A New Reduced-Complexity Channel-Resolving Model for Delta Formation

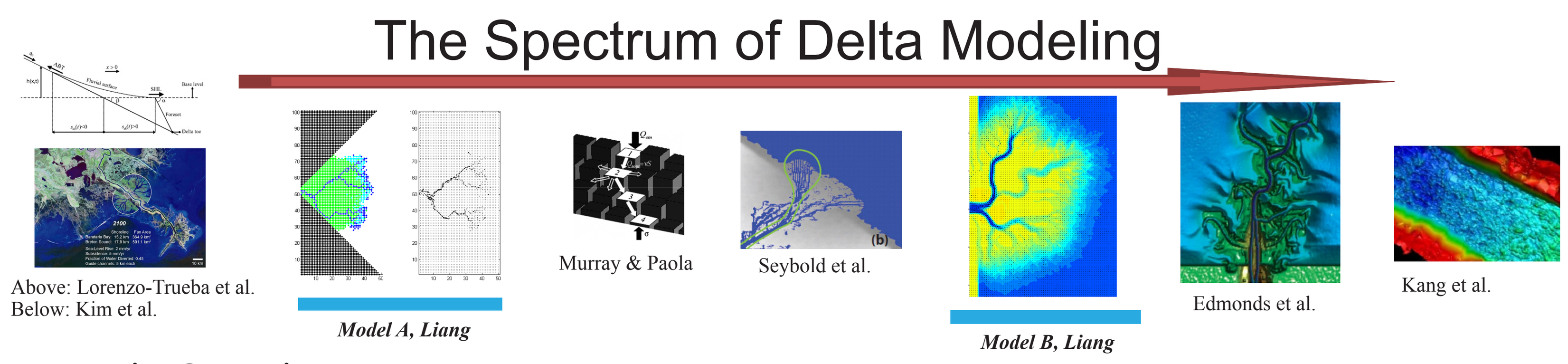
[1] Man Liang, [1] Vaughan Voller, [2] Chris Paola

[1] St. Anthony Falls Laboratory, Dept. of Civil Engineering, University of Minnesota, Minneapolis, MN

[2] St. Anthony Falss Laboratory, Dept. of Earth Sciences, University of Minnesota, Minneapolis, MN

BACKGROUND

The Spectrum of Delta Modeling



Basic Questions:

- Reduced-Complexity : what is the right/optimized level of details?
- Always ask: what is the feature you're interested in.
- How do you know the results are good or not?

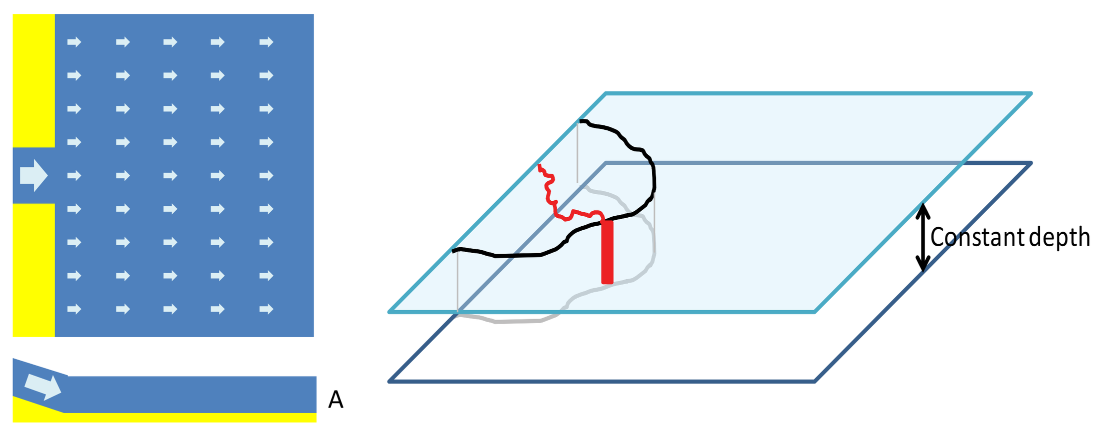
In my work, I try to model deltas at this level:

- Water flow field is resolved (or approximated with enough resolution to guide sediment transport)
- Channels emerges and changes automatically.

