Contents

Preface

This book began in February 1988 during a week long conference in the mountains of Colorado devoted to "QDS", or "Quantitative Dynamic Stratigraphy". The conference, organized by Professor Tim Cross and students of the Colorado School of Mines, was to outline the scope of QDS and to forecast its future development, including its needs and promise. As its name implies, QDS deals with stratigraphy from a quantitative and mathematical standpoint, with emphasis on dynamic computer models to represent geologic processes.

This book focuses on a major need highlighted by the conference, namely developing the skills to represent complex sedimentary depositional systems with mathematical and computational models that provide enhanced insight into processes and products. As such this book is a primer, being both a first book of instruction and a devotional manual for the laity. It represents our conviction that geologists should deal with geologic processes in terms of fundamental laws of physics or well-established empirical relationships. Our goal is to adapt these principles and relationships so they are convenient for use in creating computer simulation models of selected sedimentary systems. The book represents this conviction because it is driven by physics, centered on geologic processes, and formulated for simulating dynamic systems.

The skills emphasized here are, first, the translation of descriptions of sedimentary processes or depositional systems into dynamic models. By "dynamic model" we mean a physical-mathematical description of changes in important geological variables, such as changes in velocities in a stream or variations in thickness in a foreland clastic depositional wedge. Second, we provide instruction in obtaining numerical solutions to sets of differential equations that have been transformed into finite-difference approximations. Third, we illustrate how finite-difference approximations can be represented by FORTRAN code. Fourth, we show how components representing individual processes can be linked to create more complete sedimentary depositional systems. Finally, we discuss how different simulation experiments can be used in interpreting ancient stratigraphic sequences.

Our intended users consist of students of the earth sciences who are attempting to understand complexly coupled sedimentary systems, whether at the scale of a single-thread stream or that of a foreland basin. Some prospective users will need to restore the skills in mathematics and physics they once possessed. Most undergraduate geology curricula include year-long courses in calculus and physics, which collectively provide much of the background in mathematics and physics required for this book. Unfortunately, many geologists have allowed their skills in calculus and physics to rust through disuse. We hope this book provides rationale for reestablishing these skills and incentive for their use in an important geological context.

This book also makes major geological demands of its readers by requiring quantitative descriptions of sedimentary processes, such as sediment transport by running water. Few geologists deal with these processes directly, even though they commonly interpret ancient sedimentary features such as alluvial gravels or nearshore marine deposits in terms of depositional processes. It is our conviction that ancient deposits can be interpreted more effectively if the geologist is familiar with the physics of the processes that formed them, and the interactions and interdependencies that linked the processes together within the dynamic systems in which they formed.

The roots of this book extend back more than a quarter of a century. All three of us have used computer simulations for the past decade or two, and Harbaugh began work in process simulation in the middle 1960s. The early work was summarized in a book by Harbaugh and Bonham-Carter in 1970. Since then, the relative slowness with which geologists have utilized geologic process simulation is mostly due to an inadequate demonstration of its usefulness, but also reflects the non-mathematical outlook of many geologists. Other influences include the inadequate speed and memory of earlier computers, and the primitive state of computer graphics until recently. Meanwhile, these inhibiting influences have been greatly reduced, and process simulation is now entering the mainstream of geology.

Simulating sedimentary systems can be stimulating and enjoyable. At the least, it helps clarify "muddied" thinking, and at best it provides a new method for analyzing sedimentary basins and the dynamic systems that create them.

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October 1993

CHAPTER 1 INTRODUCTION

This book is supposed to teach you methods of mathematical modeling in sedimentary geology and to illustrate how models can help geologists understand the creation and filling of sedimentary basins. To us, modeling means the integration of mathematical equations, logical rules, and constraints in the form of computer programs. The resulting models state formal assertions in logical terms; they aid in thought; they use the logic of mathematics to get beyond intuition; they permit formulation of hypotheses for testing; and they help make evident complex outcomes, nonlinear couplings, and distant feedbacks. While modeling itself often provides insight, it is the simulations or experimental manipulations of models with computers that most often lead to true understanding. Simulations are especially relevant in sedimentary geology because they compress geologic time and permit sensitivity analyses to be undertaken with interacting geological processes. Of course, if the assumptions and mutual interdependencies in a process model are obscure, simulations with it may be no easier to understand than the actual sedimentary features themselves.

