Markov-chains and graphs for linking facies with environments and biology in space and time (Recent Arabian Gulf, Miocene Paratethys) and an ODE-based model of biotically-driven facies dynamics

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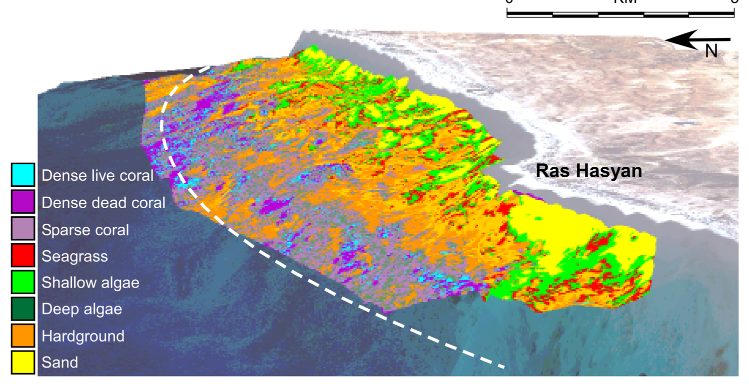
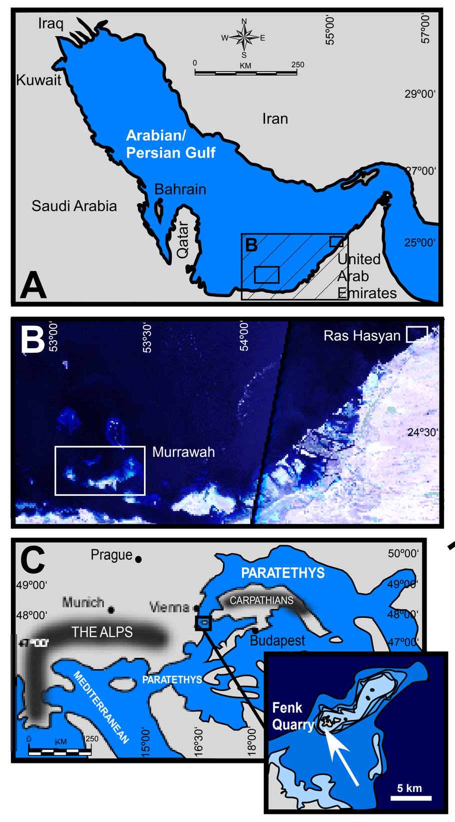
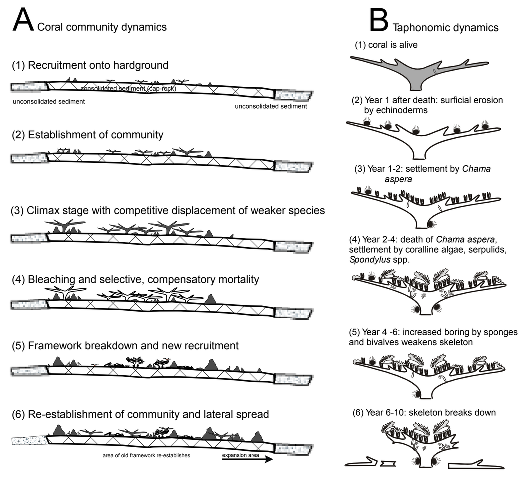
If, as comparative sedimentology maintains, knowledge of the Recent can sometimes be helpful to explain the past (and vice-versa), common quantitative denominators might exist between Recent and fossil systems. It may also be possible to describe dynamics and find linkages between space and time with a unique set of quantitative tools. To explore such conceptual links, spatial facies patterns mapped using satellite imagery were compared with temporal patterns in analogous ancient outcropping facies using Markov chains and graphs. Landsat and Ikonos satellite imagery was used to map benthic facies in a nearshore carbonate ramp (Ras Hasyan) and offshore platform system (Murrawah, Al Gharbi) in the Recent Arabian Gulf (UAE), and results were compared to the Fenk quarry outcrop in Burgenland, Austria, a carbonate ramp of the Miocene (Badenian) Paratethys. Facies adjacencies (i.e. Moore neighborhood of color-coded image pixels of satellite image or outcrop map) were expressed by transition probability matrices which showed that horizontal (spatial) facies sequences and vertical (temporal) outcrop sequences had the Markov property (knowledge of t-th state defines likelihoods of t+1st state) and that equivalent facies were comparable in frequency. We expressed the transition probability matrices as weighted digraphs and calculated fixed probability vectors which encapsulate information on both the spatial and temporal components (i.e. the frequency of the facies). Models of temporal functioning were obtained by modifying matrices (digraphs) of spatial adjacency to matrices (digraphs) of temporal adjacency by using the same vertices (facies) but adjusting transitions without changing paths. With this combined spatio-temporal model, we investigated changes in facies composition in falling and rising sea level scenarios by adjusting transition likelihoods preferentially into shallower (falling sea level) or deeper (rising sea level) facies. Our model can also be used as a numerical analogue to a Ginsburg-type autocyclic model. The fixed probability vector was used as a proxy for final facies distribution. Using Markov chains it is possible to use vertical outcrop data to evaluate the relative contribution of each facies in any time-slice which can aid, for example, in estimation of reservoir sizes and to gain insight into temporal functioning as derived from spatial pattern.

