Secondary Current Simulations in Open Channels with Different Bed Roughness Configurations

Implementation of a non-linear $k-\omega$ model

Mahdad Talebpour & Xiaofeng Liu

Penn State University, Civil & Environmental Engineering Department

mahdad@psu.edu & xliu@engr.psu.edu

Introduction

Secondary currents in open channels have been long recognized as an important mechanism to alter the path of sediment particle motion and consequently change the river and land surface evolution. As classified by Prandtl (1952), secondary currents in fluid flows are of two categories: first and second kind. The mechanism of generation behind generation and maintenance of second type of secondary current is still not fully understood (Yang et al. (2012)).

Figure 1: Left: Longitudial dunes in Medano Creek (www.wikipedia.org); Right: Longitudial dunes in a desert (*Photograph by George Steinmetz: www.georgesteinmetz.com*)

Researchers have been numerically investigating second type of secondary current in open channel flows with considering rough bed boundaries for a long time. This kind of secondary current is not observable in simulations using standard two-equation models. Standard two-equation models are based on linear assumption for modelling Reynolds stresses using Boussinesq approximation. So, as reported in literature they are not able to capture turbulence anisotropy which is believed to be the source of secondary current of second type (Kang and Sotiropoulos (2012)).

The aim of this study is to implement a new CFD non-linear 2-equation model developed by Hellsten (2004) to simulate secondary flows with different bed roughness configurations. The model use a $k-\omega$ formulation and add a non-linear term to accurately model the anisotropy of turbulence which cause the secondary current of second kind. Roughness of bed and walls of open channels play an important role in generation and continuation of secondary currents. So, After tuning the model for secondary current simulation in smooth channels, roughness is employed into the model by a new implemented boundary condition.

Main Objectives

1. Implementing a non-linear $k-\omega$ model proposed by Hellsten (2004) for studying secondary currents in open channel flows.
2. Tuning the proposed boundary conditions by Hellsten (2004) for applying roughness on bed and walls of the channel.
3. Tuning and verifying the model with the existing experimental measurements.
4. Further investigation of secondary currents in various case studies in order to gain more knowledge about mechanism behind their generation and maintenance.

Model Specifications

Hellsten (2004) proposed $k-\omega$

The Reynolds-stress anisotropy is modelled as follows:

$$a_{ij} = \frac{\overline{u_i u_j} - 2/3k\delta_{ij}}{k} \tag{1}$$

The following tensor polynomial specifies the anisotropy tensor:

$$\begin{aligned} a_{ij} &= \beta_1 S_{ij} \\ &+ \beta_3\left(\Omega^*_{ik}\Omega^*_{kj} - \frac{1}{3}II_\Omega\delta_{ij}\right) + \beta_4\left(S_{ik}\Omega^*_{kj} - \Omega^*_{ik}S_{kj}\right) \\ &+ \beta_6\left(S_{ik}\Omega^*_{kl}\Omega^*_{lj} + \Omega^*_{ik}\Omega^*_{kl}S_{lj} - \frac{2}{3}IV\delta_{ij}\right) \\ &+ \beta_9\left(\Omega^*_{ik}S_{kl}\Omega^*_{lm}\Omega^*_{mj} + \Omega^*_{ik}\Omega^*_{kl}S_{lm}\Omega^*_{mj}\right) \end{aligned} \tag{2}$$

Here S_{ij} and Ω^*_{ij} are strain-rate and vorticity tensors, respectively.

Boundary condition for roughness

The proposed boundary condition to implement roughness is based on Wilcox's rough-wall boundary condition method:

$$\omega_w = \frac{u_\tau^2}{\nu} S_R \tag{3}$$

S_R is a non-dimensional function which can be written as follows:

$$S_r = \begin{cases} \left(\frac{50}{max(k_s^+, k^+_{s,min})}\right)^2 & , k_s^+ < 25 \\ \left(\frac{100}{k_s^+}\right) & , k_s^+ > 25 \end{cases} \tag{4}$$

k_s and $k_s^+ = \frac{k_s u_\tau}{\nu}$ in equation 4 are particle roughness height and inner-scaled particle roughness height. Also $k^+_{s,min}$ is defined as follows: $k_{s,min} = min[2.4(y_1^+)^{0.85}, 8]$.

Results and Discussions

For the purpose of validation of the model *case S-75* of Wang and Cheng (2006) experiments was simulated in this study. Figure 2 shows the schematic of the experiment. Results of the the numerical simulation and Wang and Cheng (2006) experimental data are compared (figures 3 and 4). Figures 3 shows change of magnitude of transverse velocity w in cross section of the channel over four horizontal lines parallel to bed. As it can be seen the model simulated data is in a very good agreement with the experimental data. However, in the lower left figure which shows the transverse velocity at $y = 40mm$, the experimental and CFD simulation velocities have opposite signs. This can be due to the magnitude being so small in that level which result in sign being flipped very easily.

