Earth-surface Dynamics Modeling & Model Coupling

A short course

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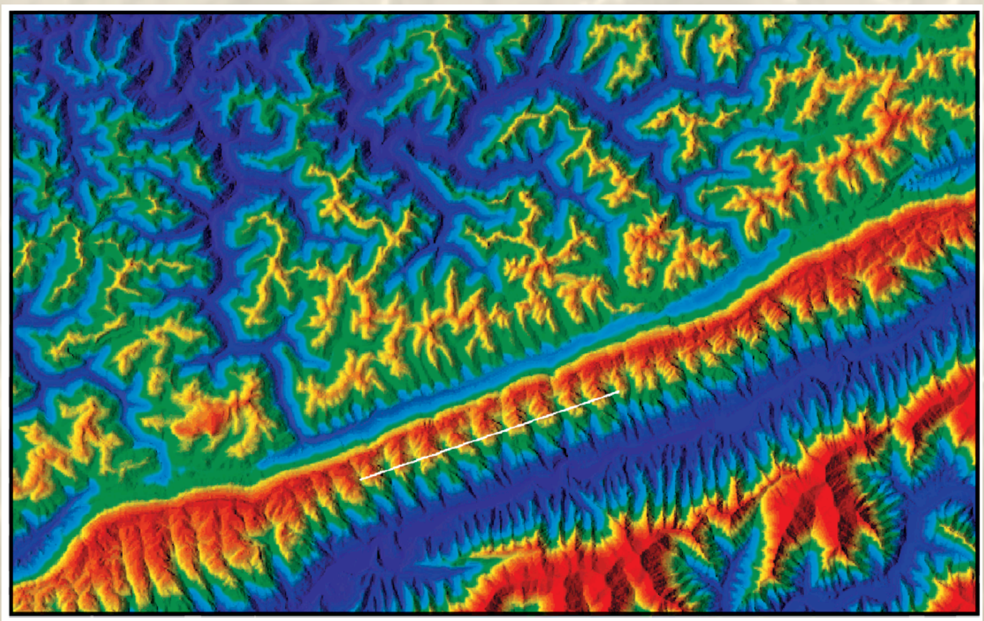
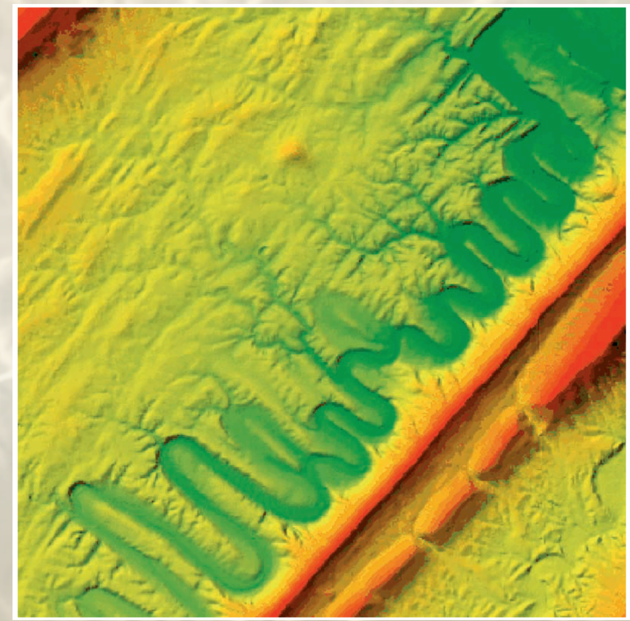
With special thanks to Alan Howard, Gary Parker, Rudy Slingerland, Greg Tucker,





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Module 3: Landscape Evolution Modeling

ref: Peckham, S.D., 2003. Fluvial landscape models and catchment-scale sediment transport. Global and Planetary Change 39: 31-51.

Intro (2)
Sediment Production (6)
Fluvial Transport
    detachment-limited (3)
    transport-limited (2)
Floodplains (2)
CHILD (4)
PHMSed Advanced Model (3)
Summary (1)

