conduitFoam: an open source one-dimensional subglacial conduit model

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Background

Increasing ice losses of the **Greenland Ice Sheet** (GIS) is contributing almost 1/4 of the global mean sea level rise[1]. Ice losses due to subglacial ice melting and ice sheet dynamics, however, are still poorly understood due to limited accessibility and thus lack of data. We here show an OpenFOAM-based one-dimensional subglacial conduit model that can be applied to evaluate the diurnal fluctuations and outburst flooding in Greenland ice sheet.

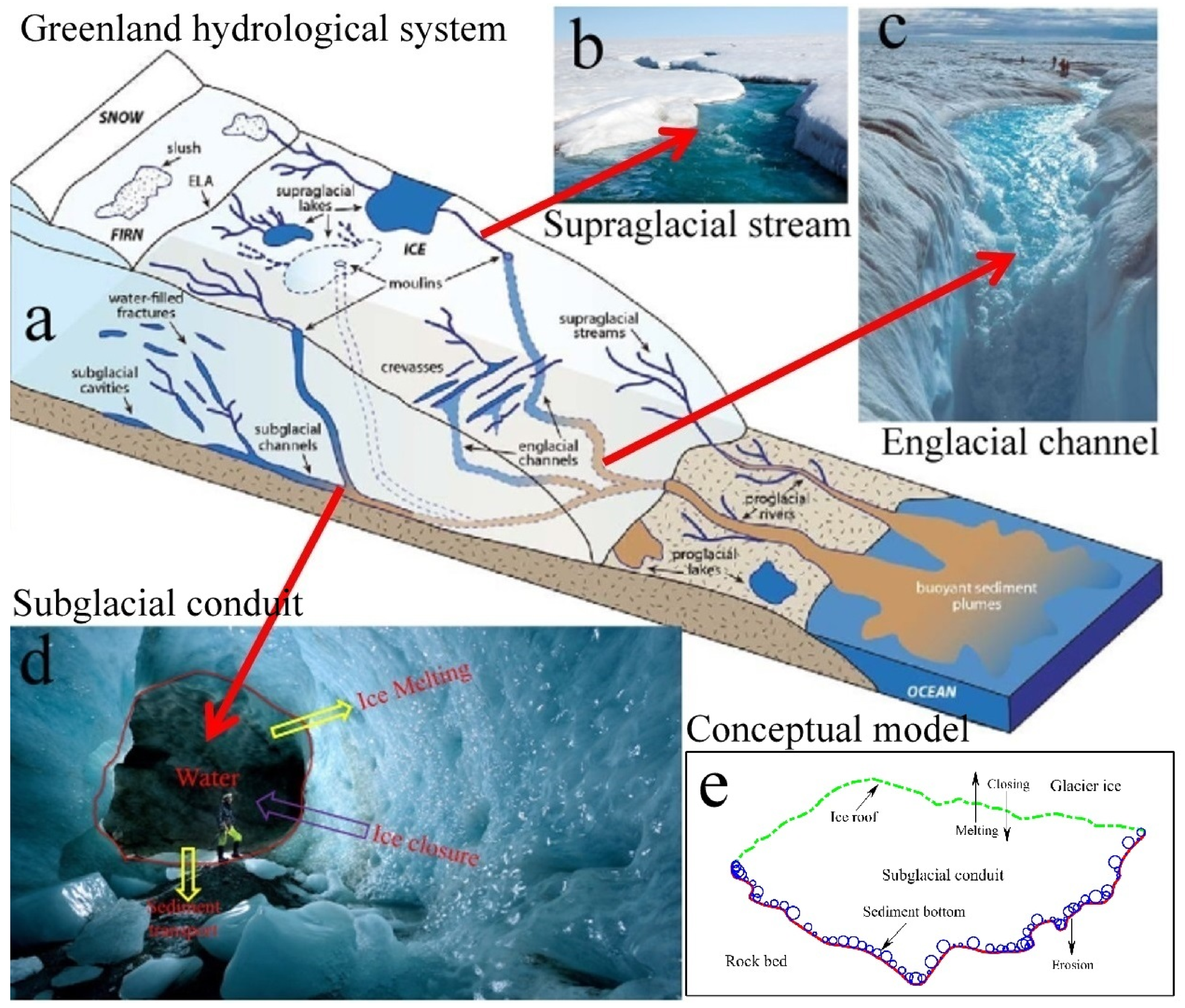

Figure 1: The hydrological system of Greenland ice sheet (a), its three components: supraglacial system (b), englacial system (c), and subglacial system (d), and an conceptual model of key physics in subglacial system (e). Figures (a-d) are modified from Cuffey and Paterson, Los Alamos, Roger Braithwaite, and Robbie Shone, respectively.