MODEL

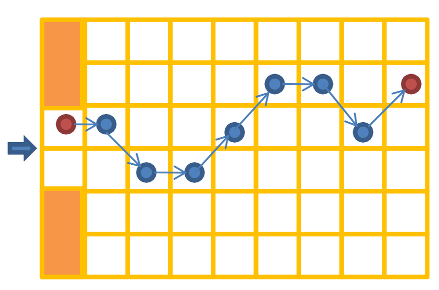
1. Setup

Current setup of the calculation domain: Constant basin depth; Normal flow channel inlet; Constant sea-level



Also constant input of water and sediment. Constant Cf = 0.01.

2. Flow



Shallow water equation

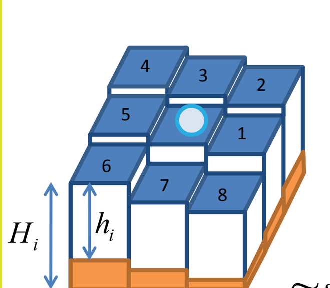
$$\nabla\cdot(h\vec{u})=0 \qquad h\vec{u}\nabla\vec{u}=-gh\nabla H-C_f\vec{u}|\vec{u}|$$

If Froude numder is small:

$$0=-gh^3\nabla H-C_f\vec{q}|\vec{q}| \quad \text{or} \quad |\vec{q}|=\sqrt{\frac{gh^3|\nabla H|}{C_f}}$$

Calculate probability of all eight walking directions with two sets of rules

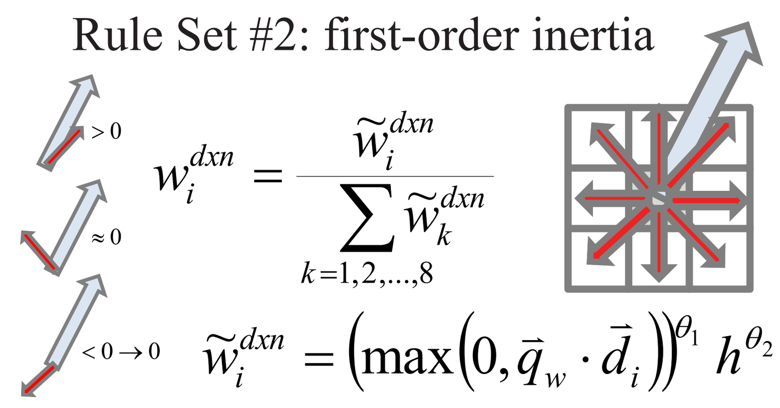
Rule Set #1: low-Fr shallow water



$$w_i^{sfc}=\frac{\tilde{w}_i^{sfc}}{\sum_{k=1,2,\dots,8}\tilde{w}_k^{sfc}}$$

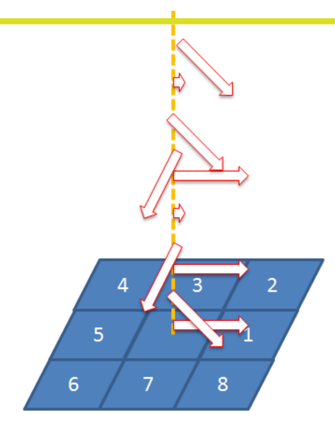
$$\tilde{w}_i^{sfc}=\sqrt{\frac{gh_i^3|\nabla H|_i}{C_f}}\Big/A$$

Rule Set #2: first-order inertia



$$w_i^{dxn}=\frac{\tilde{w}_i^{dxn}}{\sum_{k=1,2,\dots,8}\tilde{w}_k^{dxn}}$$

$$\tilde{w}_i^{dxn}=\left(\max\left(0,\vec{q}_w\cdot\vec{d}_i\right)\right)^{\theta_1}h^{\theta_2}/A$$

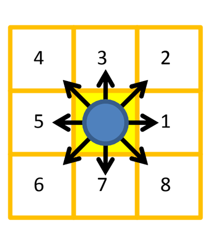


Take the linear combination of the two sets:

$$w_i=\gamma w_i^{sfc}+(1-\gamma)w_i^{dxn}$$

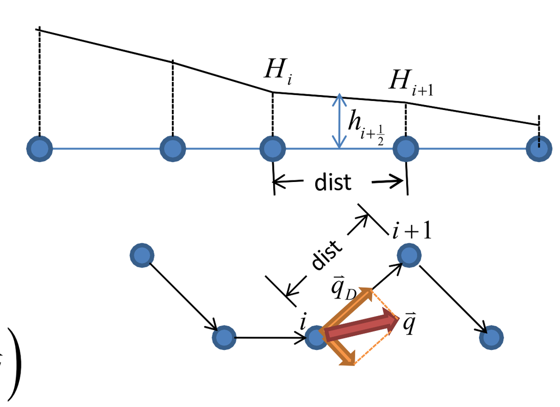
Update flow vectors

$$q_w=(1-\varpi_{qw})q_w^{old}+\varpi_{qw}q_w^{new}$$



3. Surface

Along the walk path of each water packet, use the same formula derived from low-Fr shallow water momentum equation