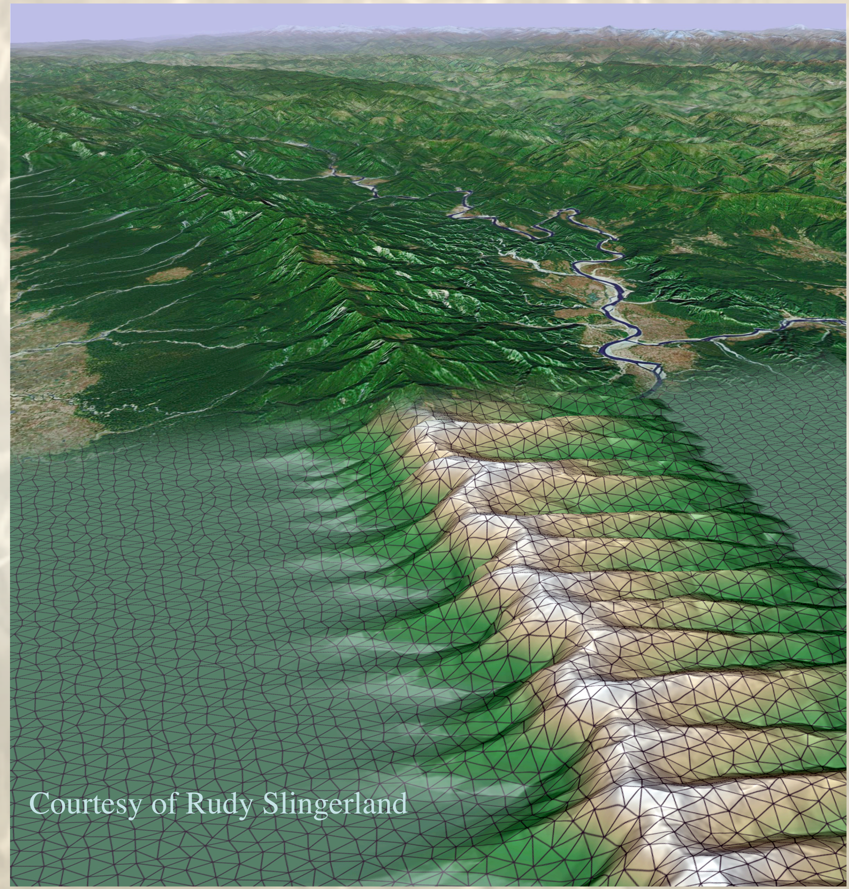
A sampling of models

Catchment Scale

- ANSWERS –Bierly et al.
- CREAMS – Alonso, Knisel, et al.
- SHESED – Wicks & Bathurst
- KINEROS – Woolhiser et al.
- EUROSEM – Morgan et al.
- InHM – Heppner et al.

Landscape Scale

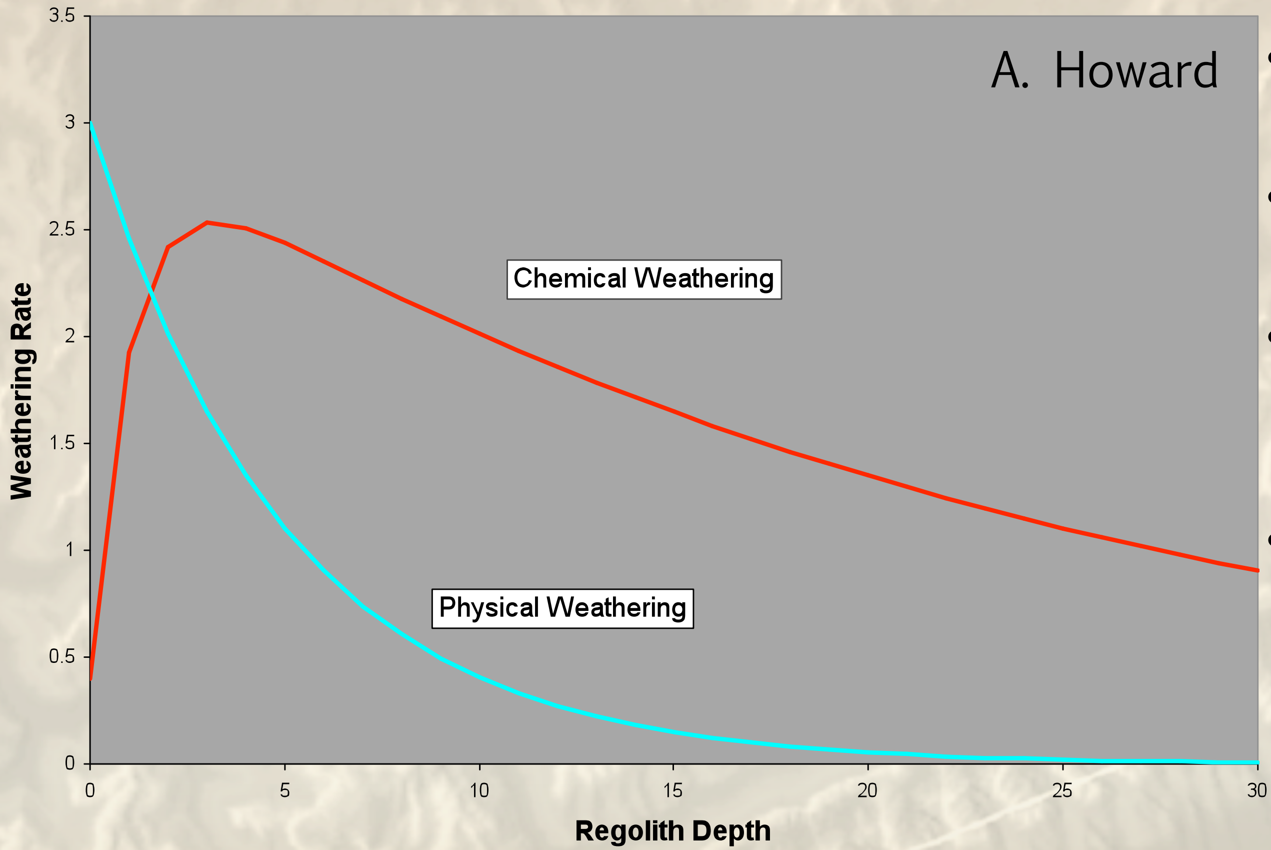
- SIBERIA –Willgoose et al. 1990
- Precipiton –Chase, 1992
- DRAINAL –Beaumont et al. 1994
- GOLEM –Tucker & Slingerland, 1994
- MARSIM –Howard, 1994
- CHILD – Tucker et al., 1999
- CASCADE – Braun & Sambridge, 1997
- CAESAR -- Coulthard et al., 1997
- ZSCAPE –Densmore et al., 1998



Drainage basin model beginnings

- Transport-limited
- Based on continuity equation
- Power-law transport capacity: $q_s \sim A^m S^n$
  - ⇒ homogeneous, cohesionless fine sediment
  - ⇒ "Geomorphically effective" runoff
- Diffusion equation for hillslope mass transport

Weathering



- Implicit -- keeps pace with erosion
- all slopes regolith mantled
- Two layer model -- regolith and bedrock
- Negative exponential or peaked weathering rate as function of regolith depth

$$Z_b = -\left(K_{r1} e^{-c_1 H} - K_{r2} e^{-c_2 H}\right)$$

Modeling mass wasting (A. Howard)

Shallow mass wasting

$$\left.\frac{\partial z}{\partial t}\right|_m = \dot{z} = -\nabla \cdot q_m$$

$$\dot{z}_m = \left[K_s \Gamma(S) + K_f \left(\frac{1}{1 - \{|S|/S_t\}^{\alpha}} - 1 \right) \right]$$

For humid temperate vegetated terrain, the Creep diffusivity, K_s = 0.0001-0.001 m^3/m-yr

For steep, vegetated slopes in semi-arid or Mediterranean climates, K_s = 0.004-0.06 m^3/m-yr

The first term is linear diffusive creep (or rain splash); second term produces threshold slopes

The threshold slope gradient, S_t, varies from 32° for noncohesive materials to >45° for cohesive regolith.