AN EXAMPLE: CLASTIC WEDGES IN FORELAND BASINS

Because all this is abstract, a concrete example is in order. Consider a source terrain whose orogenic history is deduced solely from the deposits in a foreland basin. Foreland basins form during continent-continent collisions by crustal loading on the oceanic or "outboard" side of the craton, or by combined loading and subduction of the ocean lithosphere (Jordan, 1981; Quinlan and Beaumont, 1984).

Foreland basins due solely to outboard loading are interesting because both the basin and source terrain result from thickening of the crust by overthrusting (Figure 1-1). Thickening creates a basin because the lithosphere flexes downward

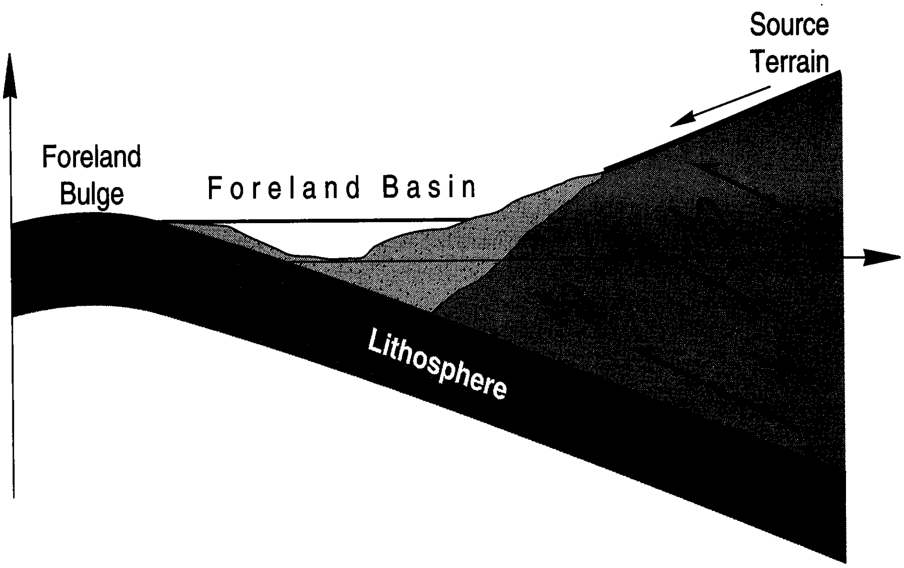

Figure 1-1 Schematic cross section of foreland basin and source terrain. Crustal thickening is represented as critically tapered accretionary wege (Dahlen and Suppe, 1988) that loads lithosphere, creating mountain range and foreland basin. Detritus from mountain range forming source terrain fills basin.

in response to increased load, subsiding laterally over hundreds of kilometers beyond the load itself. The resulting moat or basin is peripheral to the load, and the basin's geometric form depends both upon the crust's rigidity and upon the magnitude and rate at which the load is applied. Thickening also creates the source terrain because the underlying plate has sufficient strength to support the thickened crust. In addition the thickened crust is usually less dense than the surrounding lithosphere and therefore it is buoyed up by isostasy. The width and height of the source terrain thus depend upon size and shape of the loads and upon the strength of the underlying plate (Molnar, 1988; Dahlen and Suppe, 1988; Slingerland and Furlong, 1989). Meanwhile, denudation by erosion continuously modifies the source terrain and feeds sediment into the foreland basin. Thus, the filling of a foreland basin with sediment is intimately linked with the orogenic and climatic history of its source terrain. The Acadian foreland basin of the central Appalachian orogen is an excellent example (Figure 1-2).