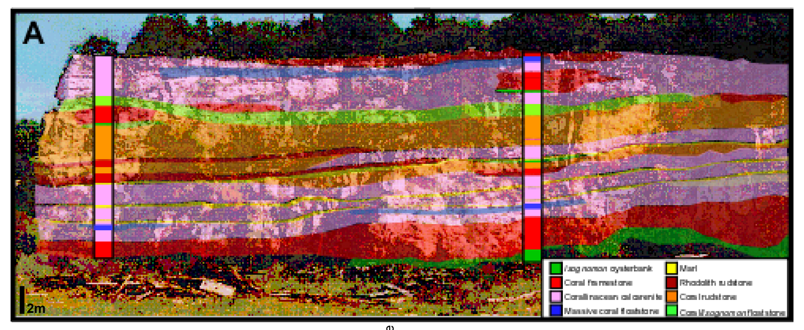
First, we took a landscape in the Arabian Gulf (UAE, Dubai) and used an Ikonos satellite iage to characterize the patterns in the benthic landscape. This is a purely carbonate landscape, thus the biology gives rise to clear facies pattern.





We had observed the dynamics of the living landscape components (corals among the most important carbonate components) for over a decade. So we had a good understanding of the life- and death- dynamics

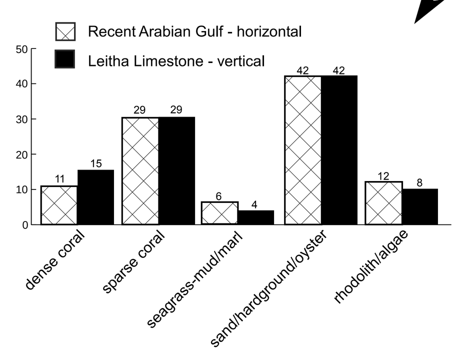




B *Leitha limestone*

	1 oyster rudstone	2 oyster/coral floatstone	3 coral rudstone	4 coral framestone	5 calcarenite	6 coral floatstone	7 rhodolith rudstone	8 marl
1 oyster rudstone	0.90	0	0.02	0.06	0.01	0	0	0.01
2 oyster/coral floatstone	0	0.91	0.04	0.01	0.04	0	0	0
3 coral rudstone	0	0.01	0.95	0	0.01	0	0.01	0.01
4 coral framestone	0.01	0	0	0.95	0.02	0	0	0
5 calcarenite	0	0	0.01	0.01	0.94	0.01	0	0.03
6 coral floatstone	0	0	0	0	0.13	0.86	0.01	0
7 rhodolith rudstone	0	0	0.06	0.01	0.03	0.01	0.87	0.02
8 marl	0.01	0	0.04	0	0.29	0	0.03	0.63