$$\nabla H=\frac{C_f\vec{q}|\vec{q}|}{gh^3}$$

$$H^{n-1}-H^n=|\vec{d}|\cos\theta|\nabla H|=\vec{d}\cdot\nabla H=\frac{C_f|\vec{q}|}{gh^3}(\vec{d}\cdot\vec{q})$$

Smooth the new surface by applying a diffuser by averaging out among neighbors

$$\tilde{H}_{i,j}^{new}\Rightarrow C_{smooth}*H_{i,j}^{new}+0.25*(1-C_{smooth})*\left(H_{i-1,j}^{new}+H_{i,j-1}^{new}+H_{i+1,j}^{new}+H_{i,j+1}^{new}\right)$$

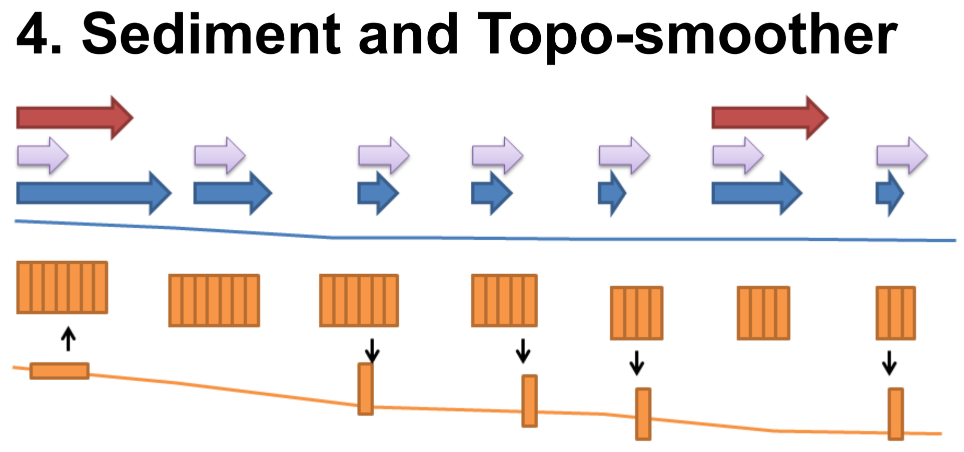
Update the free surface with under-relaxation

$$H=(1-\varpi_{sfc})H+\varpi_{sfc}H^{new}$$

Then update water depth and calculate water velocity vectors:

$$h=H-\eta \qquad \vec{u}_w=\min(u_{\max},\frac{\vec{q}_w}{h})$$

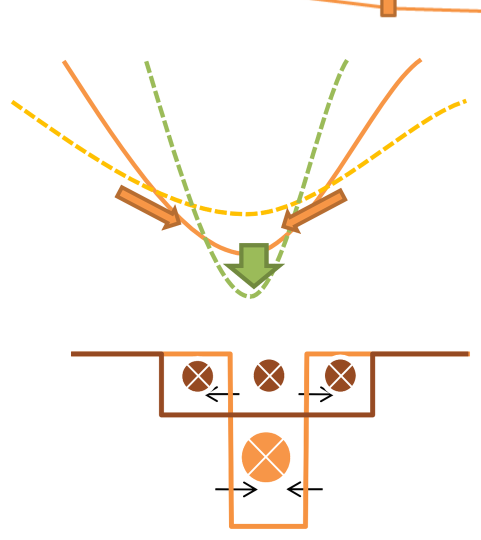
4. Sediment and Topo-smoother



A very crude set of rules for sediment deposition and erosion:

$$Q_{dep}=\lambda Q_{res}\frac{U_{dep}-U}{U_{dep}}$$

$$Q_{ero}=Q_{p\_sed}\frac{U-U_{ero}}{U_{ero}}$$



A simple smoother is applied to avoid "over-deepening" of flow. First, identify potential "digging" nodes by calculating:

$$\beta=\min\left(\beta_{\max},\frac{|\vec{q}_w|}{mean\left(|\vec{q}_w|_{neighbors}\right)}\right)$$