Equations & Boundary Conditions (1)

The subglacail model is composed by mass conservation for ice and water, momentum conservation for water, energy conservation, and empirical ice creep-closure model. Entrance pressure boundary is governed by a lake-englacial-subglacial system as shown in Fig. 2(a).

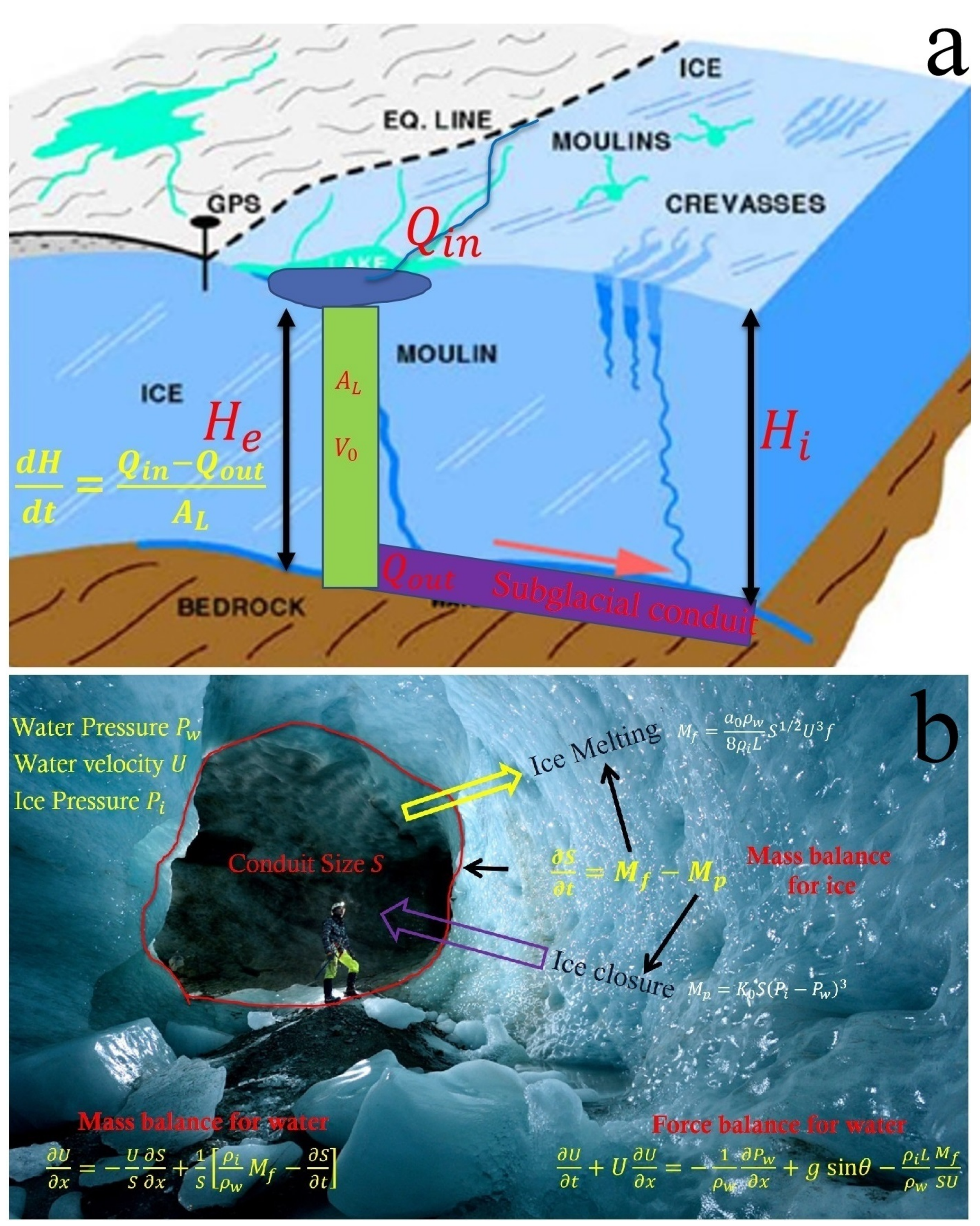

Figure 2: The entrance conditions (a) and mathematical models (b) for subglaical conduits.