Here the crux of the model: From the Transition probability Matrix, one can derive a unique Fixed Probability Vector (FPV). This expresses the likelihood with which every state is encountered independent of the state the Markov chain is started in. If the FPV expresses the likelihood of successional stages occurring in space, it should also express their likelihood of occurring through time. If a point is twice as likely to fall within stage A (or facies A), because A is twice the size of stage (facies) B (to be precise: stage A's spatial expression in facies A is twice the size of stage B's spatial expression in facies B), then through time, everything else being equal, a point will also probabilistically be encountered twice more often in stage (facies) A than in stage (facies) B. The Law of Large Numbers for regular Markov chains defines the FPV as representing the fraction of time that the process can be expected to be in state sj for a large number of steps. **From this we propose that the spatial transitions in any landscape are useful as a proxy for the temporal transitions and vice versa.**



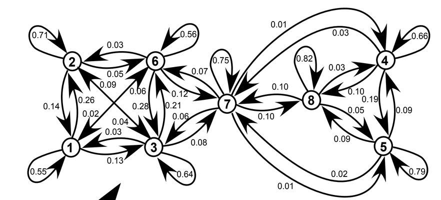
We provide evidence for this theorem by comparing the vertical (temporal) facies mosaic of a Miocene landscape in the Austrian Paratethys with the Recent Arabian Gulf landscape. Indeed, the FPVs of the facies, which are ecologically and sedimentologically equivalent, do not differ significantly.

We then counted pixel-neighborhoods on the classified satellite image. Since pixel color encodes facies, this gave information on landscape neighborhood patterns. From the raw counts we derived a transition frequeny matrix (TFM) and from that a transition probability matrix. Using our knowledge of the area's biology, we were able to arrange these neighborhood transitions in a graph that reflects the natural (ecological and sedimentological) transitions among facies.

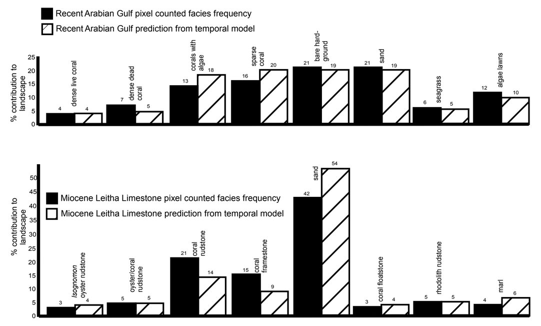
A

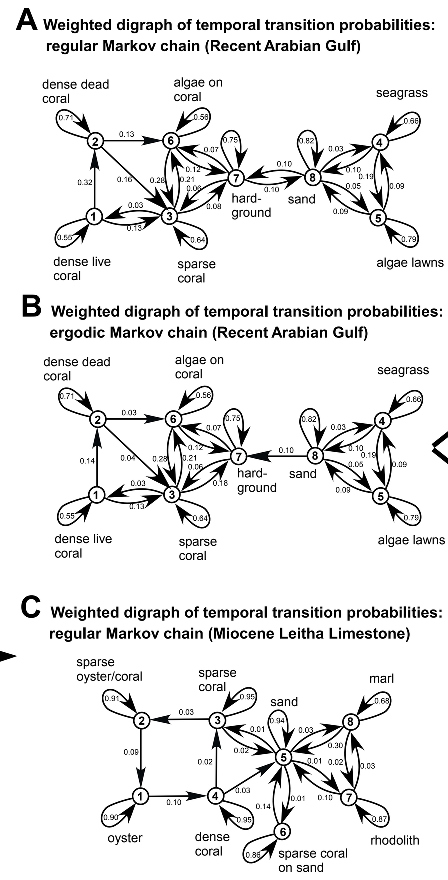
	1 dense coral	2 dense dead coral	3 sparse coral	4 seagrass	5 shallow algae	6 algae on coral	7 hardground	8 sand
1 dense coral	0.55	0.26	0.13	0	0	0.06	0	0
2 dense dead coral	0.14	0.71	0.09	0	0	0.05	0	0
3 sparse coral	0.03	0.09	0.64	0	0	0.21	0.08	0
4 seagrass	0	0	0	0.66	0.19	0	0.03	0.10
5 shallow algae	0	0	0	0.09	0.79	0	0.02	0.09
6 algae on coral	0.02	0.03	0.28	0	0	0.56	0.12	0
7 hardground	0	0	0.05	0.01	0.01	0.07	0.75	0.10
8 sand	0	0	0	0.03	0.05	0	0.10	0.82