Crustal thickening along the eastern margin of North America during the Acadian orogeny caused renewed subsidence in a foreland basin, and filled it during the Middle Devonian to Early Mississippian with a wedge of sediment 3.5 km thick on its eastern edge (Figure 1-3). This wedge, called the "Acadian clastic wedge," resulted from westward progradation of a basin plain to form a fluvial complex (Figure 1-4). The Acadian wedge is made up of two lesser wedges, a lower Catskill wedge named from the Catskill Mountains of New York State, and an upper Pocono wedge named from the Pocono Mountains of eastern Pennsylvania. Both lesser wedges are asymmetrical and are interpreted to record the same sequence of events, namely gradual progradation of the shoreline (Figure 1-5) followed by rapid submergence. Furthermore, both contain alluvial deposits that become finer downstream (Figure 1-6).

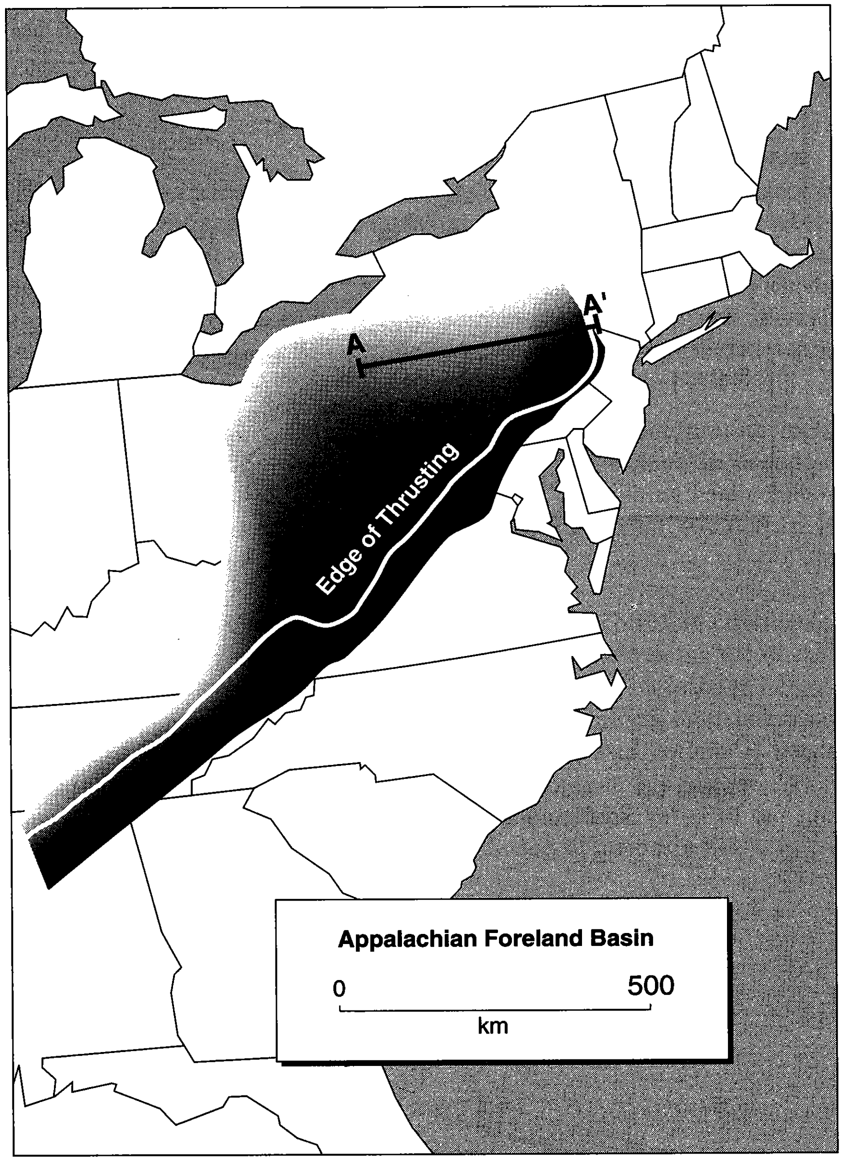

Figure 1-2 Confiuration of Appalachian foreland basin in Eastern United States (from Beaumont *et al.*, 1988). Section A-A′ is shown in Figure 1-3.