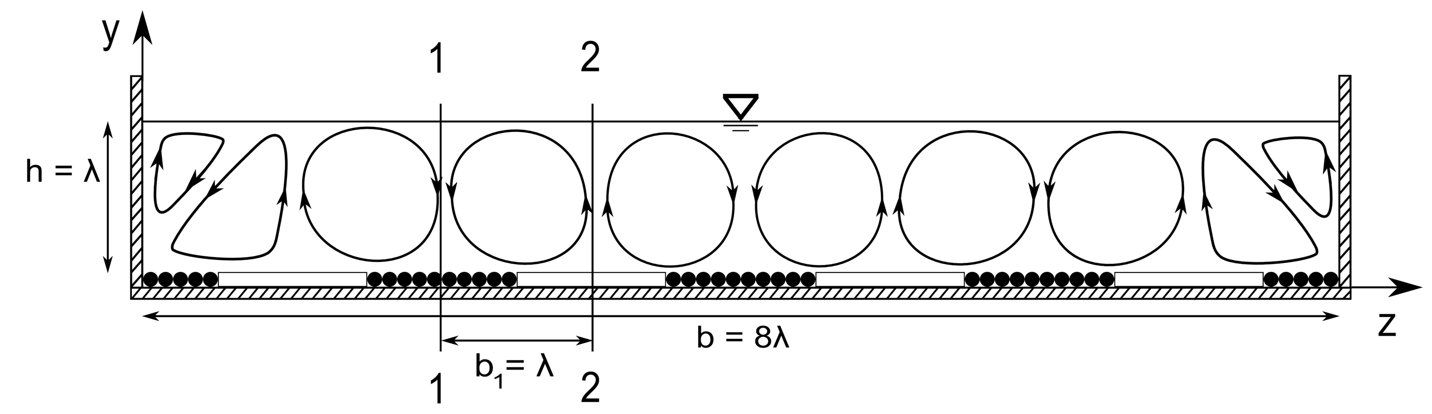

Figure 2: Schematic of Wang and Cheng (2006) case S-75 experiment setup and secondary currents. The width of and depth of channel are $b = 8\lambda = 0.6m$ and $h = \lambda = 0.075m$, respectively. The width of secondary currents is also $b_1 = \lambda = 0.075m$.