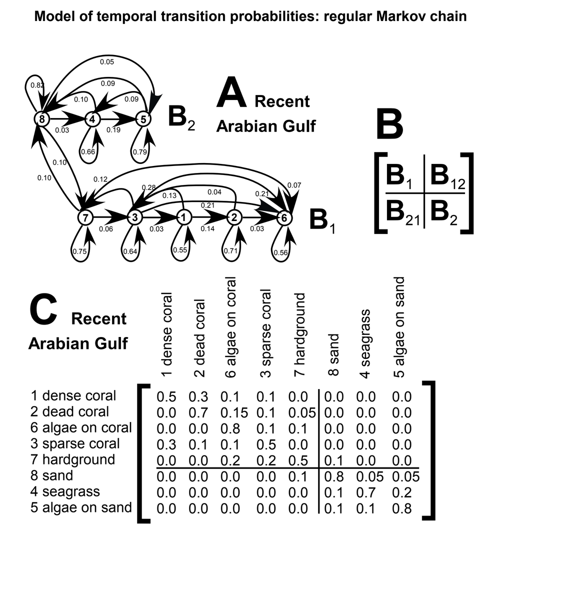
B

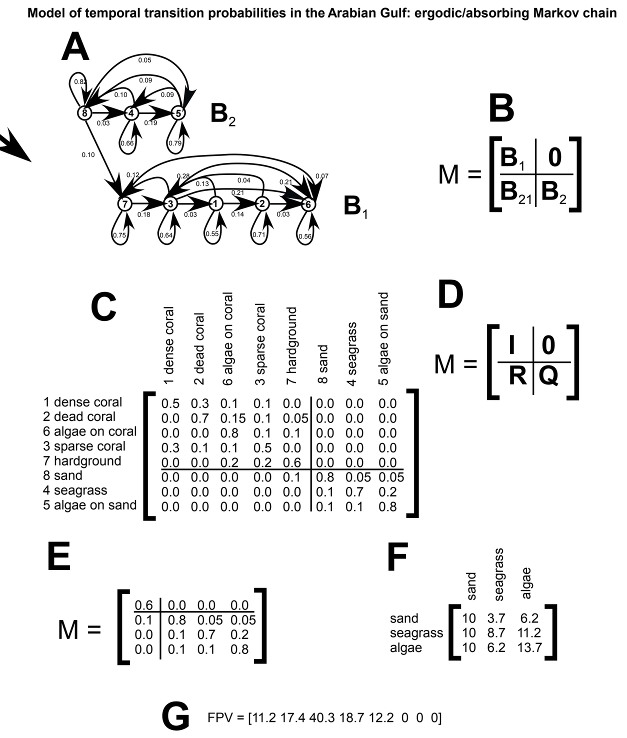


Next, using graph theory, we developed models for spatial and temporal transitions in our chosen landscapes. We then tested the FPVs of the models against the actually pixel-counted landscape transitions - and it fit quite well.