Equations & Boundary Conditions (2)

Two simplifications: (a) circular or semi-circular shape, and (b) uniform wall shear stress.

The mass balance for ice, mass balance for water, and momentum balance for water can be expressed by the following equations[2] :

$$\frac{\partial S}{\partial t} = M_f - M_p \quad (1)$$

$$\frac{\partial U}{\partial x} = -\frac{U}{S}\frac{\partial S}{\partial x} + \frac{1}{S}\left[\frac{\rho_i}{\rho_w}M_f - \frac{\partial S}{\partial t}\right] \quad (2)$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x} = -\frac{1}{\rho_w}\frac{\partial P_w}{\partial x} + g\sin\theta - \frac{\rho_i L}{\rho_w}\frac{M_f}{SU} \quad (3)$$

where S, U, and P_w are time-dependent (t) conduit cross-sectional area, water velocity, and water pressure, respectively. ρ_w and ρ_i are water and ice density, respectively. M_f and M_p denote the ice-melting rate due to viscous friction and the ice-closure rate due to viscous creep, respectively. They are defined as follows:

$$M_f = \frac{a_0 \rho_w}{8\rho_i L} S^{1/2} U^3 f \quad (4)$$

$$M_p = K_0 S (P_i - P_w)^3 \quad (5)$$

The boundary condition and initial condition are:
BC: $P_w|_{x=0} = \rho_w g H_e$, $P_w|_{x=l} = \rho_i g H_i$, zero gradient for S and U.
IC: Uniform U and S, linear distribution of P_w along channel length x.

Solutions: Numerical & Analytical

Numerical Solution:
(1) Conduit length discretization: $x_i = x_0 + i\Delta x, i = 0, 1, ..., N, \Delta x = l/N$.
(2) ICs: $P^0_{w,i} = P^0_{w,0} + i/N(P^0_{w,N} - P^0_{w,0}), U^0_i = U_0, S^0_i = S_0$.
(3) BCs: $P^j_{w,0} = \rho_w g H^j_e$, $P^j_{w,N} = \rho_i g H^j_i$, $U^j_0 = U^j_1, U^j_N = U^j_{N-1}, S^j_0 = S^j_1, S^j_N = S^j_{N-1}. H^j_e = [V_0 + (Q_0 - Q_{out})\Delta t]/A_L$, $V_0, Q_0, \Delta t, A_L$ are user-specified parameters, denoting initial water volume, upstream input discharge, iterative time interval, and value column area, respectively. Q_{out} is water discharged from the conduit to the ocean, calculated during the simulation by $Q_{out} = U^j_0 S^j_0$.
(4) Solving the coupled water velocity and pressure equations (2) and (3) to obtain $(U^{j+1}_i, P^{j+1}_{w,i})$ using Pressure-Implicit with Splitting of Operators algorithm[3] based on previous time step value S^j_i, the velocity in the right hand side uses old time step U^j_i.
(5) Updating the conduit size S^{j+1}_i using equation (1).

Numerical Solution:
Assumptions: $M_p \ll M_f$ in the initial stage and M_f is approximately a constant in the quasi-steady stage.
For initial stage:

$$S = S_0(1-\beta t)^{-4} \quad (6)$$

$$\beta = \frac{\sqrt{2}}{4} a_0 a_1^{3/2} \frac{\rho_w}{\rho_i} \frac{g^{3/2}}{L} S_0^{1/4} k_H \theta f^{-1/2} \quad (7)$$

For quasi-steady stage:

$$S = (\alpha/a_0)^{2/5} g^{-2/5} Q_0^{4/5} \theta^{-2/5} f^{2/5} \quad (8)$$

$$U = (\alpha/a_0)^{-2/5} g^{2/5} Q_0^{1/5} \theta^{2/5} f^{-2/5} \quad (9)$$

For circular shape, $a_0 = \sqrt{4\pi}$, $\alpha = 0.8 \sim 0.9$, θ, Q_0, f denote conduit slope, upstream discharge, and friction factor.

