Vulnerability maps of deltas: quantifying how network connectivity modulates upstream change to the shoreline

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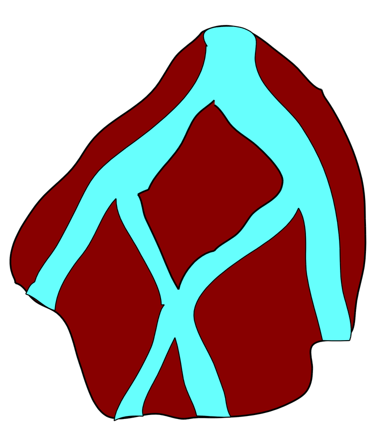
Motivation

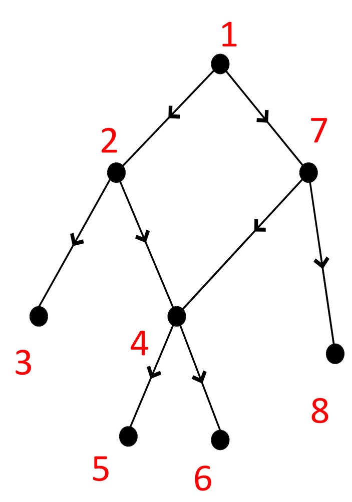
- Deltas are landforms with channels that deliver water, sediment and nutrient fluxes from rivers to oceans or inland water bodies via multiple pathways.
- A formal quantitative framework for studying delta channel network connectivity and transport dynamics, and the response to change is lacking.
- The aim is to develop a framework within which a delta channel network can be studied for:
  (1) Understanding its **connectivity** structure and **flux** transport
  (2) Understanding the response of the system to change: **Vulnerability Assessment**

Representation

Physical Delta



Graph Representation



Algebraic Representation

$$
\begin{array}{c|cccccccc|}
 & j=1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
\end{array}
\quad
\begin{array}{c}
\begin{pmatrix}
0&0&0&0&0&0&0&0\\
1&0&0&0&0&0&0&0\\
0&1&0&0&0&0&0&0\\
0&1&0&0&0&0&1&0\\
0&0&0&1&0&0&0&0\\
0&0&0&1&0&0&0&0\\
1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0
\end{pmatrix}
\end{array}
\begin{array}{l}
i=1\\2\\3\\4\\5\\6\\7\\8
\end{array}
$$

Adjacency Matrix, A

Graph Notation:

- Channels ➔ Links and Junctions ➔ Nodes
- In-degree (d^{in}): Number of links entering a node
- Out-degree (d^{out}): Number of links leaving a node

Algebraic Representation:

- All the topologic information is encoded in the Adjacency matrix (e.g. $d^{in} = \sum_{i=1}^{N} A_{ij}$ and $d^{out} = \sum_{j=1}^{N} A_{ij}$)
- Out-Laplacian and In-Laplacian matrix

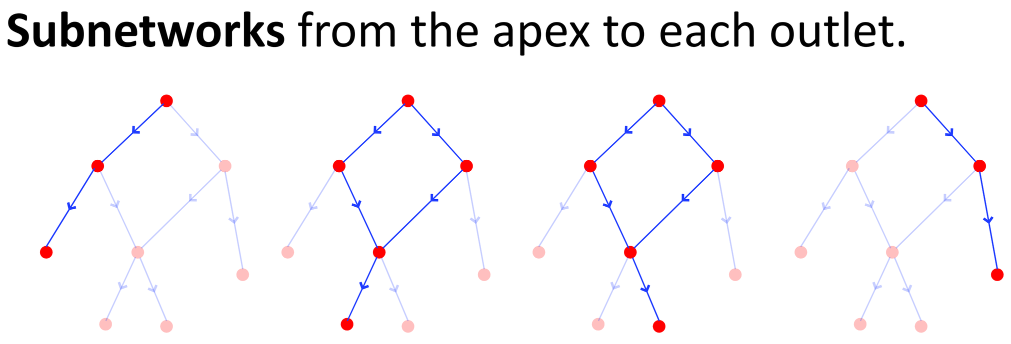
$$
L^{out} = \mathbf{D}^{out} - A = \begin{pmatrix}
2&0&0&0&0&0&0&0\\
-1&2&0&0&0&0&0&0\\
0&-1&0&0&0&0&0&0\\
0&-1&0&2&0&0&-1&0\\
0&0&0&-1&0&0&0&0\\
0&0&0&-1&0&0&0&0\\
-1&0&0&0&0&0&2&0\\
0&0&0&0&0&0&-1&0
\end{pmatrix}
$$