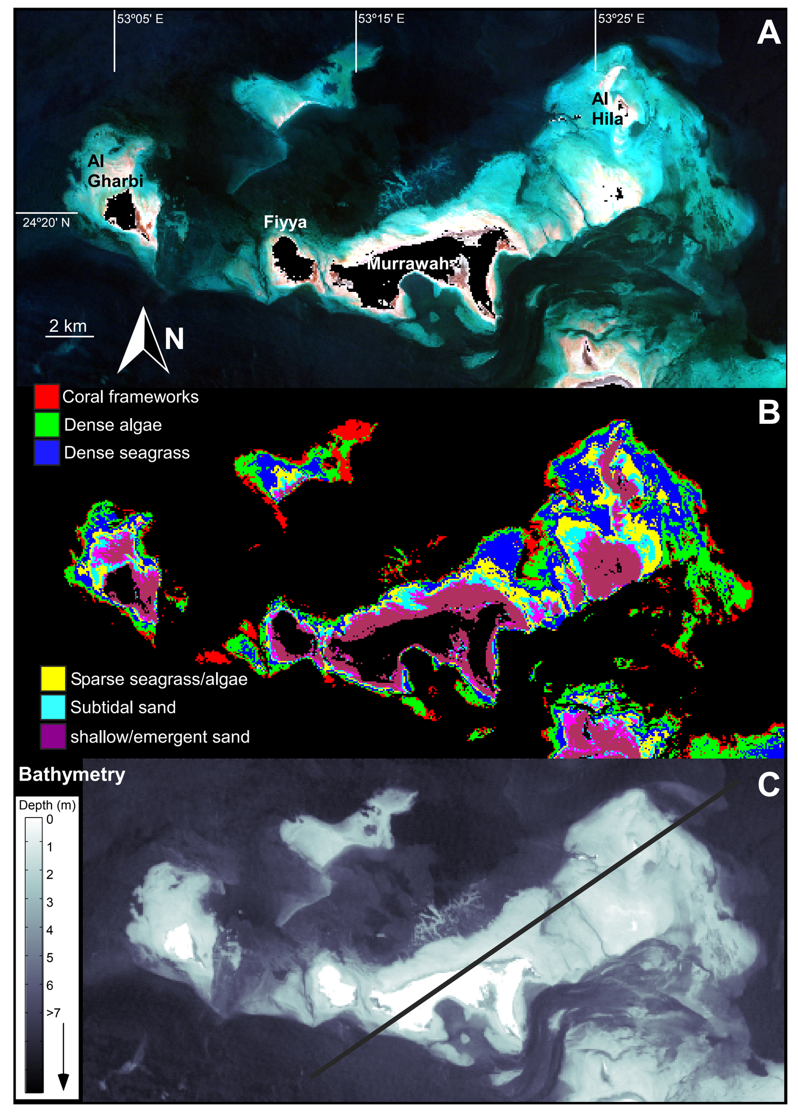






If the landscape transitions are indeed Markovian, we can look at them from two viewpoints: as regular or ergodic (or even absorbing) chains.
In a regular Markov chain, every state can transit into any other one, so there is no strictly defined directionality. In an ergodic chain, the landscape enters a loop from which it can no longer escape, thus all landscape components will eventually be drawn into this loop - or even be absorbed into a single state. Such a chain has strongly defined directionality and certain components - all those outside the ergodic loop (or absorbing state) will be lost. They will also not appear in the FPV.

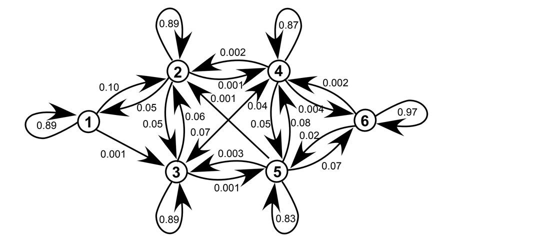
NEXT:
Let us apply this theory to another sedimentary system.
Here a part of the Great Pearl Bank in the Arabian Gulf