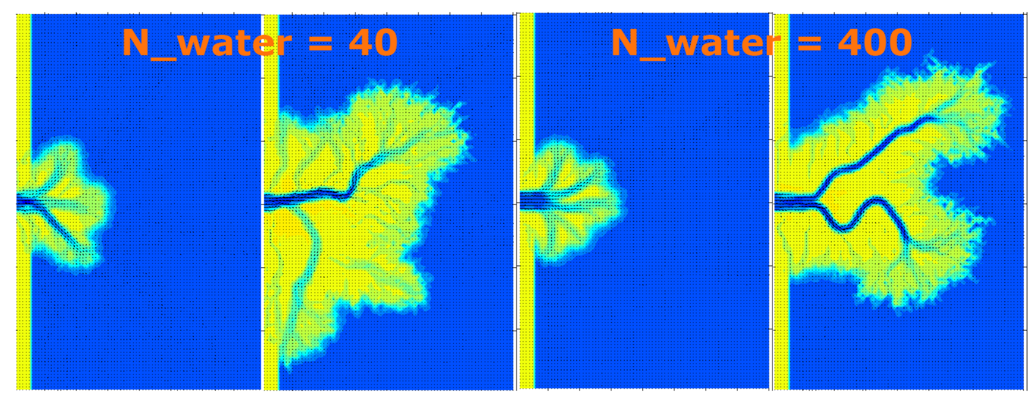
Expand flow and smooth bed topography at nodes with beta value above threshold.

$$q_w=\frac{\beta_{\max}-\beta}{\beta_{\max}}q_w+\frac{\beta}{\beta_{\max}}mean(q_w)_{neighbors} \qquad \eta=\frac{\beta_{\max}-\beta}{\beta_{\max}}\eta+\frac{\beta}{\beta_{\max}}mean(\eta)_{neighbors}$$

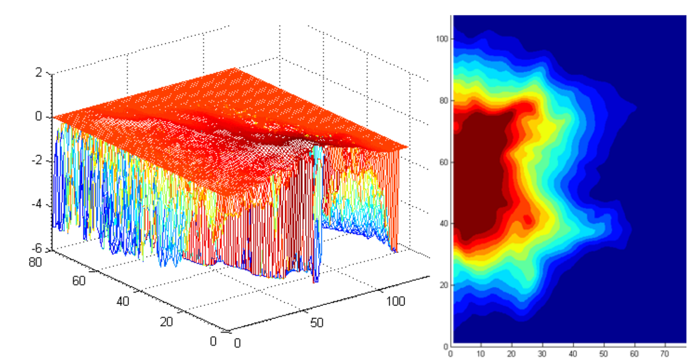
CURRENT RESULTS

Important values of the base case (model is NOT dimensionless):

- Cell size: 50m*50m
- Number of nodes: 80*120
- Channel width: 6 nodes
- Upstream inflow velocity: 1m/s
- Upstream inflow depth: 5m
- Basin depth: 5m
- Constant Cf: 0.01
- Number of water packages per timestep: 300
- Number of sediment packages per timestep: 100
- Deposition velocity threshold: 0.6m/s
- Erosion velocity threshold: 1.2m/s
- Sediment package volume: 10% unit column
- Removal rate from sediment package: 10% per step
- Gamma: 0.1