Application: Diurnal Variations

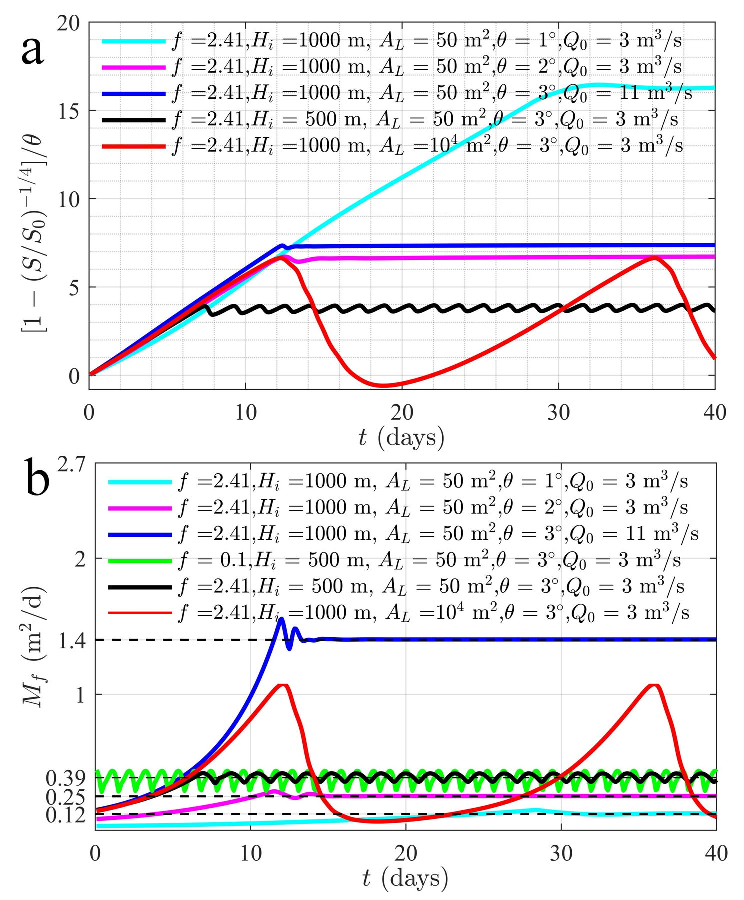

Figure 3: Validating numerical model with analytical solution (a) and test assumption (b).

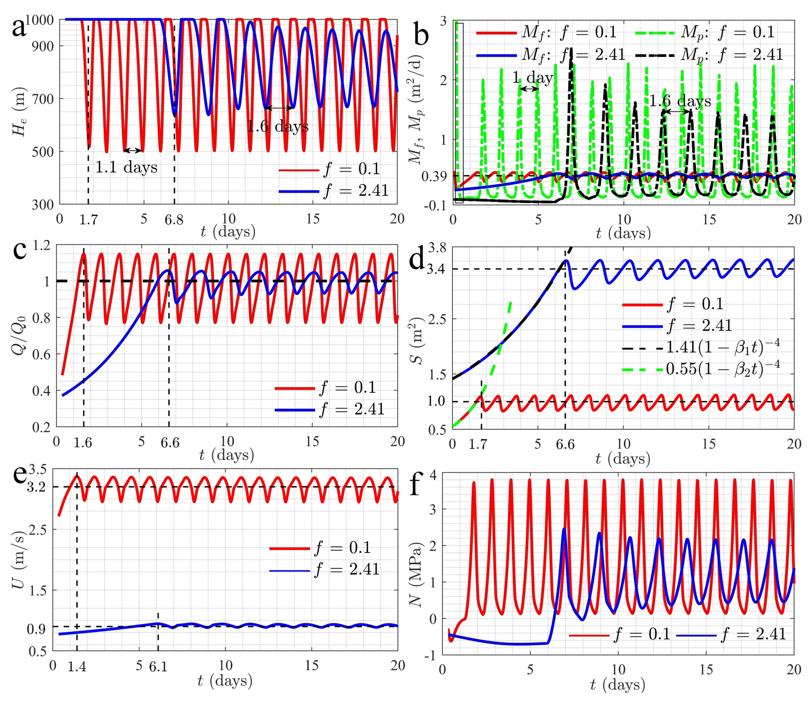

Figure 4: Impacts of friction factor on conduit properties: entrance water depth (a), ice-melting and creep-closure rate (b), discharge (c), size (d), water velocity (e), and effective pressure (f).