For over a century geologists used this sedimentary record to characterize the Acadian orogeny, much as footprints characterize a lion. While our understanding has benefited, it is still not possible to specify the tectonic and climatic conditions that led to this basin fill. The reasons for this difficulty are easy to understand.

Consider an Acadian source terrain represented as a "critically tapered" accretionary wedge (Figure 1-7). Critically tapered means that the surface slope of the wedge (angle α in Figure 1-7) increases until the compressive stress everywhere inside the wedge equals the rock strength. First, consider a steady state, critically

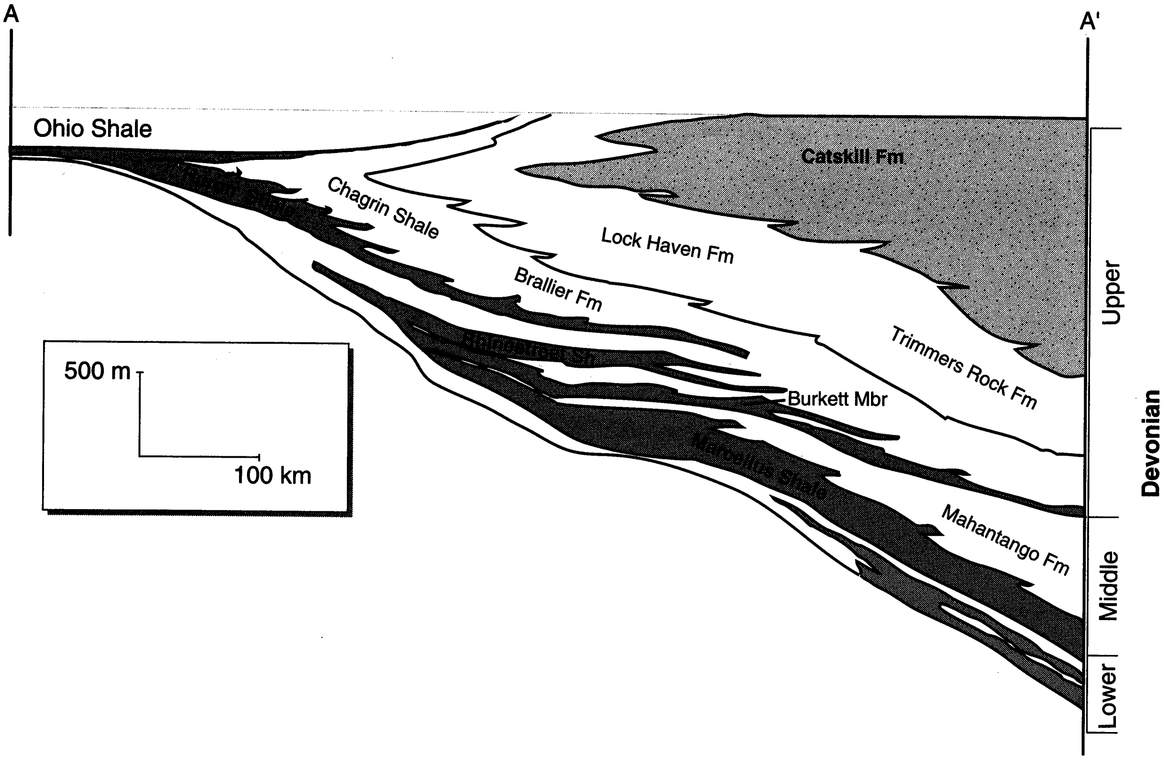

Figure 1-3 Acadian clastic wedge (from Potter et al., 1979). Line of section is shown in Figure 1-2.