- Weighted Adjacency Matrix:
  Connectivity + strength of the connection.

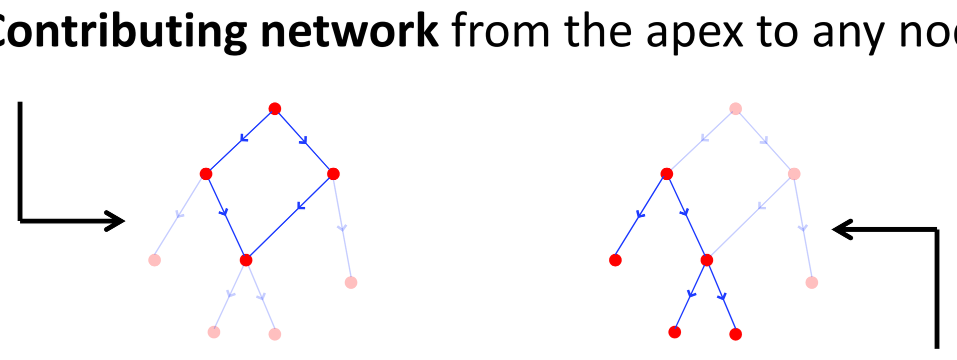
Spectral Graph theory

From the null space of the *proper* Laplacian matrix, we can compute:

1) **Subnetworks** from the apex to each outlet.

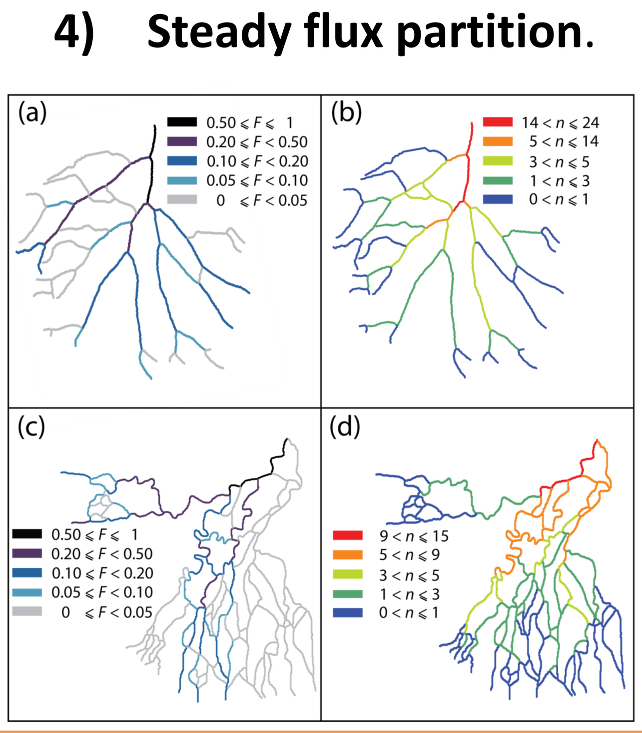


2) **Contributing network** from the apex to any node.



3) **Nourishment network** from any node to the shoreline.

4) **Steady flux partition.**



Steady state flux (a,c) and number of outlets (b,d) that a given link contributes to for (a,b) Wax Lake delta and (c,d) Niger delta. The distribution of flux among the immediate downstream links is proportional to the channel width.