The threshold parameter, K_f, is adjusted such that mass wasting rates become accelerated only a few degrees from threshold

Threshold slopes are ignored in the initial simulations (to permit large time steps).

Landsliding / mass movement

- Nonlinear diffusion (e.g. Anderson & Humphrey; Roering & Dietrich)
  $\partial z/\partial t = \partial/\partial x\, [\, -\kappa(z_x,t)\; \partial z/\partial x]$
- Threshold slope angle (Tucker & Slingerland)
- Stochastic algorithm (Densmore et al.)
- Discrete Failures (Martin)

$$F = \frac{c' + c + \left[(1-m)\rho_b g d + m(\rho_{sat} - \rho) g d\right]\cos^2\theta \tan\phi'}{\left[(1-m)\rho_b + m\rho_{sat}\right] g d \sin\theta\cos\theta}$$

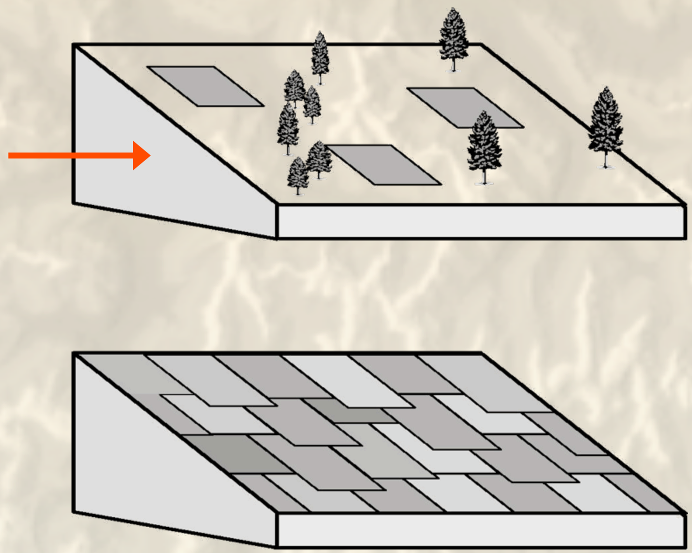
c' : effective material cohesion
c_r : pseudo-cohesion (root strength)
ρ_b and ρ_{sat}: material bulk density & saturated material density
d: depth of surface material above potential failure plane
θ: hillslope angle
m: fractional saturation (d_{sat}/d, where d_{sat} is depth of the saturated zone)
ϕ' : effective angle of shear resistance of material

Rules are required to distribute failed material downslope.

Process Specification: (after D. Martin)

Hillslope dominated by discrete failures.

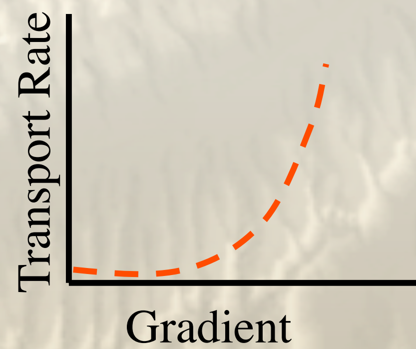
Landslide event



After > 1000 years

Use of generalized diffusive equations requires adoption of minimum time scales for which truly episodic processes can be considered continuous

$$\frac{\partial z}{\partial t} = \frac{\partial}{\partial t}\left(-\kappa\left(z_x, t\right)\frac{\partial z}{\partial x}\right) \longrightarrow$$



Slab failure model for gully sidewalls

C = 5 kPa

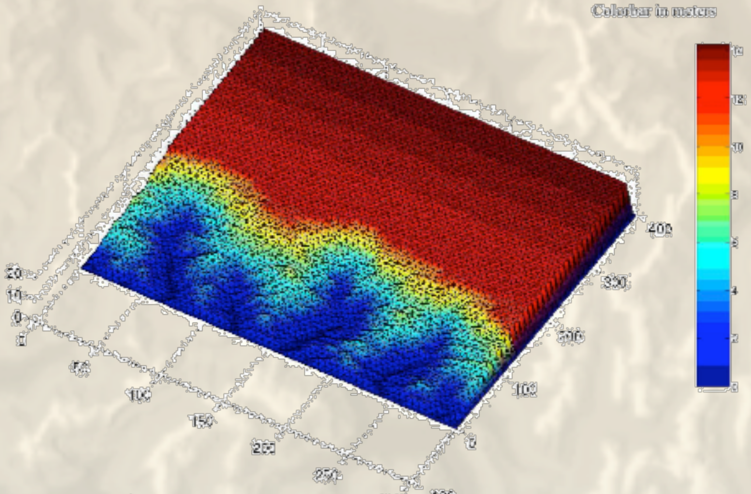


FIGURE 12 (A).

C = 10 kPa

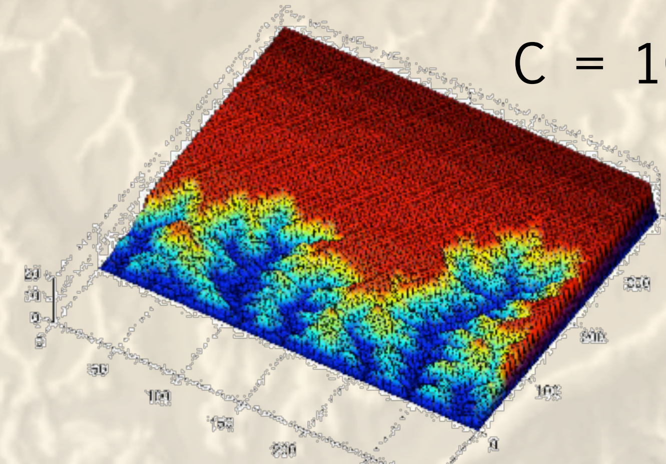


FIGURE 12 (B).