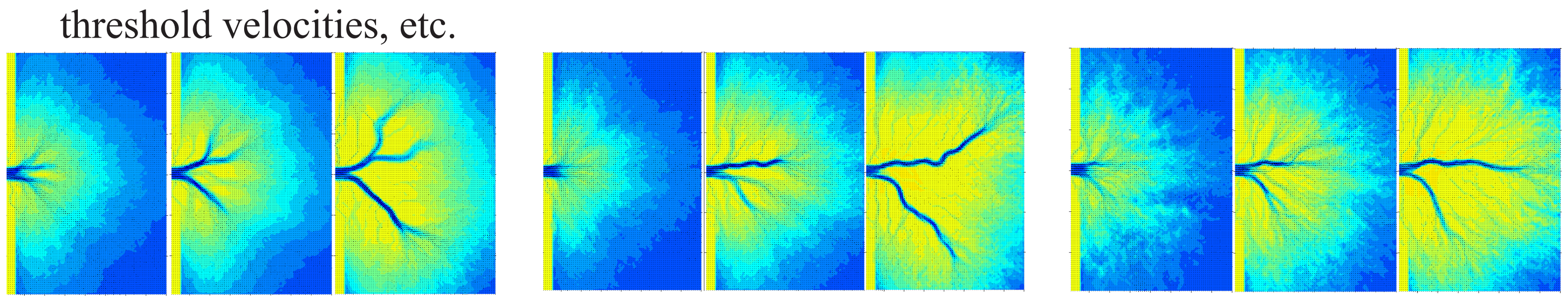


Above: Effects of changing # of water packages

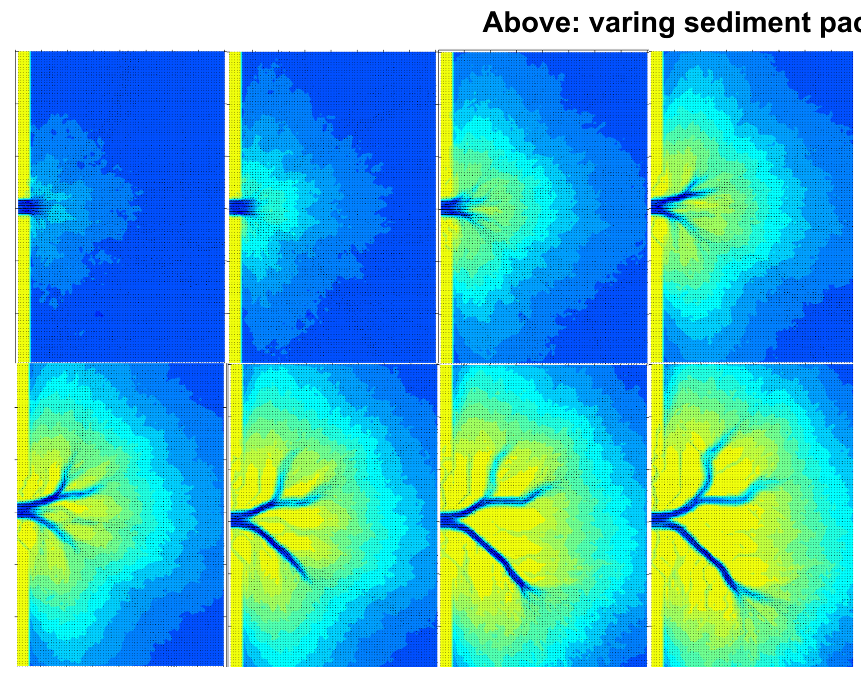


Above: free-surface profile

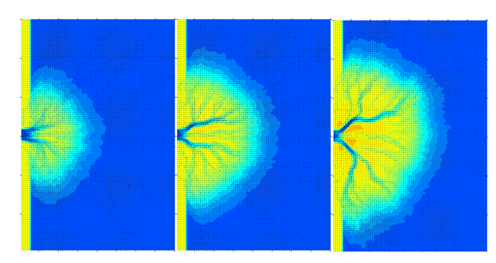
* The following results are all variations from the base case, with different cell size, depth, threshold velocities, etc.



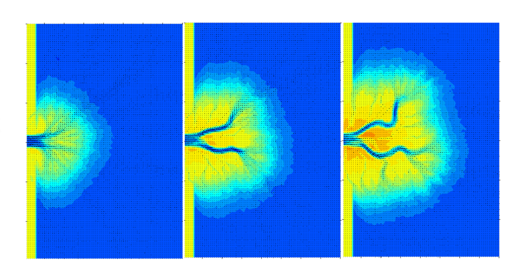
Above: varing sediment package volume (left to right: 0.1, 0.3, 0.99)



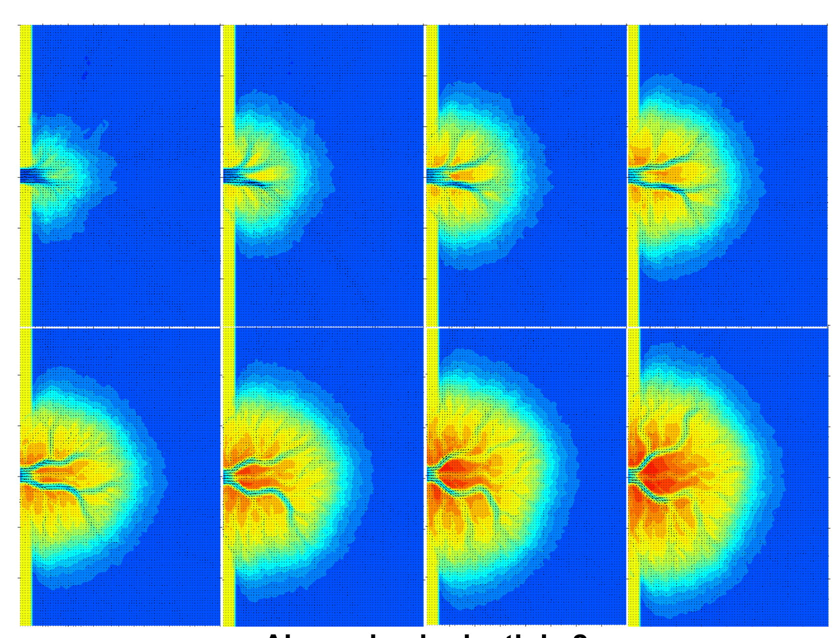
Above: sediment removal rate (2%)



