Developing and evaluating algorithms for lateral erosion of bedrock channels in landscape evolution models
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Introduction and Motivation
Theory for the vertical incision of bedrock channels is well established and is widely implemented in our current generation of landscape evolution models. However, existing models in general do not seek to implement rules for lateral migration of bedrock channel walls. This is problematic, as geomorphic problems such as terrace formation and hillslope-channel coupling depend heavily on accurate simulation of valley widening. We have begun to develop and implement a theory to represent the lateral migration of bedrock channel walls in a landscape evolution model. In a real channel, rates of lateral channel wall erosion depend on the shear stress directed at the channel walls and the resisting strength of the bedrock. Shear stress directed at the channel walls is a function of channel curvature, discharge magnitude, and sediment supply, which provides tools to abrade the walls and cover to shield the bed from erosion. Applying empirically based simple rules for lateral erosion in a gridded model results in wider valleys and more dynamic stream networks, especially in landscapes with weak bedrock. To our knowledge, these efforts represent the first attempt to incorporate lateral erosion in a network-based landscape evolution model.

Model Algorithm
We use the Landlab modeling environment to abstract these rules for lateral erosion of channel walls to a landscape evolution model.

<table>
<thead>
<tr>
<th>Boundary node</th>
<th>No erosion</th>
<th>Lateral erosion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lateral node</td>
<td>Primary cell</td>
<td>Lateral cell</td>
</tr>
</tbody>
</table>

- Calculate \( \frac{dz}{dt} \) at each node (Davy and Lague, 2009)
- Calculate \( Q_s/Q_t \) at each node
- Determine which lateral neighboring node gets eroded (figure to left)
- Determine whether lateral erosion occurs: if \( Q_s/Q_t \) is larger than a randomly drawn number between 0 and 1, neighboring cell gets all of the erosion of the primary cell (figure to left)
- Calculate stable time step size
- Erode landscape

**Lateral Erosion Theory**

The series of figures to the right shows the lateral migration of a stream over 6000 years after the model has reached steady state. In this model \( K=0.1, \alpha = 1 \). The stream is indicated by the black lines. The stream sweeps over the lower half of the model domain, smoothing the topography and creating a wide bedrock valley.

The figure below shows results from 49 model runs with a range of \( K \) values (1e-4 to 1e-1) and \( \alpha \) values (0.8-1.6). The figure shows the maximum number of times the main channel in the models shifted by at least 1 cell in 10,000 years of model time vs. the ratio of \( \alpha/K \).

**Land surface elevation:**
\[
\frac{dz}{dt} = \alpha Q_s/A - K \alpha \Gamma^{3/2} S + U
\]

**Transport-limited (\( \alpha > 1 \))/ detachment-limited (\( \alpha < 1 \))**

\[
\alpha = \frac{\rho s}{d_t + R}
\]

**Transport capacity:**
\[
Q_{s/t} = K \alpha T^{3/2}/S/\alpha
\]

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References: Johnson, J.P., and Whipple, K.X. (2010), Evaluating the controls of shear stress, sediment supply, alluvial cover, and channel morphology on experimental bedrock incision rate. JGR, 115, F02018.