A turbulence-resolving Eulerian two-phase model for coastal sediment transport applications

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**Motivation**

- The coastal zone is a very important **human habitat** of high ecological diversity and critical economic importance. Over 38% of the world’s population lives within 100 km of the coast or estuaries (1995, from *World Resource Institute*).

- **Accelerated sea-level-rise**, due to the global climate change during the past century, makes coastal zone more vulnerable to natural hazards such as storm surges.

- Studying Sediment transport is essential for **beach erosion and recovery**, however, field evidences suggest that the mechanisms critical in major storm condition were not parameterized properly by state-of-the-art sediment transport models (Foster et al., 2006; Cheng et al., 2016).
Eulerian Two-phase Model for Sediment Transport

Why two-phase model?

- Using two-phase flow equations with closures on interphase momentum transfer (e.g. drag), particle stresses and turbulence-sediment interaction, full profiles of transport can be obtained.
- Conventional bedload/suspended load assumptions are not necessary.
- SedFOAM (Cheng et al., 2016, Coastal Engineering, under revision), an Eulerian two-phase model based on a $k$-$\varepsilon$ turbulence model, is publically available via Community Surface Dynamics and Modeling System (CSDMS) model repository maintained by GitHub.
- A CSDMS clinic was hosted on SedFOAM in 2015: https://csdms.colorado.edu/wiki/CSDMS_2015_annual_meeting_Tom_Hsu
SedFOAM (Cheng et al., *Coastal Eng.*, in revision)

- SedFOAM has been validated with oscillatory sheet flow experiment (O’Donoghue and Wright, 2004)
- It has been used by many researchers for different applications such as scour problems.

**Figure.** Modeled (solid curve) and measured (symbols) concentration profiles at four instants in M5010 experiment of O’Donoghue and Wright (2004)

**Movie.** Scour downstream an apron, $u_f=3.69$ cm/s, $d=0.25$ mm
Why turbulence-resolving:

- In turbulence-averaged models, closure of turbulence-sediment interactions is highly empirical.
- For oscillatory sheet flow:
  - Turbulence-averaged two-phase model works reasonably well for medium to coarse sand.
  - Most existing models including turbulence-averaged two-phase model fail to predict enhanced transport thickness for fine sand (i.e., sand with $d_{50} < 0.15$ mm)

How can we do a better job for fine sand?

Hypothesis:

1. Turbulence-sediment interactions are critical for fine sand.
2. Typical wave conditions in coastal environment are transitionally turbulent (especially during flow reversal).

A turbulence-resolving simulation approach is needed
Filtered Eulerian two-phase flow equations:

- **Mass Conservation Equations**
  \[
  \frac{\partial \rho^f (1 - \bar{\phi})}{\partial t} + \frac{\partial \rho^f (1 - \bar{\phi}) \bar{u}_i^f}{\partial x_i} = 0, \\
  \frac{\partial \rho^s \bar{\phi}}{\partial t} + \frac{\partial \rho^s \bar{\phi} \bar{u}_i^s}{\partial x_i} = 0,
  \]
  - $\rho^f, \rho^s$: fluid and sediment density
  - $\bar{\phi}$: filtered sediment concentration
  - $\bar{u}_i^f, \bar{u}_i^s$: filtered fluid and sediment velocities

- **Momentum Equations**
  \[
  \frac{\partial \rho^f (1 - \bar{\phi}) \bar{u}_i^f}{\partial t} + \frac{\partial \rho^f (1 - \bar{\phi}) \bar{u}_i^f \bar{u}_j^f}{\partial x_j} = -(1 - \bar{\phi}) \frac{\partial \bar{p}^f}{\partial x_i} + \frac{\partial \bar{\tau}_{ij}^f}{\partial x_j} + \rho^f (1 - \bar{\phi}) g \delta_{ij} + \bar{M}_{ij}^{fs}, \\
  \frac{\partial \rho^s \bar{\phi} \bar{u}_i^s}{\partial t} + \frac{\partial \rho^s \bar{\phi} \bar{u}_i^s \bar{u}_j^s}{\partial x_j} = -\bar{\phi} \frac{\partial \bar{p}^s}{\partial x_i} - \frac{\partial \bar{p}^s}{\partial x_i} + \frac{\partial \bar{\tau}_{ij}^s}{\partial x_j} + \rho^s \bar{\phi} g \delta_{ij} + \bar{M}_{ij}^{sf}.
  \]
  - Fluid and sediment momentum coupling are dominated by **drag force**: $M_{ij}^{fs}, M_{ij}^{sf}$.
  - Fluid stresses are modeled with sub-grid turbulence model (Germano, 1991).
  - Particle stresses due to collisions and frictions are modeled with kinetic theory for granular flow and frictional stress models (Ding and Gidaspow, 1990; Srivastava and Sundaresan, 2003).
Sub-grid model:

**Turbulence-resolving:** Eulerian two-phase equations are solved in 3D with a domain size sufficiently larger than the largest eddies and high numerical resolution (on the order of grain size).