Vulnerability Assessment

Vulnerability is defined in terms of how changes in upstream parts of the system would affect the shoreline fluxes. Questions such as what links of the delta network, if altered, might affect most drastically the distribution of fluxes to the costal outlets, or where an intervention should be imposed to maintain a desired flux to a particular outlet node for land building purposes, are important components of delta management towards sustainability.

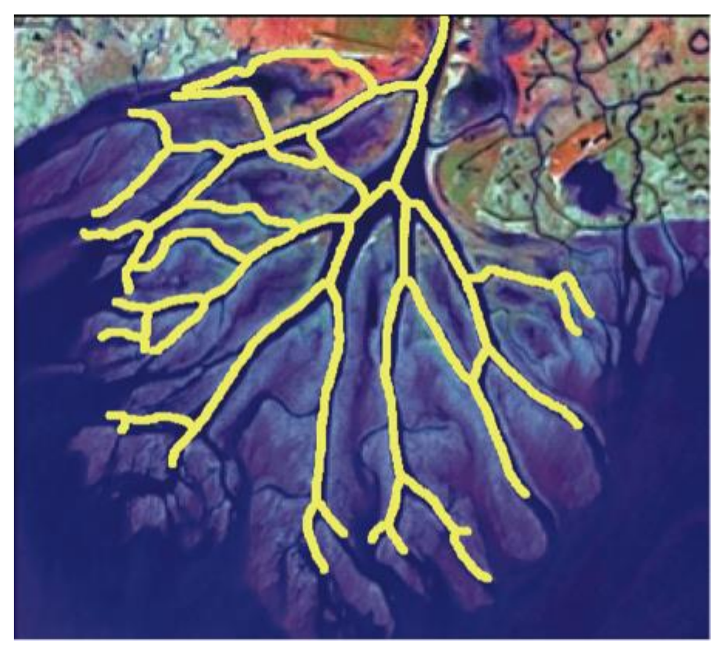
We characterize the flux reduction at outlet *i* with respect to the flux reduction at link (*vu*) by the *local vulnerability:*

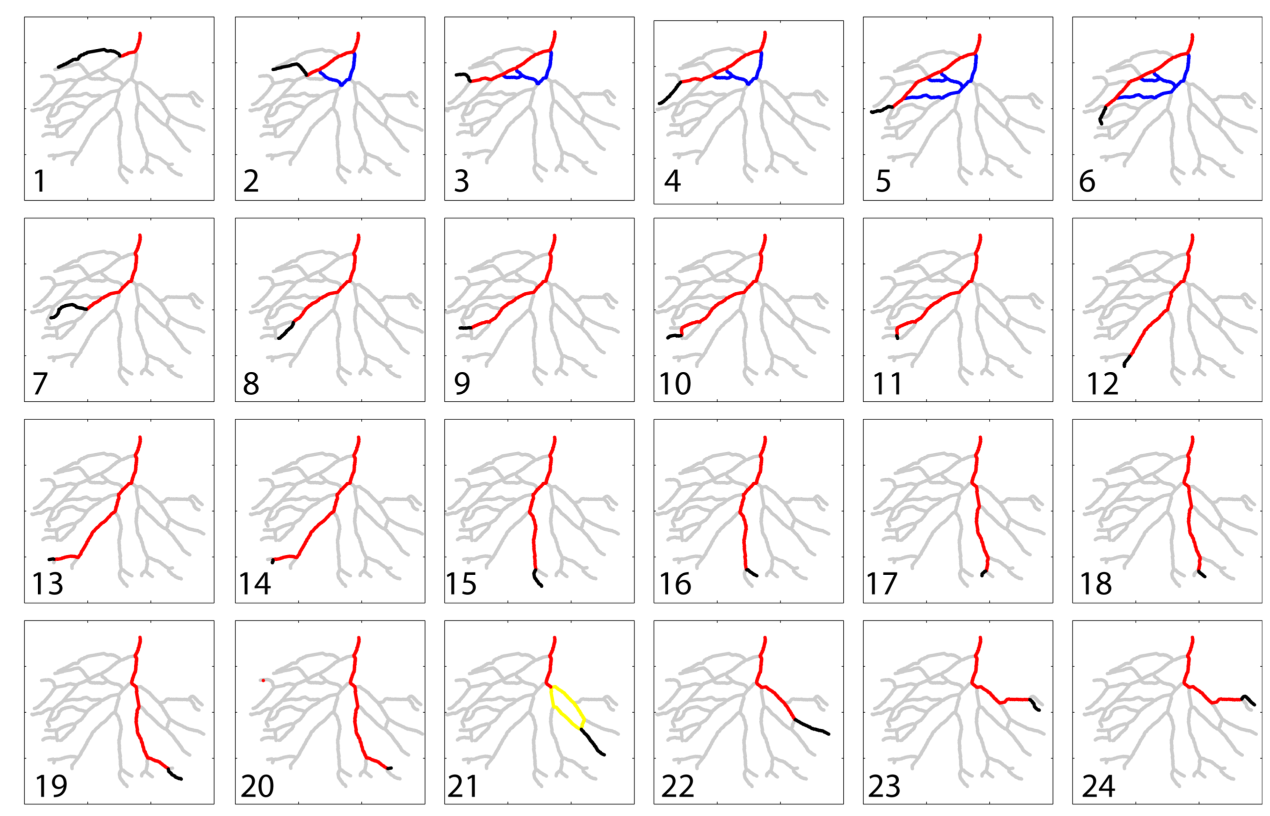
$$V_{uv}^{i} = \frac{p_{uv}^{i}}{C_{uv}^{i}}$$

