Statistically Based Morphodynamic Modeling of Dispersion of Tracer Stones

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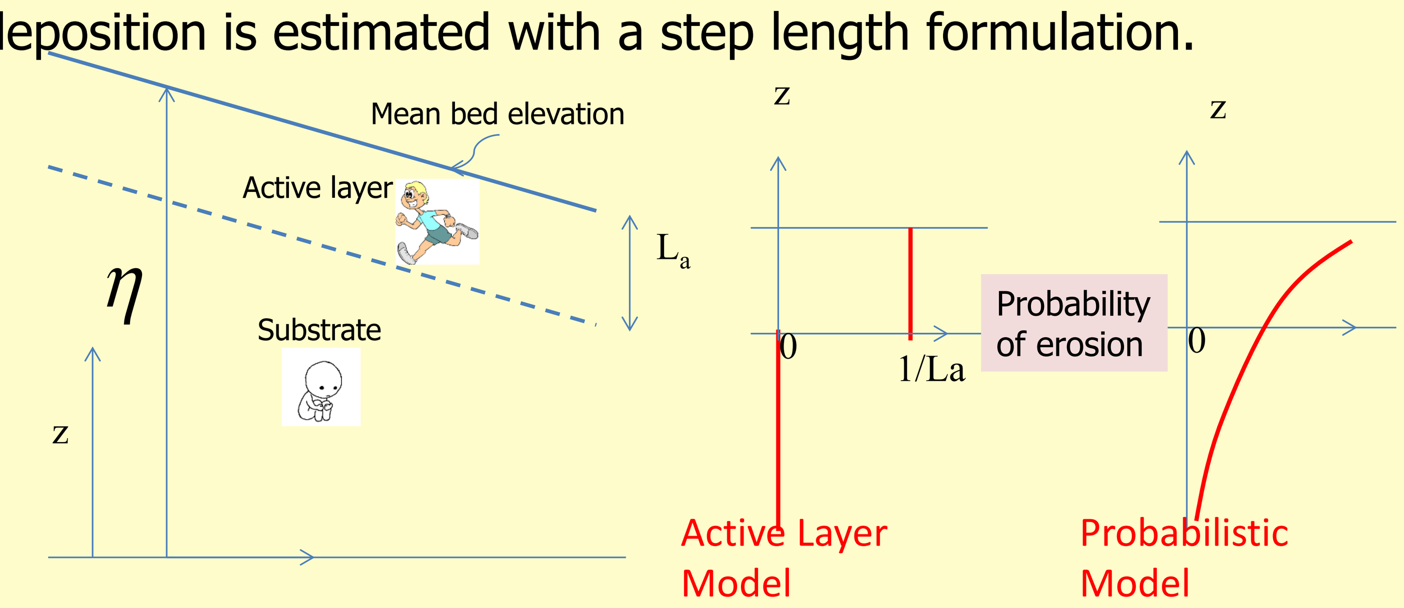
Introduction

The vast majority of the morphodynamic models that account for the non-uniformity of the bed material size are based on the active layer approximation, i.e. the channel bed deposit in two different regions. The active layer, which is the topmost part of the bed deposit, is modeled as mixed layer whose particles can interact with the bed material transport. Particles can be exchanged with the bed material only when the channel bed aggrades or degrades. Morphdynamic formulations based on the active layer approximation, however, have well known limitations:

1) they neglect the vertical fluxes within the deposit associated with e.g. bedform migration;
2) they cannot capture the infiltration of fine sediment tracer stone dispersal;
3) the statistical nature of sediment entrainment is neglected.

To overcome these limitations, Parker and coauthors in 2000 introduced a continuous, i.e. not layer-based, morphodynamic framework based on **a stochastic description of the bed surface elevation, of the entrainment and deposition**. In this framework particle entrainment rates are computed as a function of the flow and sediment characteristics, while particle deposition is estimated with a step length formulation.

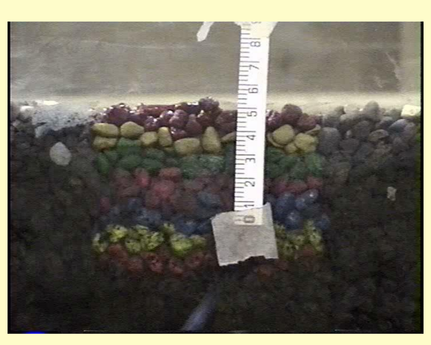


Research Goal

Here we present one of the first implementation of the continuum framework at laboratory scale and its validation against laboratory experiments on tracer stones dispersal.