C = 20 kPa

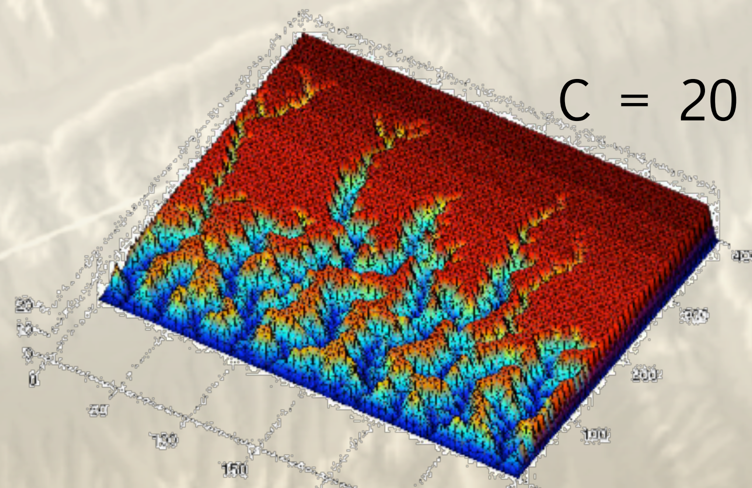


FIGURE 12 (C).

Istanbulluoglu et al., 2005,
JGR-Earth Surface

Sediment Production: CHILD simulations: G. Tucker, 2002

SOIL CREEP

THRESHOLD LANDSLIDING

SATURATION-EXCESS RUNOFF

PORE-PRESSURE DRIVEN LANDSLIDING

Fluvial erosion, transport and deposition after A. Howard

Detachment-limited – assumes rate of erosion is some function f of sediment load, q_s, and some function g of flow intensity, ϑ.

$$\dot{Z}_c = f(q_s)g(\vartheta)$$

The role of sediment in detachment-limited erosion is ignored (or assumed to scale with ϑ), and a power function relationship is assumed, possibly with a critical flow intensity, ϑ_c:

$$\dot{Z}_c = -K_t(\vartheta - \vartheta_c)^n$$

Critical shear stress, ϑ_c, depends both on sediment properties (cohesion) and erosional resistance afforded by vegetation.

Typical values (in N/m^2):

Submerged shelf sediments (Wiberg): 0.1 – 1.0; Bare upland soils: 10-40
Poorly-vegetated soils: 60-80; Grass-covered soil: 100-240; Forest soils: 300-3000

Fluvial erosion, transport and deposition

Assumes steady, uniform flow, consistent downstream hydraulic geometry. This results in an erosion rate a function of local gradient, S, discharge, Q, a climatic precipitation index, P, and the critical flow intensity, ϑ_c:

$$\dot{Z}_c = -K_t\left[(K_z P^d Q^f S^h - \vartheta_c)\right]^n$$

Proxies of flow intensity are shear stress, τ, or stream power per unit width, ω.

$$\vartheta = \tau = \rho_f g R S \qquad \vartheta = \omega = \rho_f g R S V$$

Downstream hydraulic geometry equations are used to parameterize areal variations in flow intensity:

$$Q = PA^e \qquad Q = K_p R W V$$

$$V = K_n g^{1/2} R^{2/3} S^{1/2} / N \qquad W = K_w Q^b$$

Example: Appalachian Streams

- Downstream hydraulic geometry from Appalachia (Brush, 1961) (mks units, Q is mean annual flood), A is drainage area:

$$w = 2.0Q^{0.56} \qquad\qquad Q = 0.000017A^{0.8}$$

- Flow resistance and channel cross-section parameterization:

$$K_n g^{1/2} = 1; \quad N = 0.03; \quad K_p = 1$$

- Measurement of 10yrs of detachment-limited channel erosion in an unvegetated borrow pit in Virginia in Coastal Plain sediments (Howard and Kerby, 1983) resulted in

$$\dot{Z}_c = -0.11A^{0.44}S^{0.68}$$

- Assuming $\vartheta = \tau$, the exponent n =1, and previous equations, the bedrock channel erosion rate (in m/yr) is:

$$\dot{Z}_c = 0.015(932.0Q^{0.26}S^{0.7} - \tau_c)$$

Fluvial erosion, transport and deposition after A. Howard

Transport-limited channels: Erosion is proportional to divergence of sediment flux. Generally alluvial bed at low flow. Simplest model assumes single grain size, d.

q_{sb} is volumetric bed sediment transport rate, μ is porosity, S_s is sediment specific gravity

$$\dot{Z}_c = -\nabla q_{sb}$$

$$\Phi = K_e \left\{ \frac{1}{\psi} - \frac{1}{\psi_c} \right\}^p$$

$$\Phi = \frac{q_{sb}}{G g^{1/2} d^{3/2} (S_s - 1)^{1/2} (1 - \mu)}$$

$$\frac{1}{\psi} = \frac{\tau}{\rho_f g (S_s - 1) d}$$

E.g. assume Einstein-Brown total load sediment relationship (with $1/\psi_c=0$):

$$\Phi = 40 \left\{ \frac{1}{\psi} - \frac{1}{\psi_c} \right\}^3$$

Fluvial erosion, transport and deposition after A. Howard

- The use of a Meyer-Peter like formula with K_e=8 and p=1.5, and an explicit critical shear stress is an alternative if coarse sediment is assumed.
- Assume a single grain size for bed sediment, therefore no sorting or selective deposition.
- Assume d=0.2 mm, S_s=2.65, μ=0.5, G=2/3, and previous hydraulic geometry, and solve for total load transport capacity in a channel, Q_{sb}, (m^3/yr) as a function of gradient and discharge, assuming that an effective discharge is the mean annual flood occurring 0.5 days/year. The resulting equation is:

$$Q_{sb} = 15.0 Q^{0.56} (288.0 Q^{0.26} S^{0.7})^3$$

- This equation is solved for the channel gradient in equilibrium with the sediment supplied from upstream from bedrock channel erosion, and this supplied sediment is deposited (or eroded) at that gradient.