where p_{uv}^{i} is the fraction of the steady flux in link (*vu*) that drains to outlet *i* and C_{uv}^{i} is related to the ratio of the steady flux at the outlet F_i and the steady flux at the link (*vu*) F_{uv}.

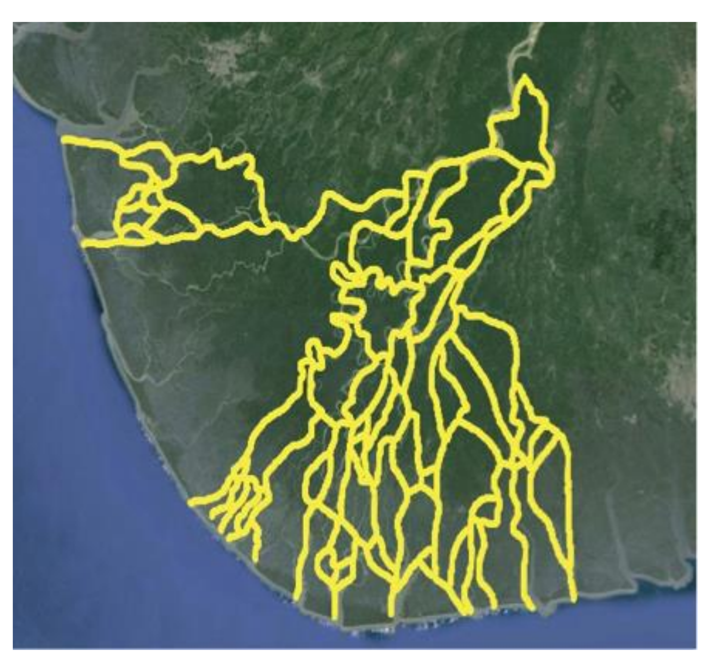
Vulnerability maps

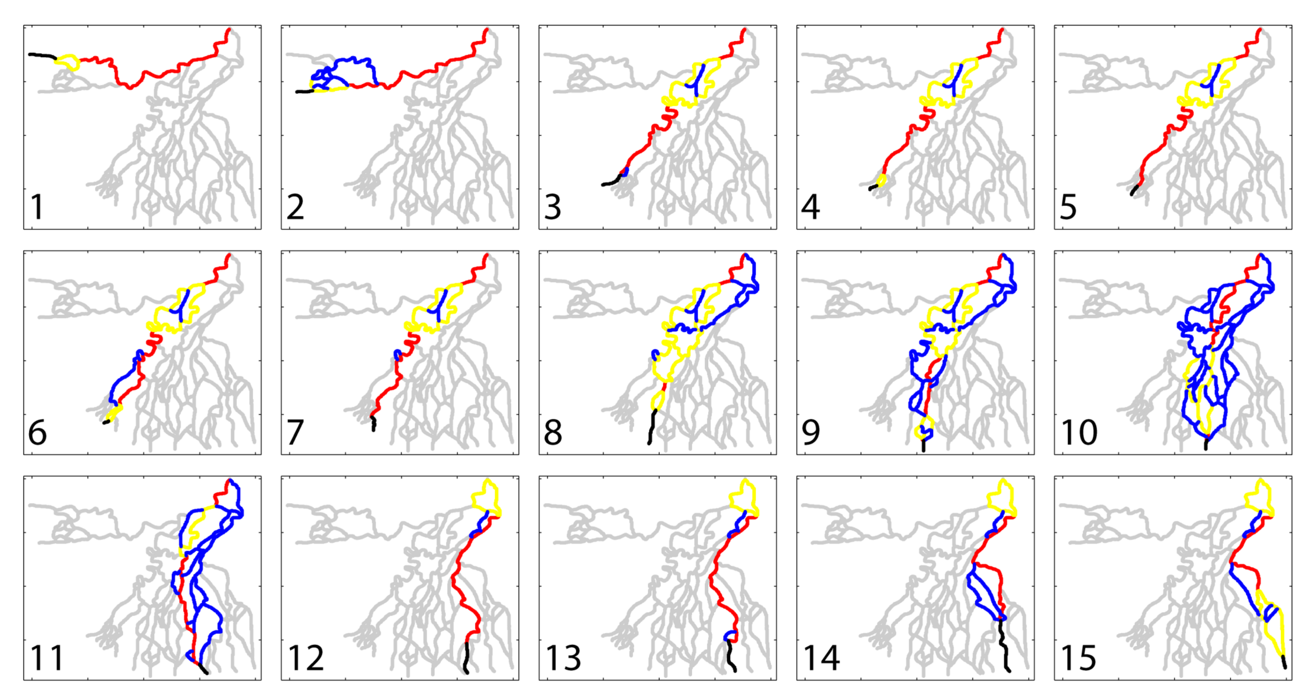
1. Wax Lake Delta





2. Niger Delta





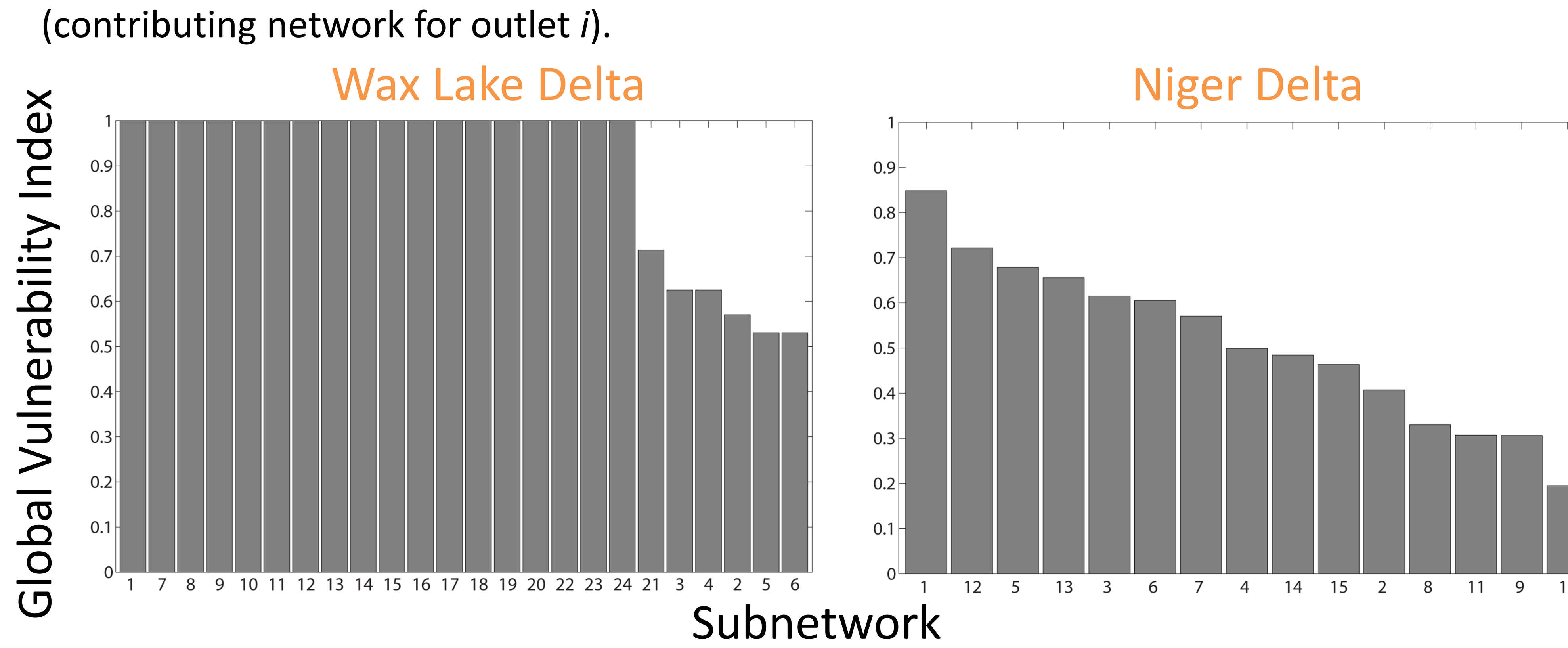
Each panel highlights the contributing network for a single outlet. Red, yellow, and blue links represent high $\left(V_{uv}^{i} > 0.75\right)$, medium $\left(0.25 < V_{uv}^{i} \le 0.75\right)$, and low $\left(V_{uv}^{i} \le 0.25\right)$ values of the local vulnerability index. Shoreline outlets are shown in black.

Global Vulnerability

The *global vulnerability* of outlet *i* is defined as the average of the local vulnerabilities over all links in the subnetwork that drains to outlet *i* (edges of graph *G*):