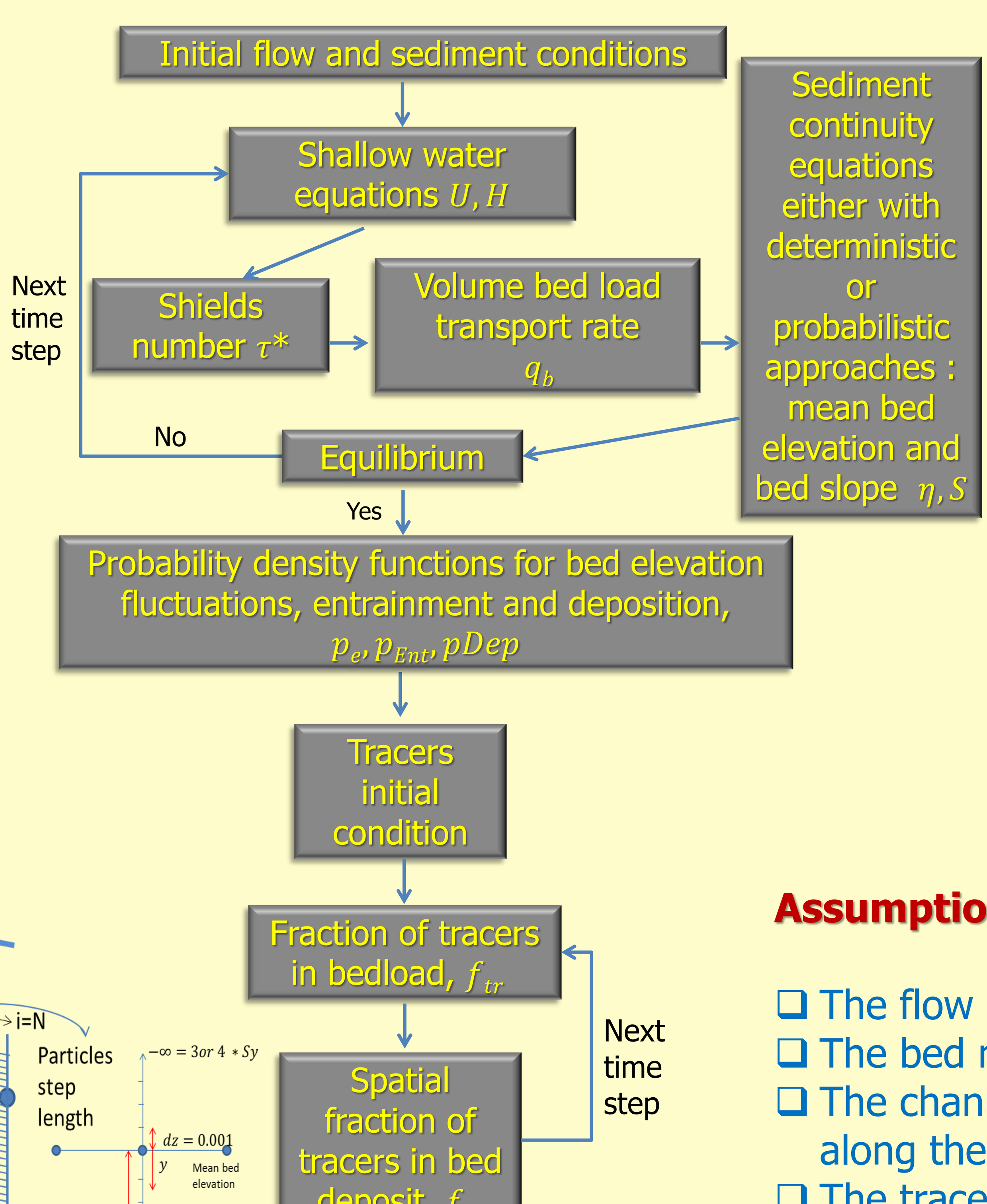
Numerical Model

Input information	Run 1
Water discharge, Q (m3/s)	0.093
Width of the flume, B (m)	0.508
Length of the flume, L (m)	22.5
Sediment supply rate, Q_{feed} (kg/s)	0.150
Median grain size of bed sediment, D_{50} (m)	0.0071