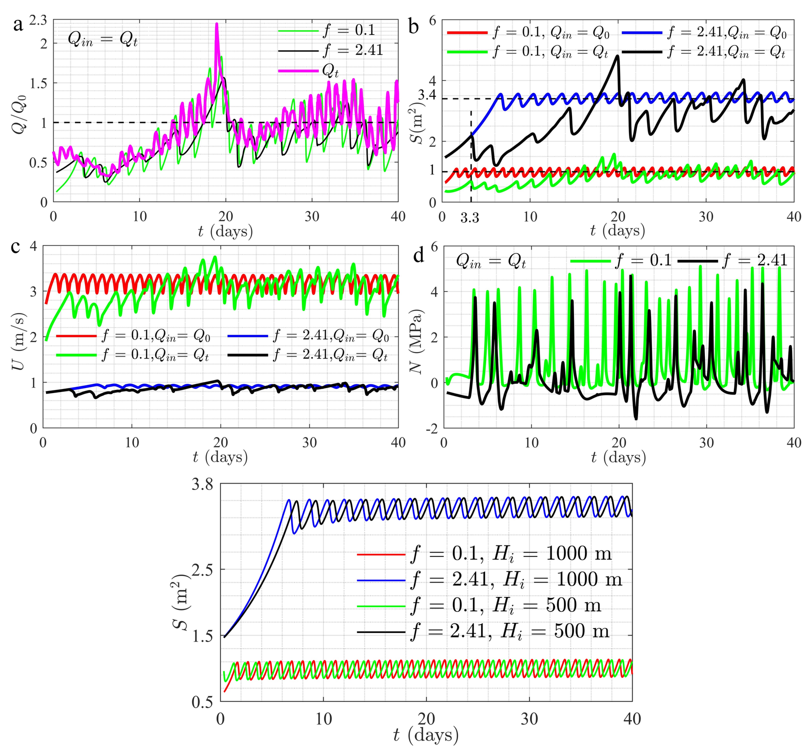

Figure 5: Impacts of time variable input discharge Q_t on conduit discharge (a), size (b), water velocity (c), and effective pressure (d), and the effects of ice thickness on conduit size (f).

Application: Outburst Flood

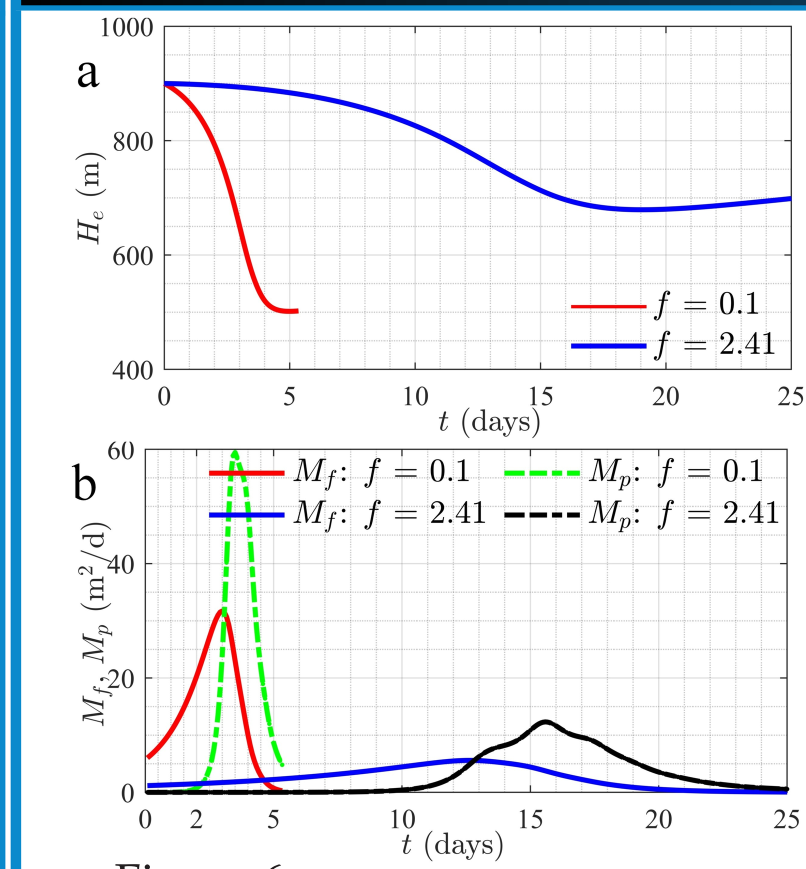

Figure 6: Impacts of friction factor on entrance water head (a) and ice-melting/creep closure rate (b).