$$V_i = \frac{1}{|E_i|} \sum_{(vu)\in E} V_{uv}^{i},$$

Where $|E_i|$ denotes the number of links in the subnetwork that drains to outlet *i* (contributing network for outlet *i*).



Average vulnerability indices V_i of contributing subnetworks arranged in decreasing order for Wax Lake (left panel) and Niger (right panel) deltas. Each subnetwork is labeled by its outlet number. A subnetwork composed of a single path from the apex to the outlet is more vulnerable to a flux change than a subnetwork that includes several interconnected paths.

References:

1. Tejedor, A., Longjas, A., Zaliapin, I. and Foufoula-Georgiou, E. (2015), Delta channel networks: 1. A graph-theoretic approach for studying connectivity and steady state transport on deltaic surfaces. Water Resour. Res.. Accepted Author Manuscript. doi:10.1002/2014WR016577
2. Tejedor, A., Longjas, A., Zaliapin, I. and Foufoula-Georgiou, E. (2015), Delta channel networks: 2. Metrics of topologic and dynamic complexity for delta comparison, physical inference, and vulnerability assessment. Water Resour. Res.. Accepted Author Manuscript. doi:10.1002/2014WR016604

Acknowledgements:

This work is part of the BF-DELTAS project on “Catalyzing action towards sustainability of deltaic systems” funded by the Belmont Forum and the forthcoming 2015 “Sustainable Deltas Initiative” endorsed by ICSU. The research is also supported by the FESD Delta Dynamics Collaboratory EAR-1135427 and NSF grant EAR- 1209402 under the Water Sustainability and Climate Program. The data in our article can be provided upon request.