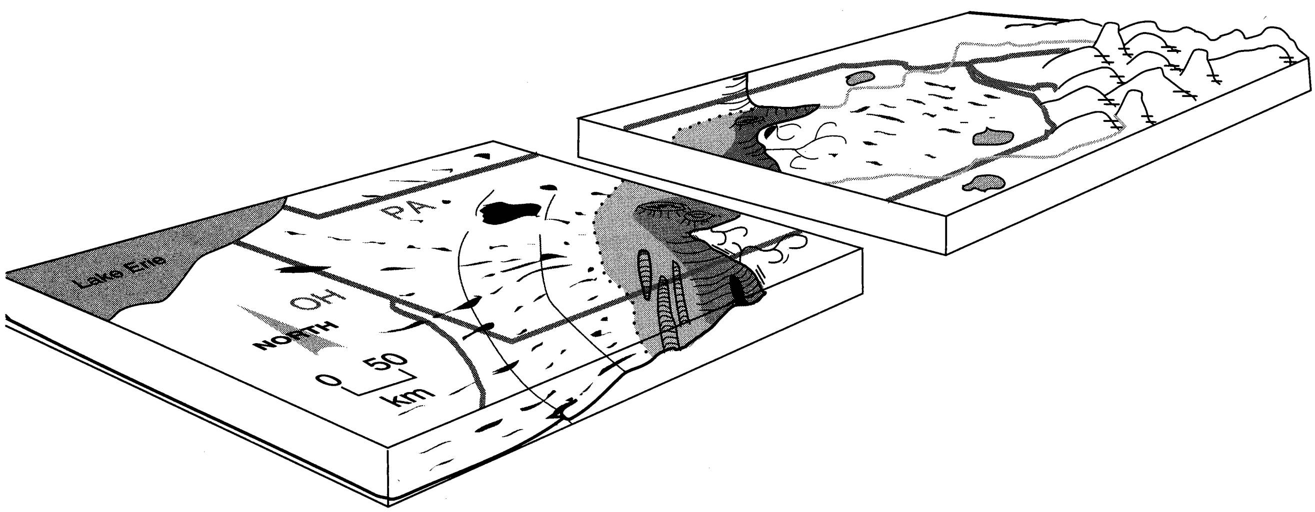

Figure 1-4 Perspective of Devonian shoreline 6 shown in Figure 1-5 (from Slingerland and Beaumont, 1989).

tapered wedge. If denudation rate is proportional to elevation (Ahnert, 1970; Pinet and Souriau, 1988), then for a particular mass flux into the toe of the wedge, crustal duplication will cause the wedge's height and width to be increased until the mass-per-unit time entering is balanced by the mass-per-unit-time eroded from the wedge's upper surface, creating a steady state geometrical form for the wedge (see Dahlen and Suppe, 1988, for details, and Beaumont, Quinlan, Hamilton, 1988, or Slingerland and Furlong, 1989, for application to the Appalachians). The height and width of the wedge multiplied by its average density yields the steady state load-per-unit-length applied to the lithospheric plate. In response, the lithospheric plate deforms to produce a foreland basin whose geometry changes continuously as it receives sediment eroded from the wedge. If the lithosphere flexes viscoelastically (Quinlan and Beaumont, 1984) and redistributed sediment is not considered, the basin will follow the changes depicted in Figure 1-8.

This scenario presents the first complication in interpreting tectonics from sediment fill. Even in this simplest case, with no change in denudation rate or mass load, filling of the basin is influenced by a changing basin shape caused by relaxation of bending stresses over spans ranging from 100,000 to 100 million years (Quinlan and Beaumont, 1984).

Now consider an increase in the rate at which colliding continents converge, thereby increasing the rate at which sediment is added to the toe of the accretionary wedge. The system (Figure 1-1) responds with an increase in height and width of the source terrain, which in turn increases both crustal load and sediment yield. In turn, these increases cause the rate at which basin volume is created and the rate at which sediment is delivered to the basin both to increase. We also expect the sediment supply to become coarser as topographic relief of the source terrain increases. How will these events be manifested in the basin fill?