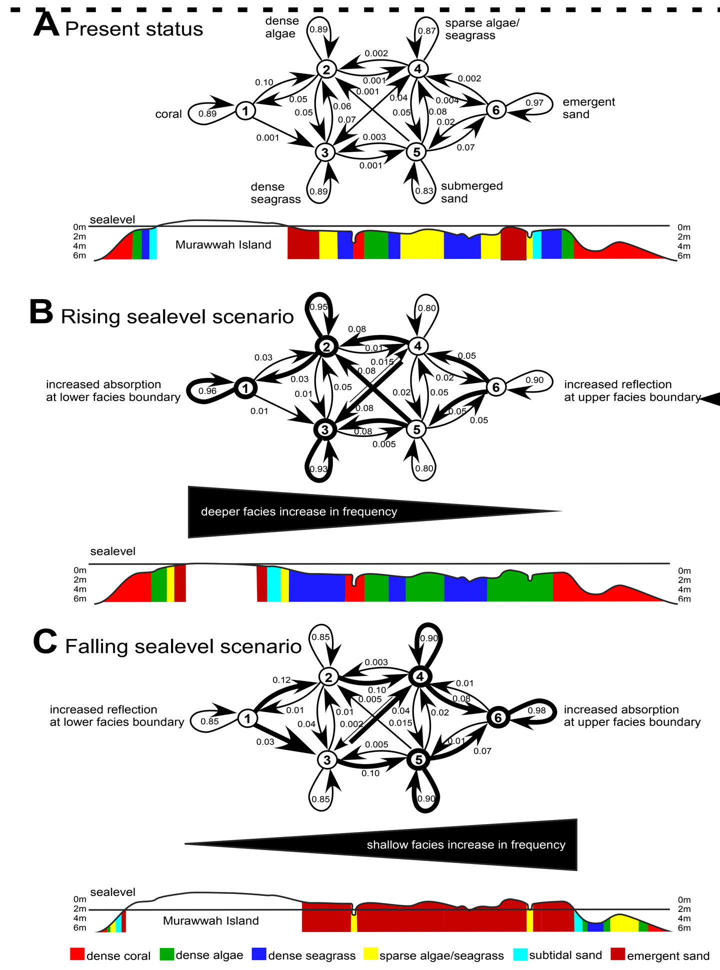
A

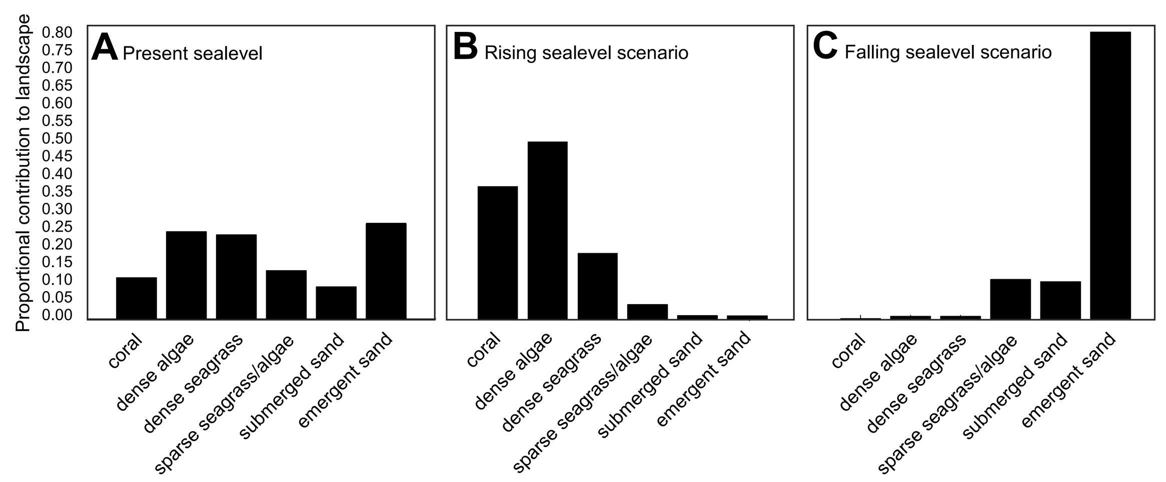
	1 dense coral	2 dense algae	3 dense seagrass	4 sparse seagrass/algae	5 submerged sand	6 emergent sand
1 dense coral	0.897	0.101	0.001	0	0	0
2 dense algae	0.048	0.894	0.056	0.001	0	0
3 dense seagrass	0	0.060	0.899	0.040	0.001	0
4 sparse seagrass/algae	0	0.002	0.071	0.869	0.054	0.004
5 submerged sand	0	0.001	0.003	0.083	0.837	0.077
6 emergent sand	0	0	0	0.002	0.026	0.972

