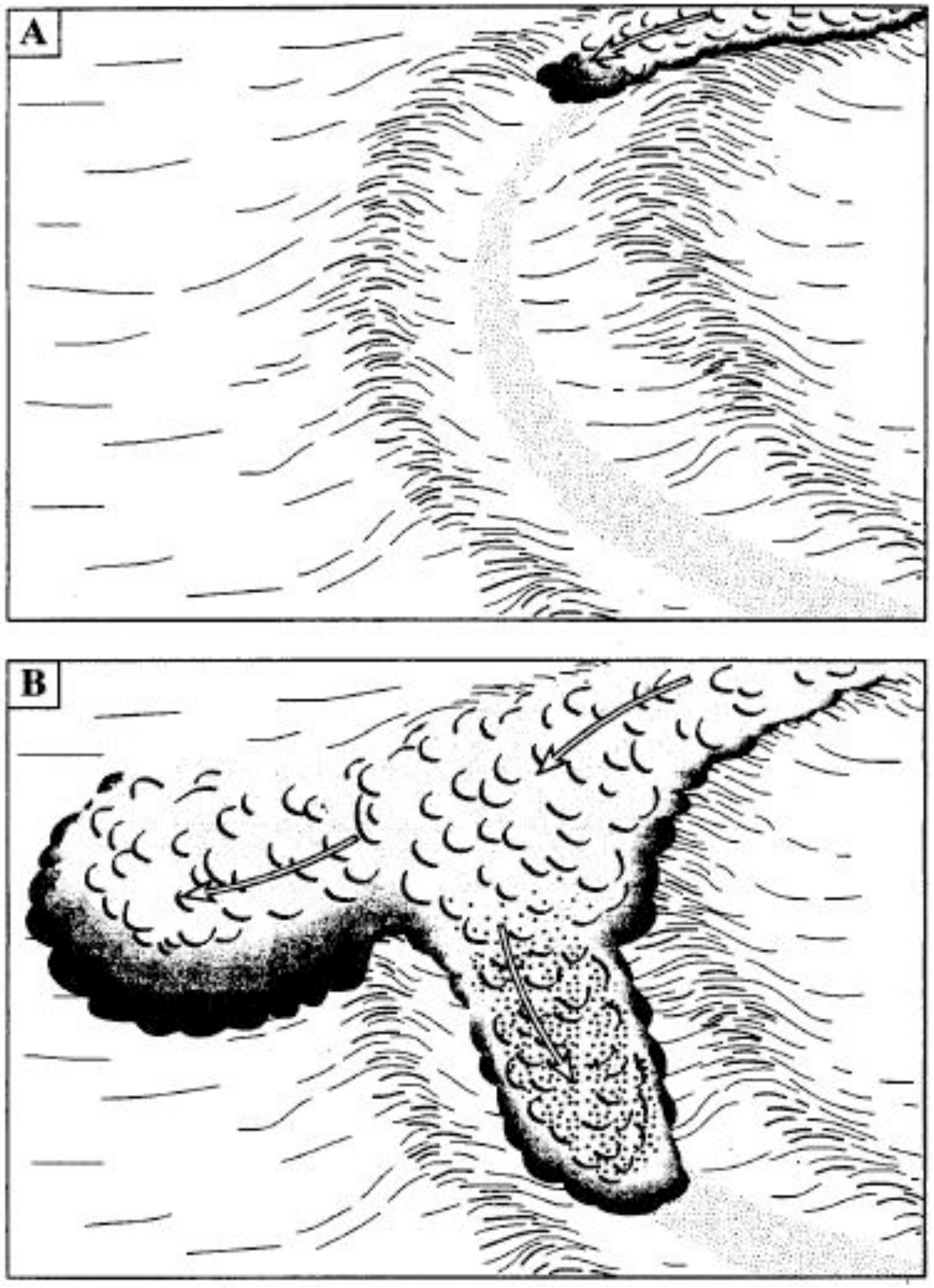
Comparison with experiment

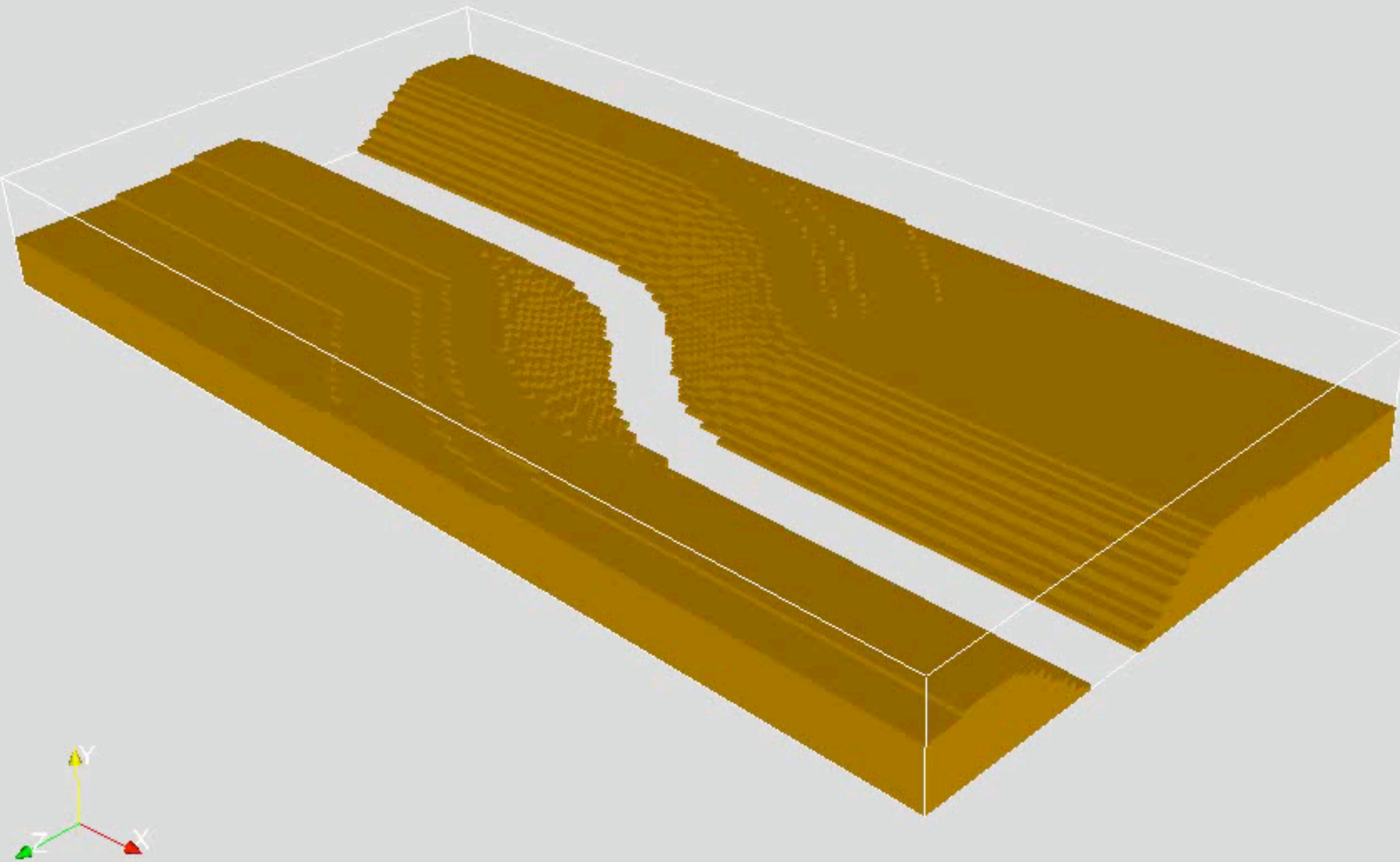
Experiments/Simulations by Kubo, 2004

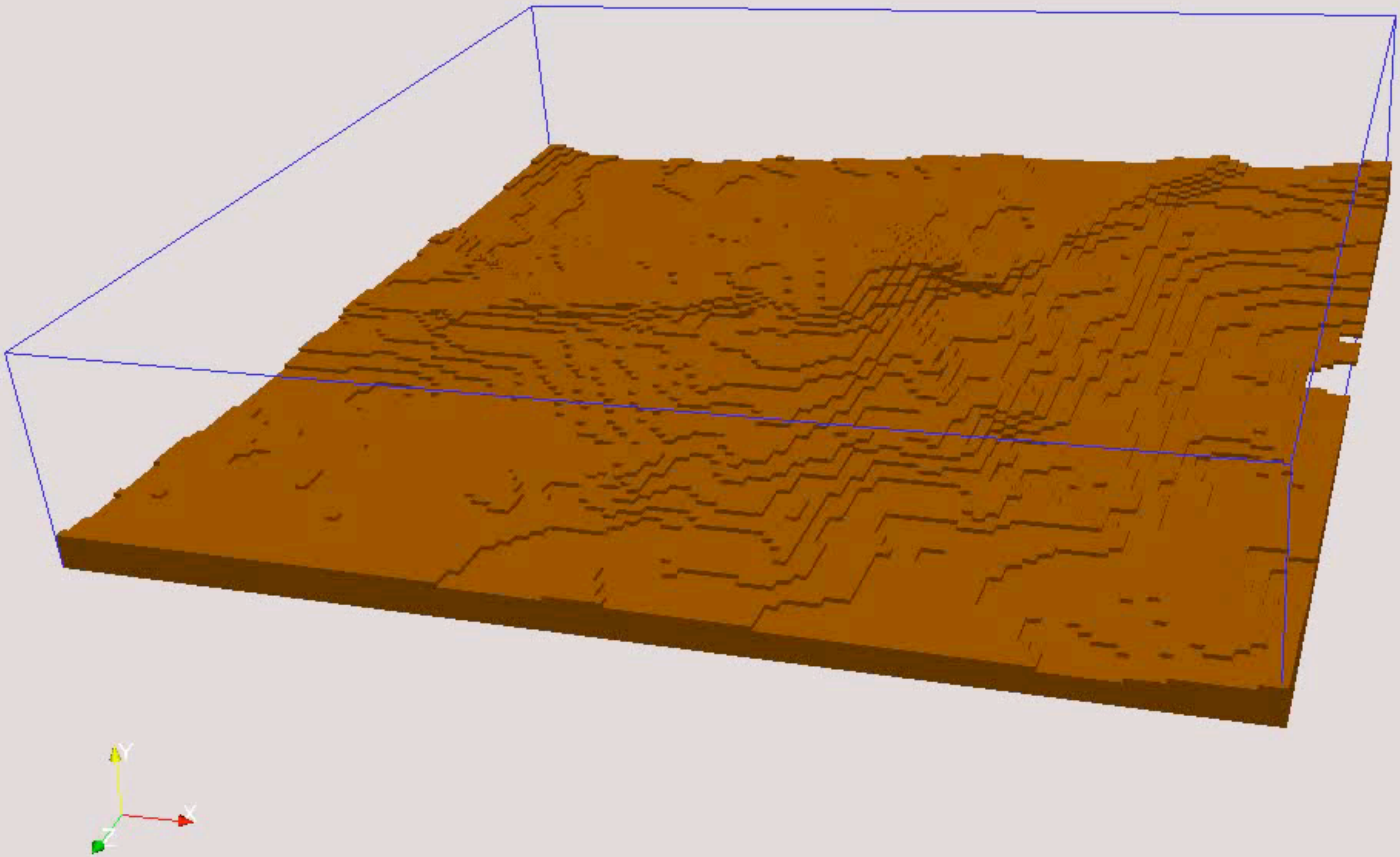



Y
Z
X

Flow stripping in channel bends




Y
Z
X


Y
X
Z

In summary

- Developed a 2D & 3D nonlinear simulation that includes the effect of an arbitrary boundary

- Validation by comparison with experimental results

- Currently working on realistic geometries, more accurate interface treatment (eg. immersed boundaries)