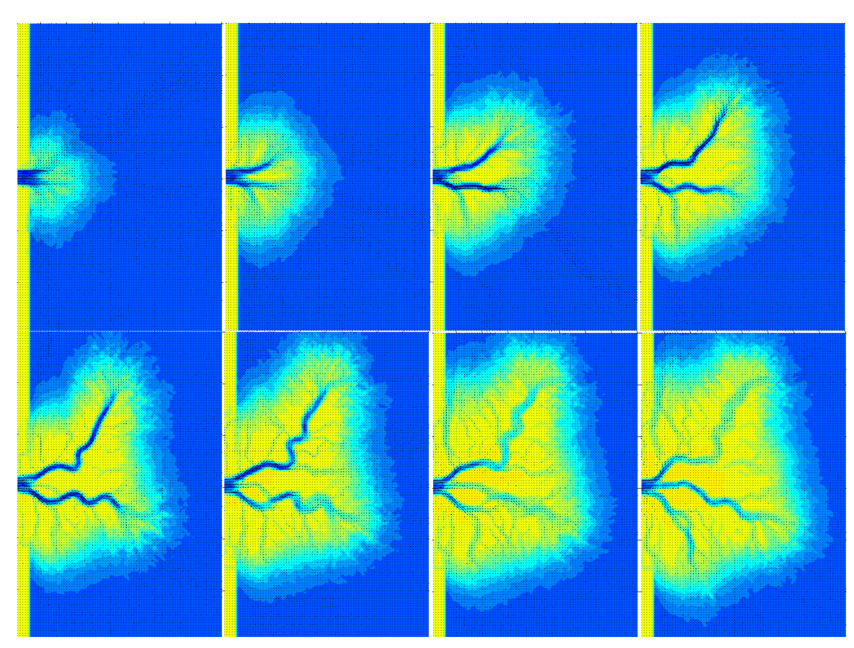
Left: basin depth is 5m, but increase input Fr#



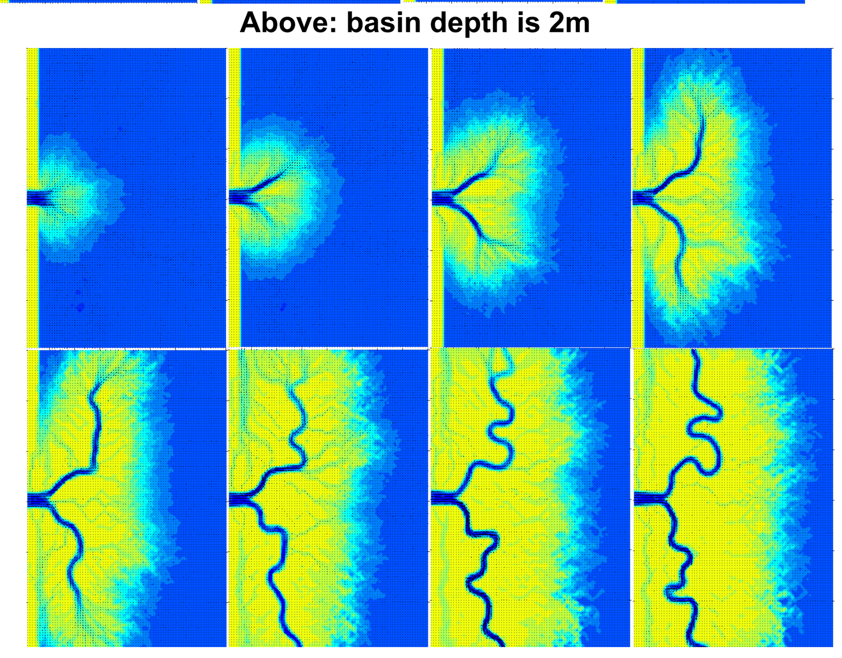
Right: basin depth is 2m, but decrease input Fr#