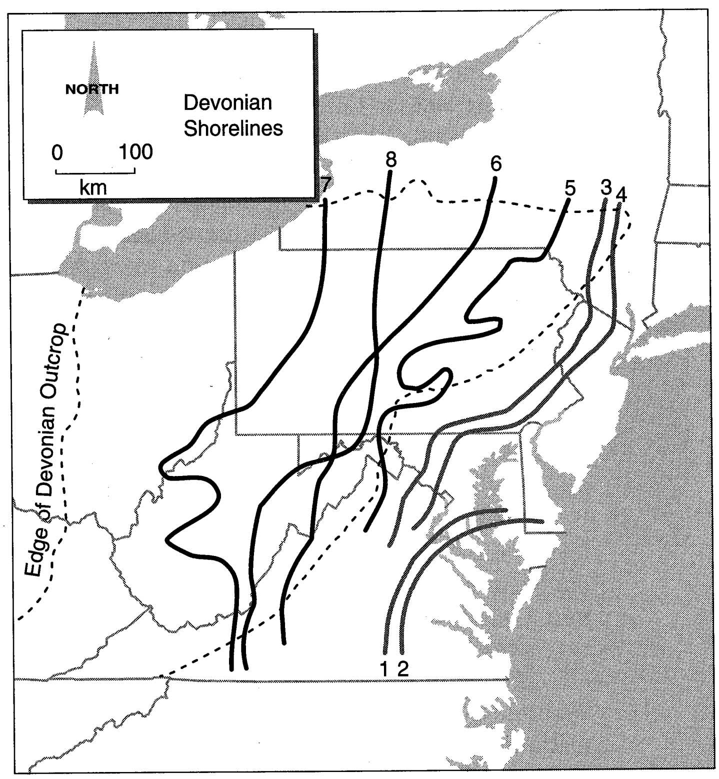

Figure 1-5 Devonian shorelines in central Atlantic states (from Dennison, 1985).

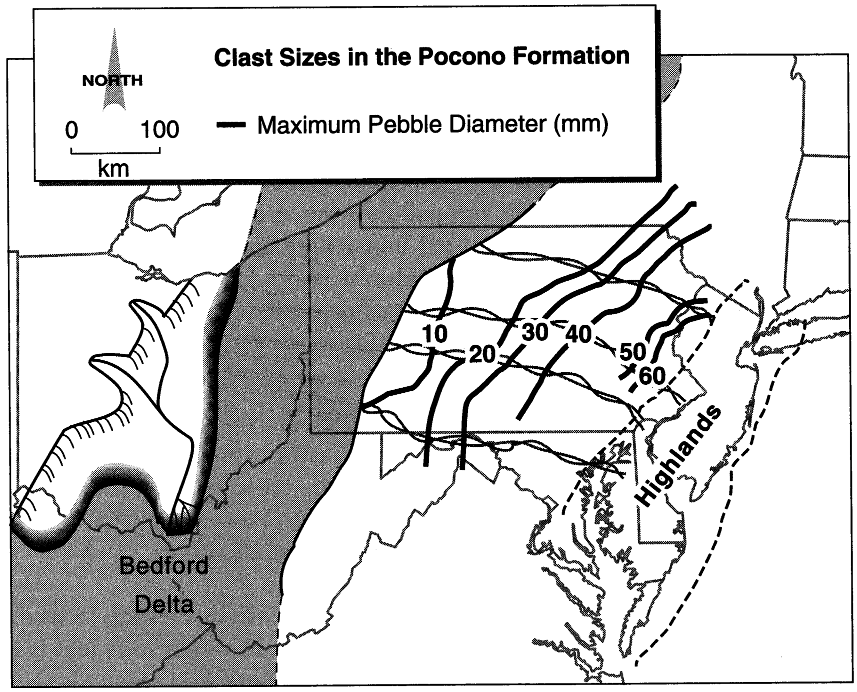

Figure 1-6 Paleogeography of central Atlantic states during Kinderhookian Stage (Early Mississippian). Alluvial deposits of Pocono Sandstone (now called Rockwell and Burgoon Fms.) become finer downstream, following nonlinear function (from Pelletier, 1958; Donaldson and Shumaker, 1981).