CONSERVATION OF BED SEDIMENT: TRANSVERSE AS WELL AS STREAMWISE BEDLOAD TRANSPORT (2D)

After Gary Parker

y = transverse coordinate [L]

$q_b \rightarrow q_{bx}$

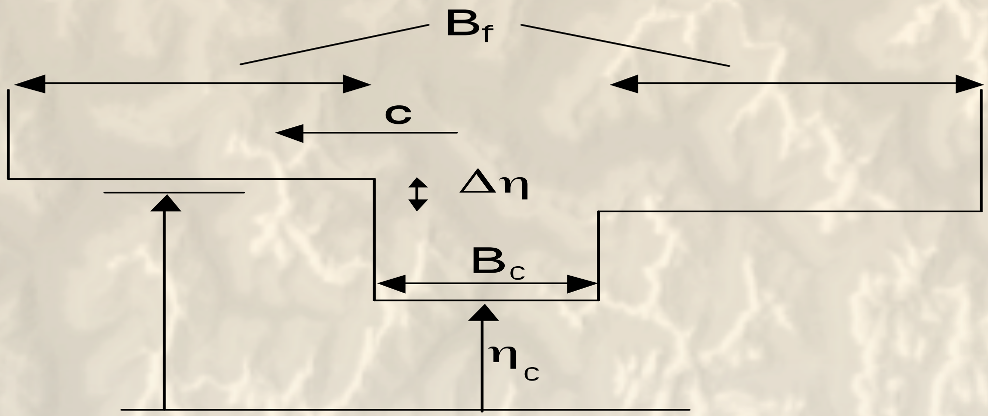
q_{by} = transverse volume bedload transport rate per unit normal distance [L^2/T]

Jamuna River, Bangladesh

$$(1-\lambda_p)\left(\frac{\partial\eta}{\partial t}+\sigma_b\right) = -\frac{\partial q_{bx}}{\partial x} - \frac{\partial q_{by}}{\partial y} + D_s - E_s$$

SEDIMENT CONSERVATION FOR FLOODPLAINS

After Gary Parker



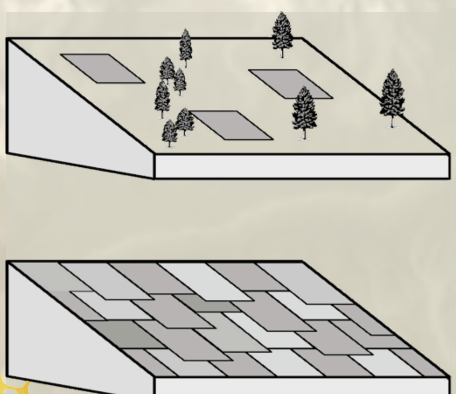
B_f = floodplain width
B_c = channel width
η_f = mean floodplain elev.
η_c = mean channel bed elev.
c = mean channel migration speed

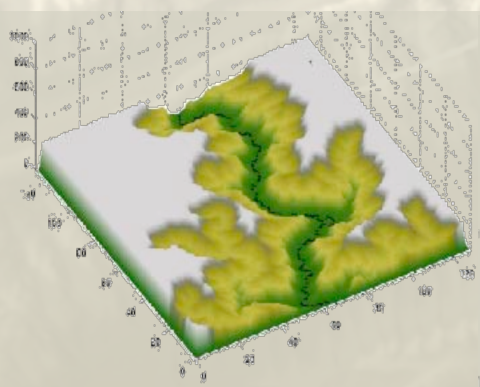
$\Delta\eta$ = elev. diff. due to channel migration
F_{fi} = floodplain fractions
f_{ci}, f_{fi} = exchange fractions
q_{oi} = mean normal overbank sediment export rate
e_i = efficiency coefficient

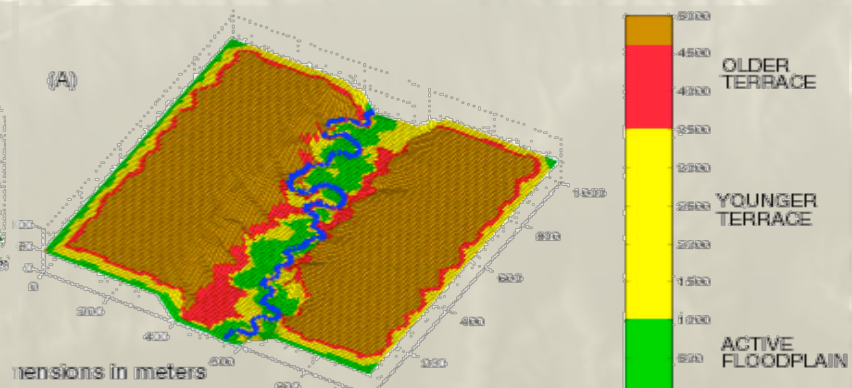
$$(1-\lambda_{pf})B_f\left[f_{fi}\left(\frac{\partial\eta_c}{\partial t}+\sigma_b\right)+\frac{\partial}{\partial t}F_{fi}(\eta_f-\eta_b)\right]=e_iq_{oi}-c\,\Delta\eta f_{ci}$$

CHILD after G. Tucker et al.