Large eddies/structures are directly **resolved**, and effect of small eddies/structures on large scale motions are **modeled** with sub-grid closures:

**For fluid and particle sub-grid stress:**

\[
\frac{u_i' u_j'}{\Delta} - \frac{\bar{u}_i' \bar{u}_j'}{\Delta} = -2s_{sgs}^{ij} S_{ij}'
\]

\[
\nu_s' = C_s (\Delta)^3 |S'|
\]

where, \( |S'| = \sqrt{2S_{ij}' S_{ij}'} \)

\( \Delta = (\Delta_x \Delta_y \Delta_z)^{1/3} \)

The coefficient \( C_s \) is determined using a dynamic procedure (Germano, 1991), and similar closures are used for sediment sub-grid stress.

**For sub-grid contribution of drag:**

Ozel et al. (2013): mesoscale structures of sediment particles such as **streamer** and **clusters**, may not be resolved by the mesh size, and they can have a dramatic effect on the overall sediment dynamics.

The effect of unresolved mesoscale structure can be accounted by a sub-grid drag correction:

\[
\phi(u_i' - u_i^*) = (1 + K) \phi(\bar{u}_i' - \bar{u}_i^*)
\]

The coefficient \( K \) depends on grid size and sediment concentration, and it’s determined by using a dynamic procedure (Ozel et al., 2013).
Model Validation - Sheet flow in steady channel flow

LEGI experiment (Revil-Baudard et al., 2015, *JFM*):

Colocated two-component velocities \((u, w)\) and sediment concentration \((\phi)\) are measured.

Flow condition: \(u_*=5 \text{ cm/s}, \ h=0.13 \text{ m}\)

Sediment properties: \(s=1.192, \ d=3 \text{ mm}, \ W_s=5.59 \text{ cm/s}\)

Shields parameter: \(\theta = \frac{u_*^2}{(s-1)gd} \approx 0.5\)

Numerical simulation:

\[
L_x = 0.844 \text{ m} \quad L_y = 0.422 \text{ m} \quad L_z = 0.175 \text{ m} \quad (z_b = 0.045m)
\]

\[
\Delta x = \Delta y = 6.6 \text{ mm} \quad \Delta z = 0.4 \sim 2.2 \text{ mm}
\]

Domain size and grid resolution are verified by velocity fluctuation correlation and energy spectrum.
3D view - Sheet flow in steady channel flow

$\phi \geq 0.08$
**Ensemble-averaged flow statistics**

Symbols: measure data (Revil-Baudard et al., 2015, *JFM*)
Curves: simulation results
Ensemble-averaged flow statistics

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Evidence of attenuated fluid turbulence by sediments

Rough-wall log law velocity profile:

\[ \langle \bar{u}' \rangle(z) = \frac{u_s}{\kappa} \ln \left( \frac{z - z_{lb}}{z_0} \right) \]

Reduction of von Karman constant:

Measured: \( \kappa = 0.23 \)  <  Clear fluid: \( \kappa = 0.41 \)

Modeled: \( \kappa = 0.2 \)

Sediment-induced density stratification can damp fluid turbulence
Evidence of attenuated fluid turbulence by sediments

Rough-wall log law velocity profile:

$$\langle \bar{u}^f \rangle (z) = \frac{u_*}{K} \ln \left( \frac{z - z_b}{z_0} \right)$$

Reduction of von Karman constant:

Measured: $\approx 0.23$  <  Clear fluid: $\approx 0.41$
Modeled: $\approx 0.2$

Sediment-induced density stratification can damp fluid turbulence, however, it’s playing a minor role comparing to drag induced turbulence damping effect for this flow condition and sediment ($W_s/u_* = 1.1$).
Preferential concentration

Heavy particles are preferentially biased to regions of **high strain rate** and **low vorticity** (Wang and Maxey, 1993), and second invariant $Q$ is used to identify these regions (Chakraborty et al., 2005):

$$Q = \frac{1}{2} (|\Omega|^2 - |S|^2)$$

Similar phenomenon are also reported by Cheng et al. (2015, *Computers & Geosciences*).

Plane-view of $Q = 250$ (gray color) and concentration fields.

Iso-surface of $Q = 250$ along with a plane cut of sediment concentration at $(z-z_d)/d=4$, $\langle \phi \rangle=0.2$

$Q>0$ $\rightarrow$ low $\phi$

Sediment clusters
**Intermittency**

Turbulent motions:

\[
\begin{array}{c|c|c}
& w' > 0 & w' > 0 \\
\hline
u' < 0, w > 0 & outward interact & u' > 0, w' > 0 \\
\hline
u' < 0, w' < 0 & inward interact & u' > 0, w' < 0 \\
\end{array}
\]


(Revil-Baudard et al., 2015, *JFM*)

- Red: Ejection
- Blue: Sweep
- Increase of bed level
- Drop of bed level
**Intermittency**

Turbulent motions:

- \( u' < 0, w' > 0 \) \rightarrow \text{ejection}
- \( u' > 0, w' < 0 \) \rightarrow \text{sweep}


- \text{Ejection}
- \( \text{Increase of bed level} \)
- \text{Sweep}
- \( \text{Drop of bed level} \)

![Diagram showing bed level fluctuations over time](image-url)
Concluding Remarks

Findings:
- Turbulence-resolving two-phase Eulerian model is developed and validated with LEGI steady sheet flow experiment.
- Drag-induced turbulence attenuation becomes more important than the density stratification in LEGI experiment.
- Model is able to capture sediment preferential concentration.

Further investigations:
- Streamwise velocities and sediment suspension is under-predicted in the dilute region.
- The inward/outward interaction events are under-predicted by our model (not reported).
- More quantitative analysis of bed intermittency are needed.
Thank you!

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