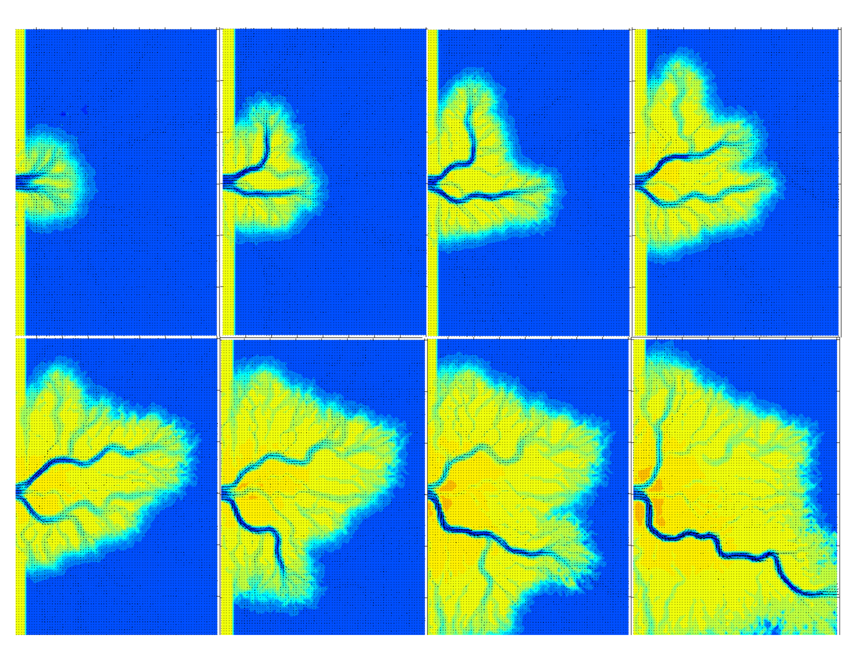
Above: basin depth is 2m



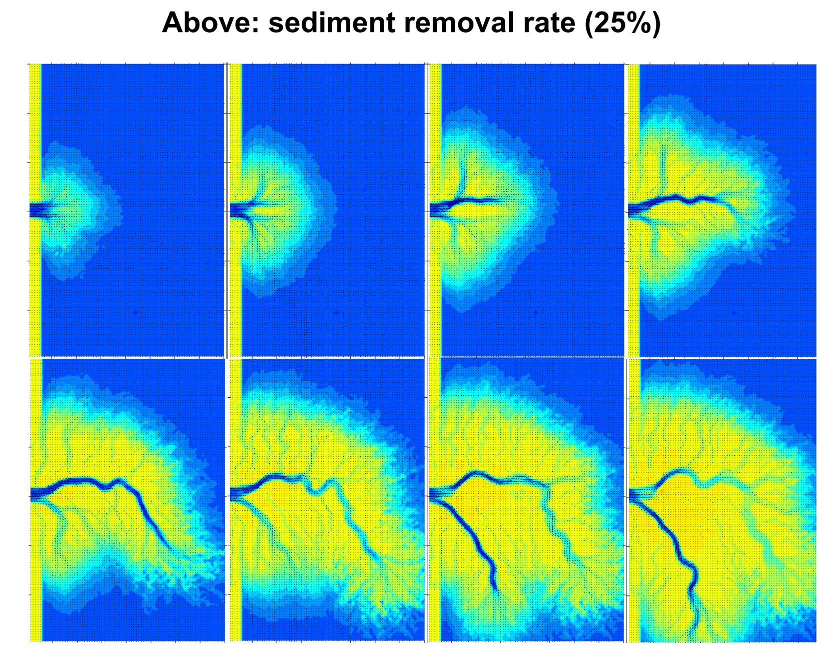
Above: base case



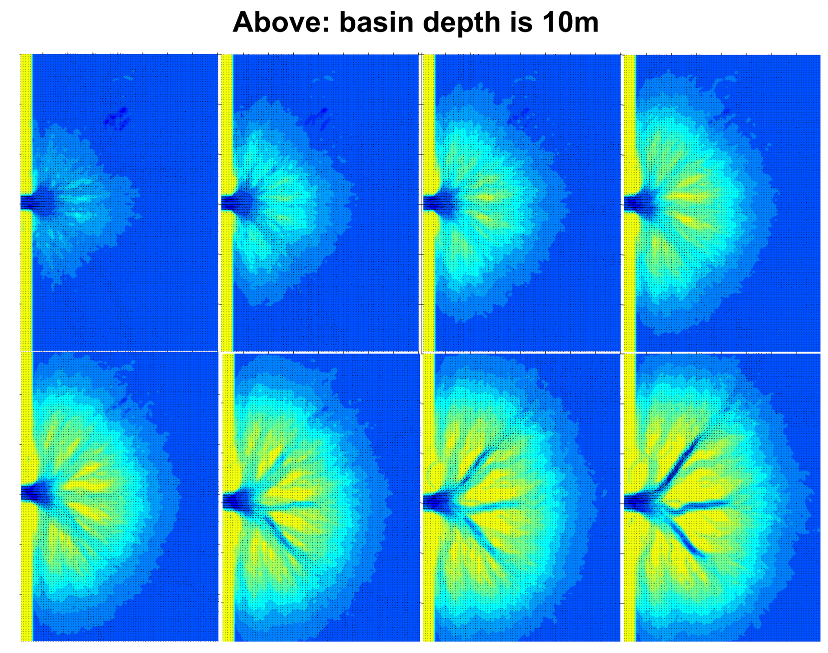
Above: basin depth is 10m



Above: sediment removal rate (25%)

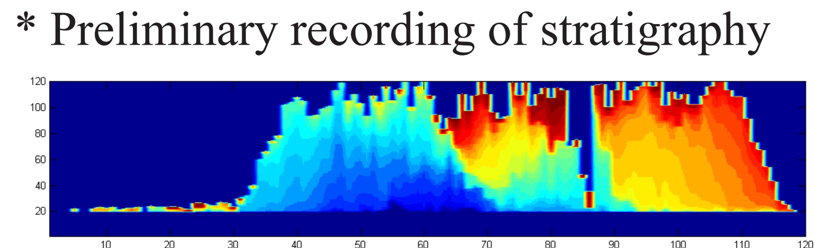


Above: another base case

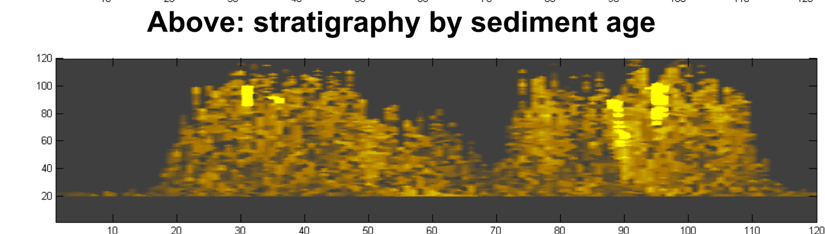


Above: deposition threshold is 0.2m/s

* Preliminary recording of stratigraphy