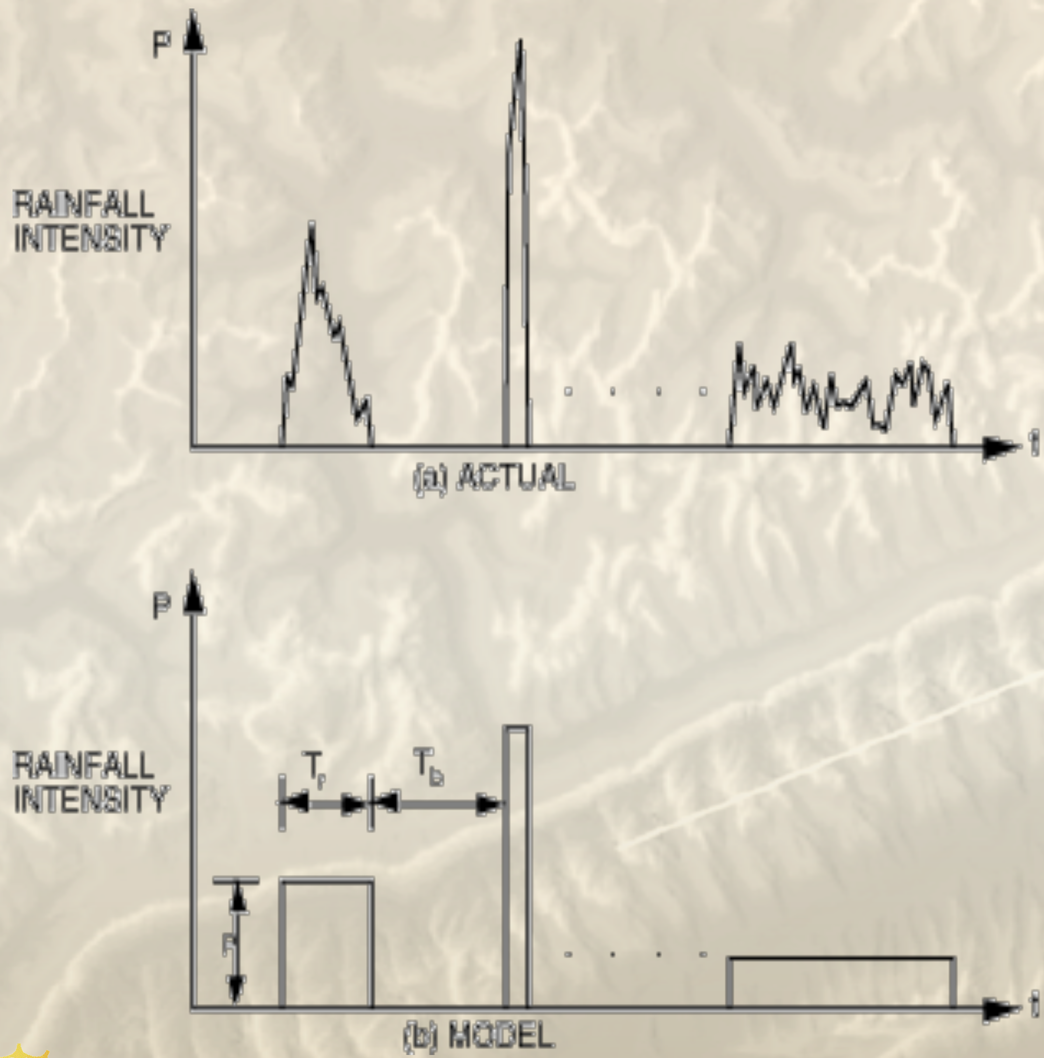
1. CONTINUITY LAWS	2. CLIMATE & HYDROLOGY	3. SOIL CREEP & VEGETATION
Sediment: $\frac{\partial z}{\partial t} = U - \nabla \tilde{q}_s$ *Water:* $-\nabla \tilde{q} = R(x,y,t)$	Stochastic, event-based storm sequence Steady infiltration-excess or saturation-excess runoff	*Creep:* $\tilde{q}_{cr} = -K_d \nabla z$ Optional vegetation dynamics module
4. SHALLOW LANDSLIDING (1) Nonlinear diffusion: $\frac{\partial z}{\partial t} = \frac{\partial}{\partial t}\left(-\kappa(z_x,t)\frac{\partial z}{\partial x}\right)$ (2) Event-based approach $\tilde{q}_{ls} = \frac{K_d \nabla z}{1-(\lvert\nabla z\rvert / S_c)^2}$	5. FLUVIAL TRANSPORT & EROSION / DEPOSITION $\tilde{q}_f = f(q, S, D_{50}, q_s)$ 6 alternative transport laws 4 detachment-transport laws	6. GRIDDING & NUMERICS *Space:* irregular discretization using Delaunay triangulation; finite-volume solution scheme *Time:* event-based with adaptive time-stepping







CHILD model after Tucker
Poisson rainfall model (after Eagleson, 1978)