A traditional answer to our question might be that coarser facies would migrate basinward, producing a prograding, coarsening-upwards sequence both near and at intermediate distances from the source terrain (e.g., Krynine, 1942; Armstrong and Oriel, 1965; Potter and Pettijohn, 1977; Wiltchko and Dorr, 1983; DeCelles *et al.*, 1987). But this is not always true because the overall geometric form of the basin fill depends upon the relative rates at which the basin's volume is created and sediment is deposited. Jordan, Flemings, and Beer (1988) and Flemings and Jordan (1989) argued that it is not yet possible to relate facies migration to changes in deformation rates. If the basin subsides faster than sediment is delivered, then increased accretion rates at the source result in relative deepening of the basin margin accompanied by facies transgression, followed by shallowing of the basin and facies regression as the rate of influx of sediment surpasses subsidence.

Thus we see that previous explanations are not adequate because we need to know the functional relationships among all the components, particularly if other factors such as changes in sea level and climate are considered. For example, climatic change in the source region will affect the denudation rate, in turn affecting the basin's fill by changing the quantity and proportions of types of sediment fed into a basin, and also changing the steady state height and width of the accretionary wedge and subsidence rate. Even order-of-magnitude estimates of these effects are difficult unless we calculate them as components of an integrated dynamic system. Thus, inferences about the tectonic, paleogeographic, and climatic conditions that produced a particular sedimentary sequence (such as that shown in Figure 1-4) require use of a dynamic mathematical system that links both creation of foreland basins and their filling.