B



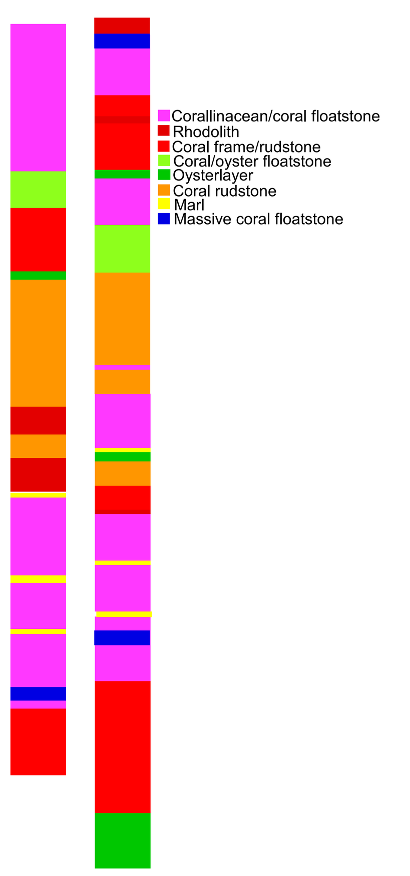
Just as above, we pixel-count transitions, calculate the TPM and turn it into a graph. The graph is linked to the underlying TPM. We can now simulate changed conditions by modifying individual transition liklyhoods. This is shown by bold lines on the graphs to the left.





From the modiofied TPMs, we obtain changed FPVs, reflecting the changed landscape in response to different conditions. We can simulate the changes in facies-extent under, for example, different sea-level regimes.

This is thee facies-sequence in the Miocene Paratethys outcrop.
What is encoded?
Environmental variability or interspecific (=inter-facies) competition?



To get a better feeling, we build a model of the interactions:

The coral equation:

$$\frac{dC}{dt} = \frac{\lambda C}{R}(R - C) + \sigma\theta$$

The oyster equation:

$$\frac{d\theta}{dt} = \frac{\varsigma\theta}{\kappa}(\kappa - \theta - \alpha C)$$

The branching red algae equation:

$$\frac{dN}{dt} = \frac{rN}{K}(K - N) - \gamma\Re - \varepsilon C$$

The rhodolith equation:

$$\frac{d\Re}{dt} = \frac{\lambda\Re}{\aleph}(\aleph - \Re) - \nu\theta$$

Model Assumptions:

We assume logistic growth in all facies (this has been frequently used for corals and oysters and is at least plausible for red algae and rhodoliths).

Corals benefit from oysters for extra substratum.

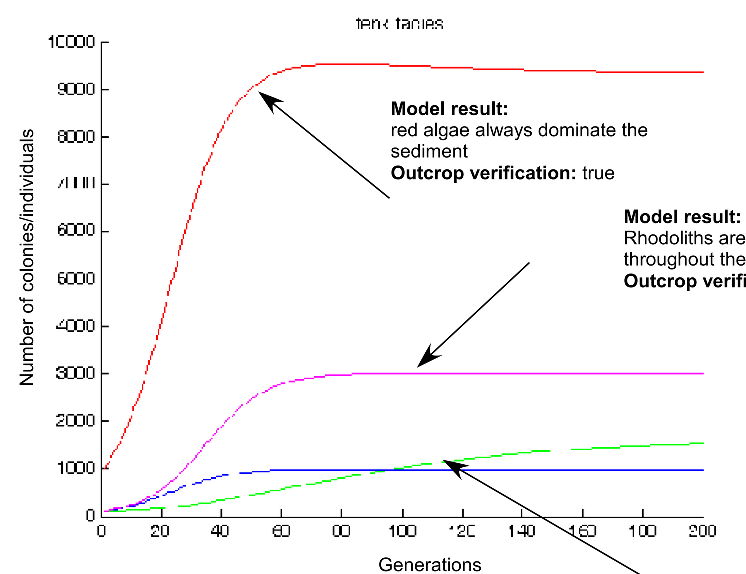
Oysters suffer mortality from coral overgrowth.

Red algae compete with corals for space but not with oysters.

Rhodoliths are limited by oysters.

Neither corals nor oysters are limited by rhodoliths or branching red algae.