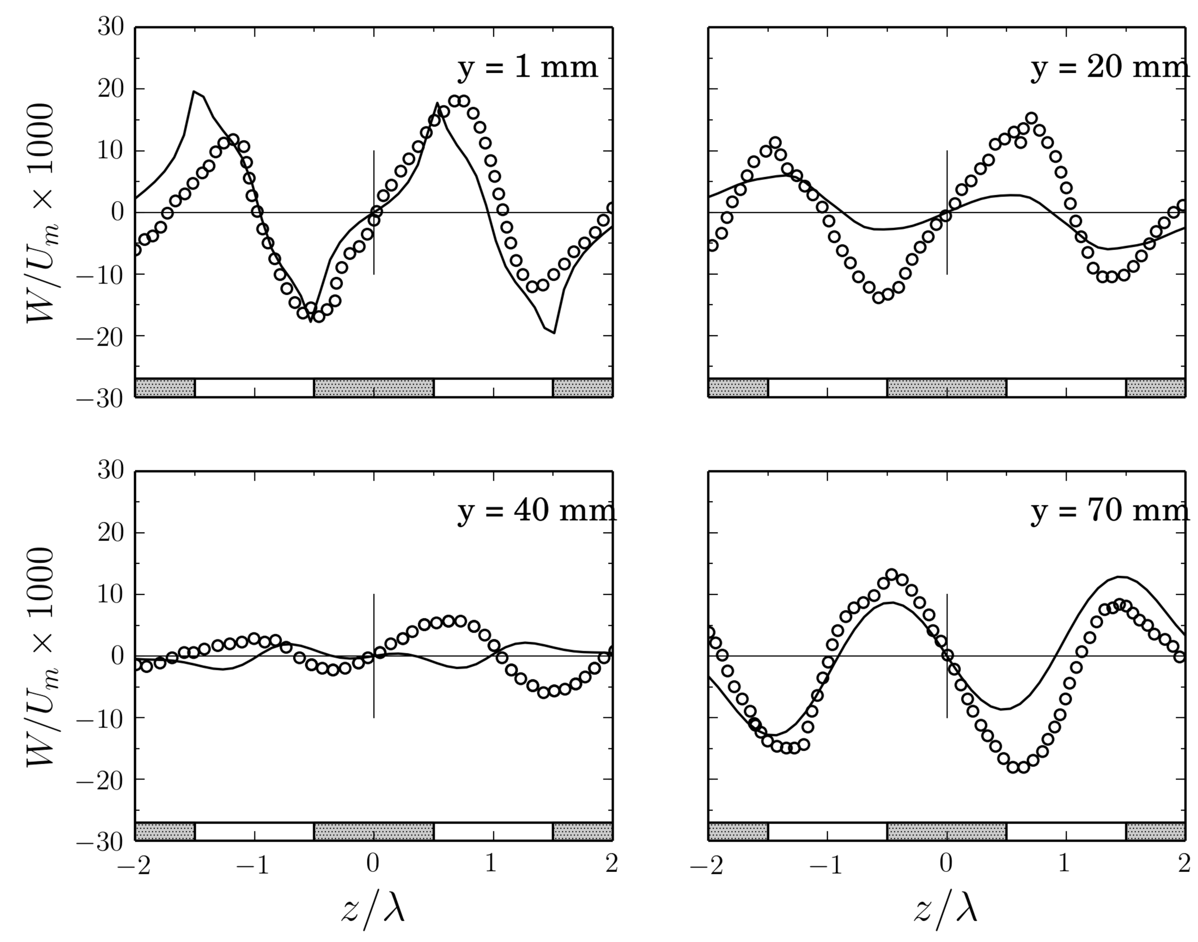

Figure 3: Comparison between measured and simulated transverse velocity W along width of the channel at four different height.

Figure 4 shows a different comparison of results of CFD simulation results versus Wang and Cheng (2006) experimental results. In this figure the transverse velocity (w) magnitude is demonstrated on ten vertical lines. As it can be seen the overall pattern is in a good agreement between simulated and experimental data. But a difference in pattern is noticeable in the lower section in the central lines of the figure between experimental and simulation results. This can be due to the implementation of the roughness on the bottom. Especially the extreme change in ω on the bottom in the intersection of rough and smooth boundary might cause this problem. Further investigation is needed to address this matter.

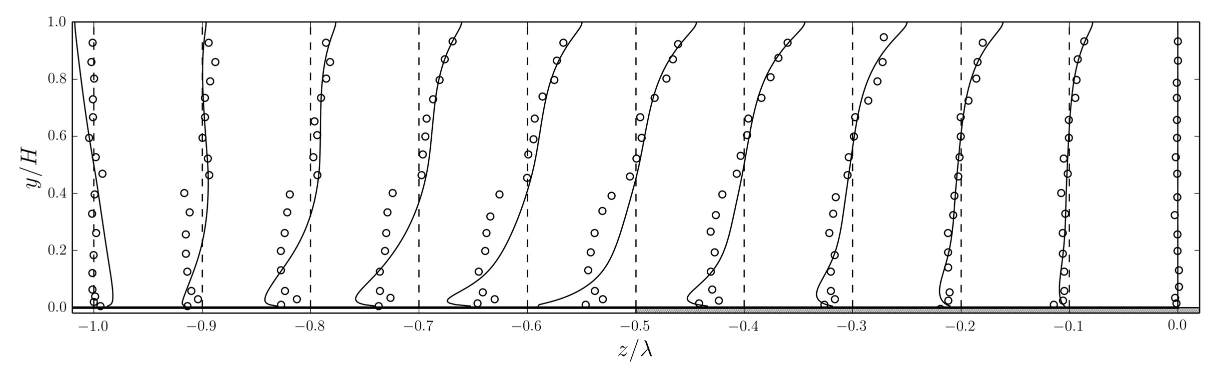

Figure 4: Comparisons between measured and simulated transverse velocity W along 10 vertical line sections.

Conclusions

- The new model as demonstrated in this study is capable of providing almost accurate results for simulation of secondary flows in open channel flows.
- Roughness can be implemented in the new model, but further investigation is needed to tune the model for accurate simulation of rough boundaries.
- This model can be used in future studies of numerous case studies of secondary currents in open channel flows with various configuration to investigate the mechanism behind its generation and continuation in the channel.

Forthcoming Research

- The proposed model by Hellsten (2004) is validated for a $2-D$ flow. More investigation is needed to tune the model for a $3-D$ flow.
- Further investigation is needed for calibrating the boundary condition for drastic change in roughness.

References

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