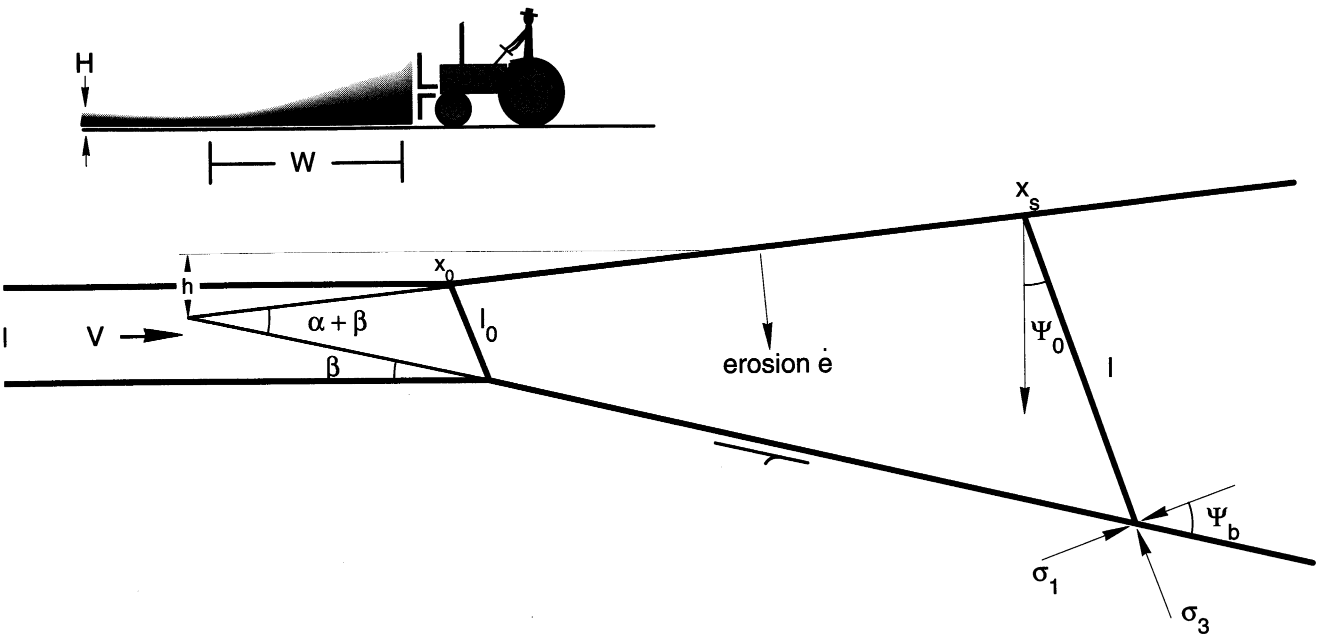

Figure 1-7 Schematic diagram of accretionary wedge where H is thickness of material entering toe, V is convergence velocity, β is basal slope of wedge, α is topographic slope, h is height of wedge surface above vertex, l_0 is initial length at x_0, l is stretched length, $\dot{e}$ is erosive flux off wedge surface, Ψ_0 and Ψ_b are acute angles between axes of principal compressive stress (σ_1) and topographic and basal surfaces, respectively (from Dahlen and Suppe, 1988).

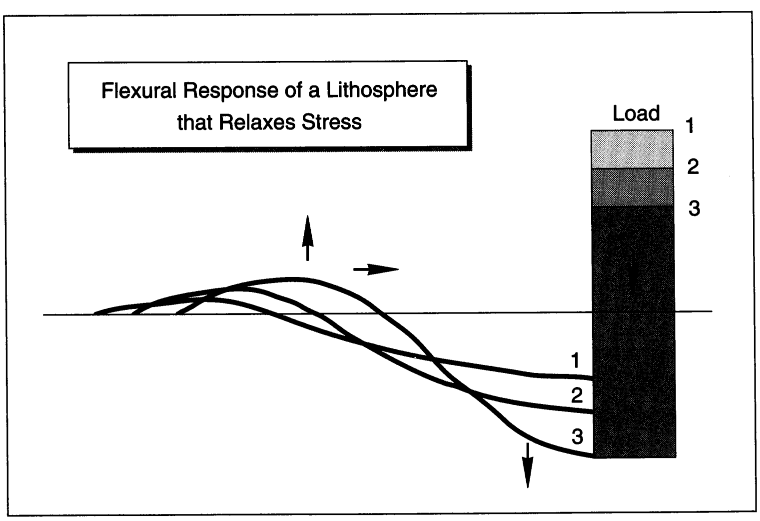

Figure 1-8 Qualitative representation of loading and unloading response of model lithosphere that releases stress by thermally controlled creep (Slingerland and Beaumont, 1989).

STEPS IN MODEL BUILDING

> *"...And believe me, if I were again beginning my studies, I would follow the advice of Plato and start with mathematics."* ---*Galileo*

Throughout this book we will try to follow some logical steps in model development derived from the classroom notes of Leonard Austin, Professor of Mineral Processing at Penn State. First, *get the physical picture clearly in mind.* One must define the physical processes to be treated and the boundaries of the model. Second, *write down the physical laws to be used.* These will be laws such as conservation of mass, Fick's law, and so on. Third, *put down very clearly the restrictive assumptions made.* If one assumes that sediment transport on hillslopes can be modeled as a diffusion process, write that assumption here. Fourth, *perform the balance, first in words, and then in symbols.* The balance could be a force balance or a mass balance, for example. Fifth, *check units.* Sixth, *write down initial and boundary conditions.* By initial conditions are meant the values of the dependent variables at the start of the calculations. Boundary conditions are the values of the dependent variables at the edges of the spatial domain of interest. Last, *solve the mathematical model.* Often no analytical solutions will be available, and this step will require converting the equation set into a numerical form amenable for solution on a computer.

ABOUT THE COMPUTER PROGRAMS

We do not pretend to offer a balanced treatment of finite-difference techniques. These are codes for teaching purposes and for gaining initial insight. Research caliber codes would use implicit difference schemes for economy and stability and would be streamlined for vector processors.

We presume that users of the book can read FORTRAN. Our FORTRAN is not ANSI standard FORTRAN-77, because we use the DO-ENDDO structure, for example. But we think it is compatible with FORTRAN 90, the new standard FORTRAN. We explicitly define the types of all variables so that integer or floating-point variables can start with any letter.