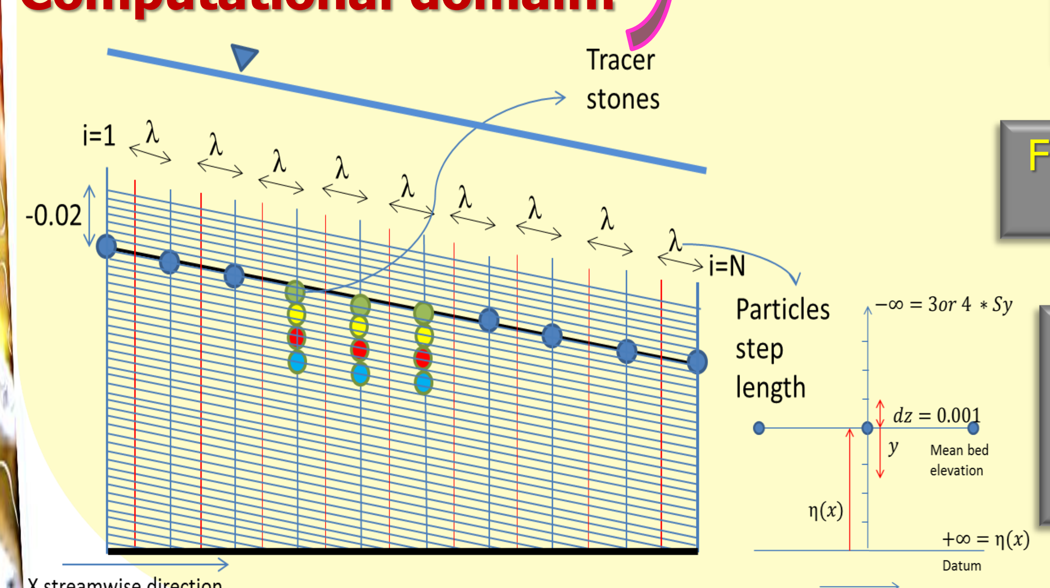


Wong and Parker, 2005

Model framework:



Computational domain:



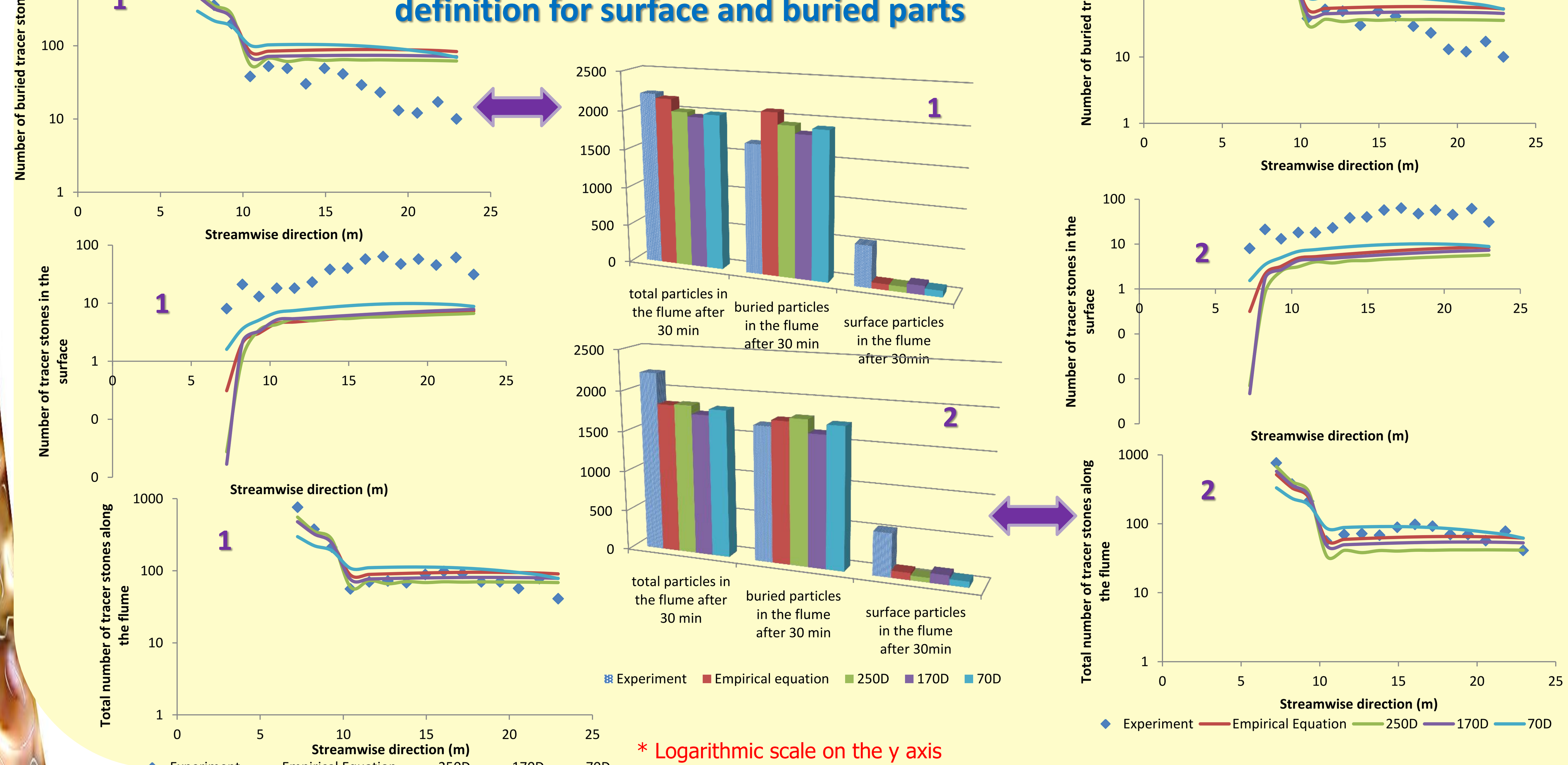
Equations and notations		
Shallow water	1) $q = UH$ 2) $S = C_f \frac{U^2}{gH}$ $\tau^* = \frac{\tau_b}{\rho R g D_{50}}$	U: flow velocity (m/s) H: water depth (m) η: Bed elevation (m) R: Submerged specific gravity g: Acceleration of gravity (m/s²) λ_p: Bed material porosity C_f: Friction coefficient τ^* : dimensionless bed shear stress (or Shields number) τ_b : bed shear stress τ^*_c : reference Shields number for particle incipient motion (0.0549) E: Entrainment rate D: Deposition rate from bedload to the bed standard deviation of bed elevation fluctuations: $s_y = D_{50} * 3.09 (\tau^* - 0.0549)^{0.56}$ Offset characterizing mobile-bed equilibrium: $y_0 = 0.25 * D_{50}$ Additional offset generated by non-equilibrium condition: $y_1 = 0$
Exner	$(1-\lambda_p)\frac{\partial \eta}{\partial t} = D - E - (1-\lambda_p) A_1 \frac{\partial \tau^*}{\partial t}$, $A_1 = \int_{-\infty}^{\infty} \frac{\partial P_s}{\partial \tau^*} dy$	
Entrainment and deposition rate	$E = \sqrt{RgD_{50}} * 0.05 (\tau^* - 0.0549)^{1.85}$ $D(x) = E(x - \lambda)$	
Probability of bed elevation Fluctuation, deposition and entrainment	$p_e(y) = \frac{1}{\sqrt{2\pi} s_y} \exp\left[-\frac{1}{2}\left(\frac{y}{s_y}\right)^2\right]$ $p_{Ent}(y) = \frac{1}{2s_y} \exp\left(-\frac{\lvert y - y_0 + y_1 \rvert}{s_y}\right)$ $P_s(y) = \int_{-\infty}^{y} p_e(y) dy$ $p_{Dep}(y) = \frac{1}{2s_y} \exp\left(-\frac{\lvert y - y_0 - y_1 \rvert}{s_y}\right)$	
Particle step length	$\lambda \cong D_{50} * 53.2 (\tau^* - 0.0549)^{-0.35}$	

Assumptions and approximations:

- The flow is assumed to be uniform and also the quasi-steady
- The bed material is uniform and transports as bedload.
- The channel cross section is rectangular and the channel width is constant along the channel.
- The tracer stones have the same size and density of the bed material

Results

Comparison model results with experimental data: Applying different step lengths and different definition for surface and buried parts



* Logarithmic scale on the y axis

Observation and Future Plan

- In overall with both averaging approach the number of particles in surface is lower compare to the experimental results.
- Adding aggradation and degradation instead of equilibrium bed elevation condition to see the effect of bed forms.

References

Parker, G., Paola, C. & Leclair, S., 2000. Probabilistic Exner sediment continuity equation for mixtures with no active layer. Journal Hydraulic Engineering, 126(11), 818-826.
Wong, M. & Parker, G., 2005. Flume experiments with tracer stones under bedload transport. Proc. River, Coastal and Estuarine Morphodynamics, Urbana, Illinois, 131-139
Blom, A., 2003. A vertical sorting model for rivers with non-uniform sediment and dunes. PhD thesis, University of Twente, the Netherlands, 267 pp.