(Tucker and Bras, 2000)

$$Q = A\,(p - I)$$

$$f(t_b) = 1/T_b \exp(-t_b / T_b)$$

$$f(t_r) = 1/T_r \exp(-t_r / T_r)$$

$$f(p) = 1/P \exp(-p / P)$$

CHILD model after Tucker

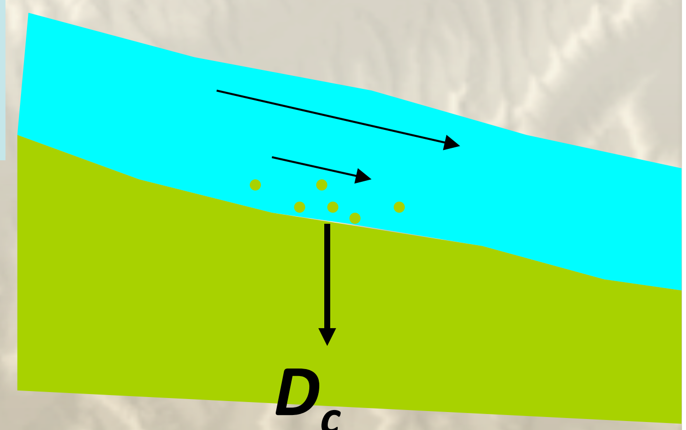
Theory: instantaneous rates

Particle detachment: $E = K_d(\omega - \omega_c)$

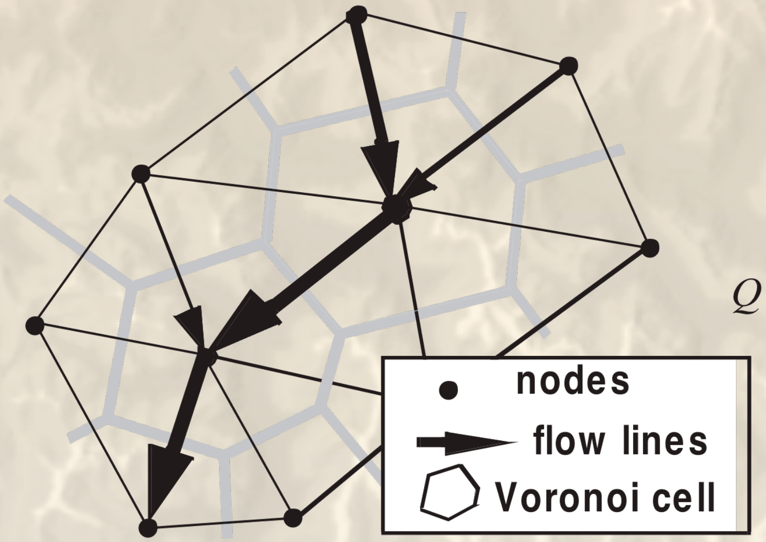
Bedload transport: $C_s = f(\tau_* - \tau_{*c})^{3/2}$

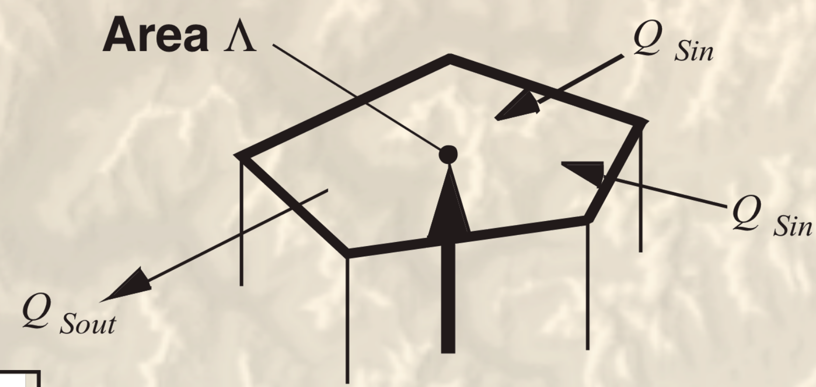
Assumptions:

1. Steady, uniform flow
2. Bankfull width ~ sqrt(discharge)
3. Discharge ~ drainage area



CHILD model after Tucker





RUNOFF TRANSPORT/EROSION:

$$\left.\frac{\partial z}{\partial t}\right|_{water} = \begin{cases} U - E, & E \leq \nabla Q_s \\ U - \nabla Q_s, & E > \nabla Q_s \end{cases}$$

$$\nabla Q_s = C_s - \sum_{j=1}^{N} Q_{S_{jin}}$$

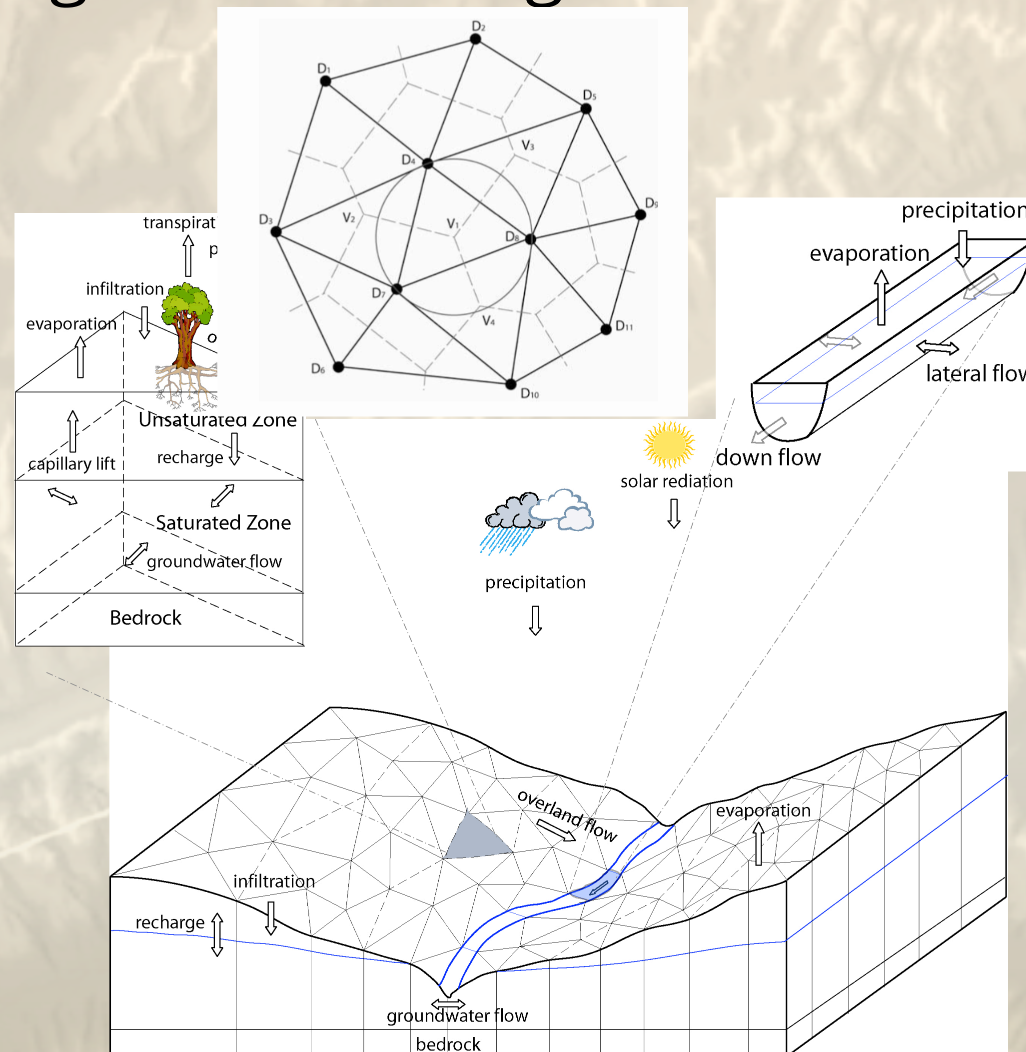
SOIL CREEP TRANSPORT/EROSION:

$$\left.\frac{\partial z_i}{\partial t}\right|_{creep} = K_d \sum_{j=1}^{N_i} \lambda_{ij} \left(\frac{z_j - z_i}{L_{ij}} \right)$$

CHILD
MODEL

A new strategy for integrated hydrologic and landscape modeling after R Slingerland

1) Use GIS tools to decompose horizontal projection of the study area into Delauney triangles (i.e., a TIN)
2) Project each triangle vertically to span the “active flow volume” forming a prismatic volume
3) Subdivide prism into layers to account for various physical process equations and materials
4) Use adaptive gridding



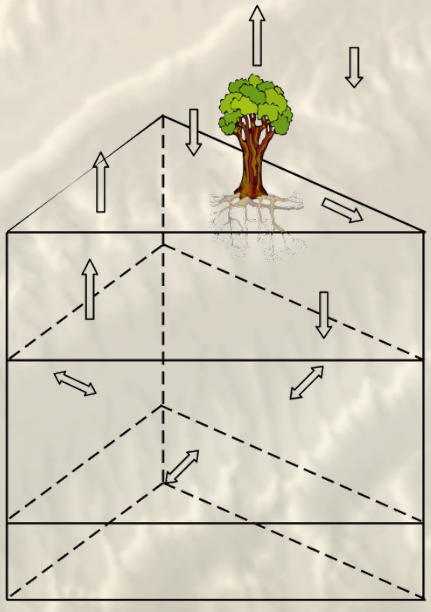
A new strategy for integrated hydrologic and landscape modeling after R Slingerland

5) Employ hillslope & channel equations

6) Use semi-discrete finite volume method to transform PDEs into ODEs
   - For small-scale numerical grids, FVM yields contiuum constitutive relationships
   - For larger grids the method reflects assumptions of semi-distributed approach, but with full coupling of all elements

7) Assemble all ODEs within a prism, each associated with its appropriate layer(s)

- Combine the local system over the domain of interest into a "global system"
- Solve global system by i) SUNDIALS (SUite of Nonlinear and DIfferential/ALgebraic equation Solvers) or ii) PETSc (Portable, Extensible Toolkit for Scientific computation)



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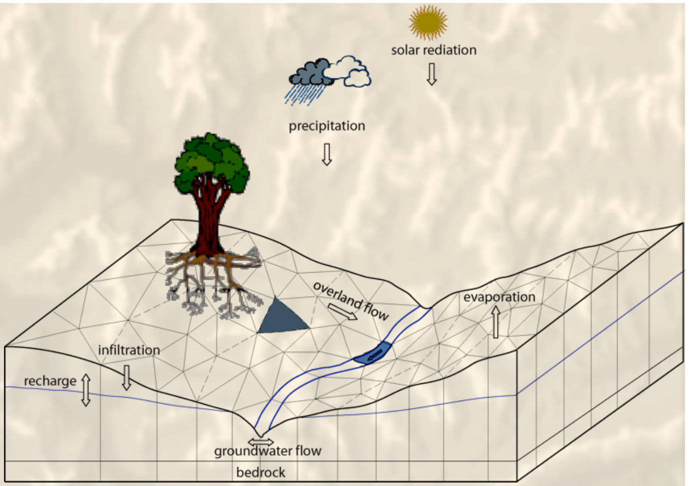
One Possible Realization: PIHMSed (after R Slingerland)



- Canopy-interception: bucket model
- Snowmelt runoff: temp. index model
- Evapotranspiration: Pennman-Monteith Model
- Subsurface unsaturated flow: Richard Model
- Subsurface saturated flow: Richard Model
- Surface overland and channel flows: Saint Venant Model
- Sediment transport and bed evolution: Cao et al. [2002] Model

Landscape Evolution Modeling Summary

Sediment Production: Weathering (regolith, bedrock, physical, chemical); Mass Wasting (continuous vs discrete, creep, thresholds, non-linear, geotechnical properties); Fluvial erosion

Fluvial Transport (erosion, transport, deposition): i) detachment-limited (proportional to sediment load & flow intensity), critical shear stress, hydraulic geometries; ii) transport-limited (proportional to divergence of sediment flux)

Floodplains: Exner Equation with lateral transport rate, bed elevation

CHILD: modular landscape evolution model, event scaling, theory, assumptions, delaunay triangulation grid (TIN)

PHMSed Advanced Model: A new strategy for integrated hydrologic and landscape modeling, PDEs to ODEs, SUNDIALS, PETSc