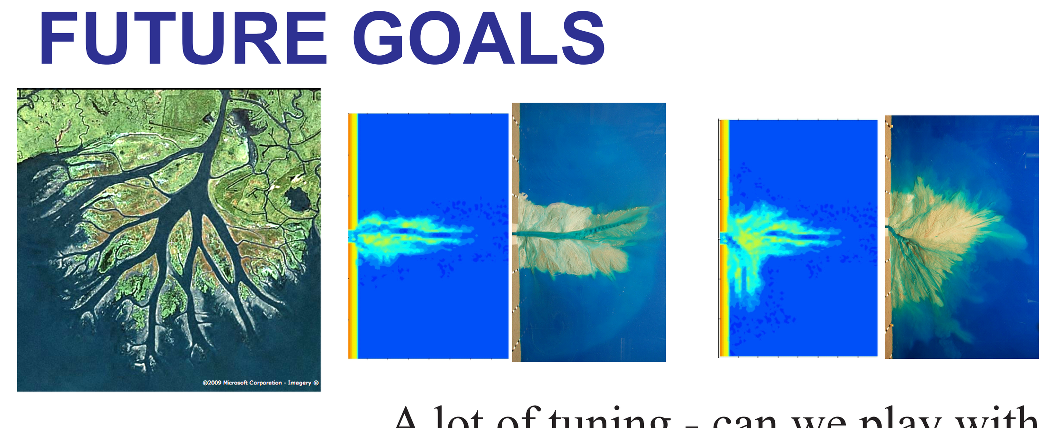


Above: stratigraphy by sediment age



Above: stratigraphy by flow volocity at depotion (a "false" grain size)

FUTURE GOALS



Flow or Sediment? - a steady dendritic pattern is missing.

A lot of tuning - can we play with parameters and match case by case?

And: methods to analyze models!

To add (by rules or parameters) Wave, vegetation, sea-level, floods... etc.