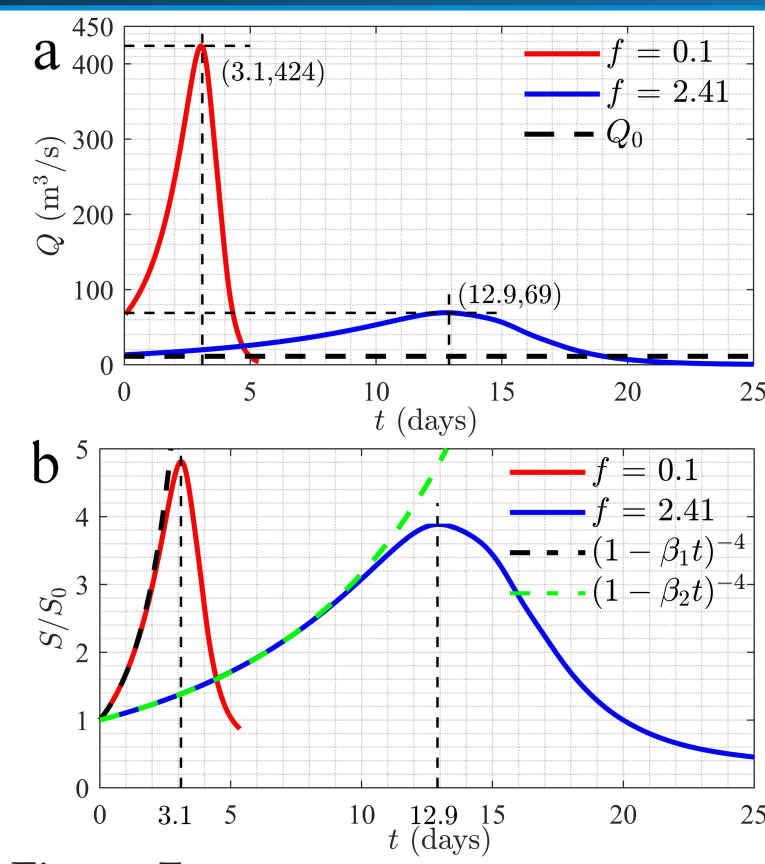

Figure 7: Impacts of friction factor on conduit discharge (a) and size (b). $\beta_1 = 0.122$ /day and $\beta_2 = 0.025$/day.

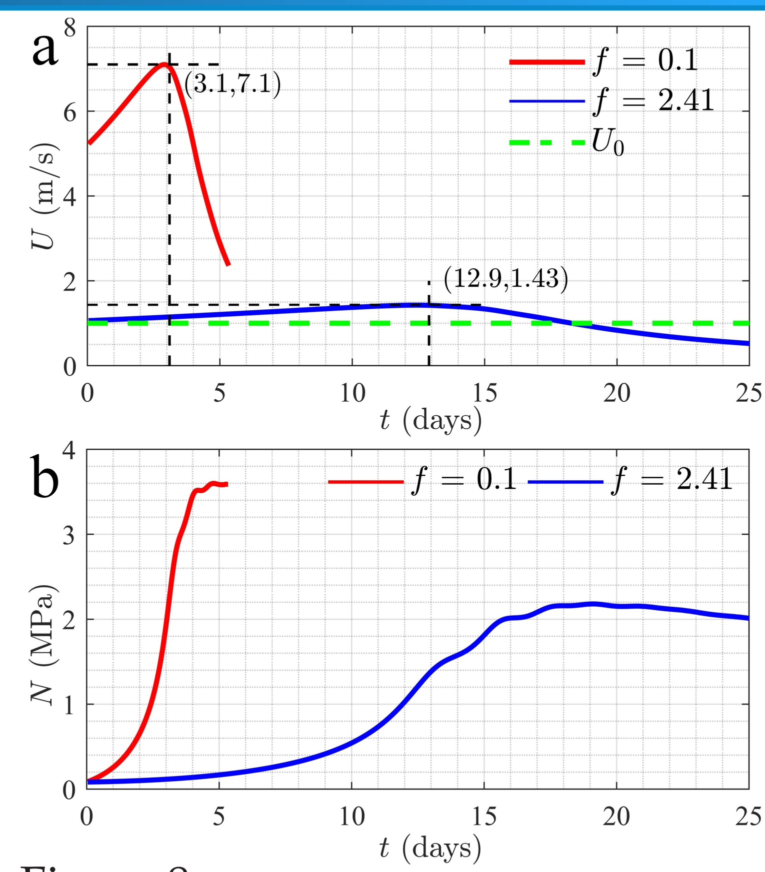

Figure 8: Impacts of friction factor on water velocity (a) and effective pressure (b).

References

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