Rhodoliths exsert mortality on branching red algae

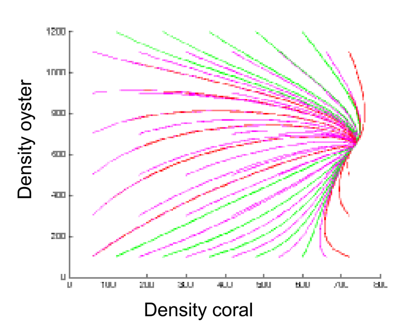


Model result:
red algae always dominate the sediment
Outcrop verification: true

Model result:
Rhodoliths are common throughout the sequences
Outcrop verification: true

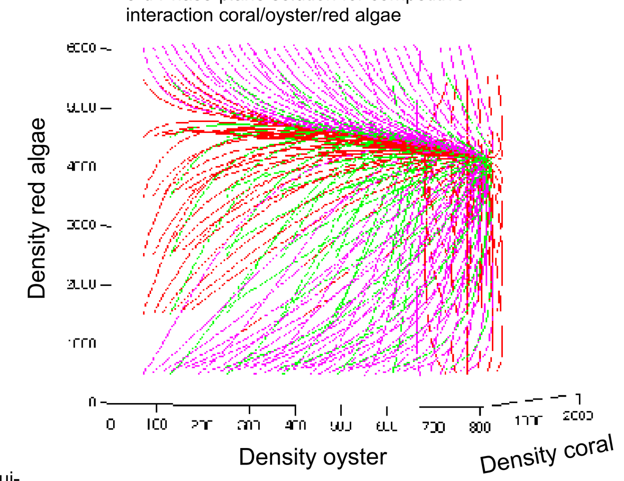
Model result:
corals outcompete oysters, thus no environmental change is necessary for switch in dominance. Oysters and corals occur regularly next to each other.
Outcrop verification: true

2-d Phase-plane solution for competitive interaction coral/oyster



The phase-plane solution for the trajectories of corals in dependence of oysters and vice versa confirms the result - corals dominate oysters. Furthermore, since oysters serve as substratum for corals, the final equilibrium point is actually above the set original environmental carrying capacity. As long as there are oysters around, coral space limitation is mitigated against. Hence the sharp point to the right of the phase-plane.

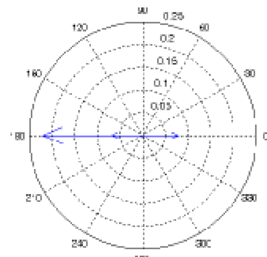
3-d Phase-plane solution for competitive interaction coral/oyster/red algae



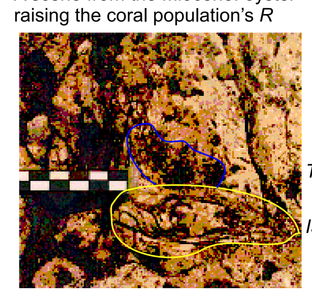
The phase-plane solution that includes the red algae shows that competition with corals and oysters does not allow red algae to realize their full carrying capacity However, being the most common taxon, they still dominate the sediment.

The equations solve to several feasible internal equilibria (nor shown here, since long and cumbersome), and the phase-plane solutions suggest that a relatively well-defined stability region exists.
Indeed, the eigenvalues of the system's Jacobian confirm this. The two dominant eigenvalues are real and negative - the requirement for stability.
Only one eigenvalue is positive, suggesting some oscillation around the equilibrium.

Eigenvalues of Jacobian:
All real, only one positive.



A scene from the Miocene: oyster raising the coral population's *R*



The moral of the model's story:

- oysters aid as substratum-builders for corals and coral framestones develop on a basis of oyster.
- given enough time, corals will outcompete the oysters, but they occur together. No environmental changes need to be invoked for changes from oyster- to coral-dominance.
- since corals and oysters competitively disadvantage the red algae, strong environmental perturbances (probably cold-events) must be invoked to explain the temporary absence of corals and oysters in the sediment and its complete dominance by red algae.
- without oysters, corals are strongly density limited (due to lack of substratum) and are limited to forming float- and rudstones. Framestones develop with the help of oysters (an instance where ecological facilitation becomes sedimentological facilitiation!).
- Since Markov chains are often used to model ecological succession, this plausible ODE model can be used to refine the Markovian assumptions presented above.