A Graph-Theoretic Approach to Studying Deltaic Systems: Quantifying Complexity and Self-Organization

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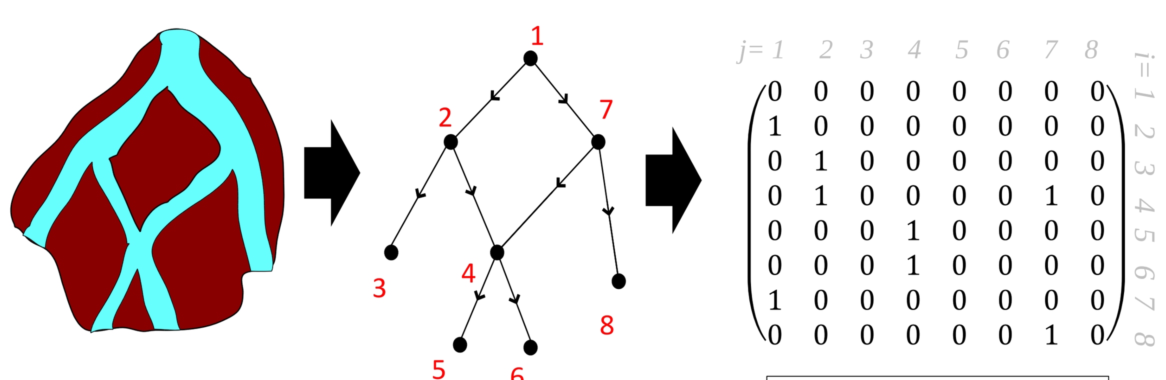
Motivation

- River deltas are intricate landscapes with complex channel networks that self-organize to deliver water, sediment, and nutrients from the apex to the delta top and eventually to the coastal zone.
- A formal quantitative framework for studying delta channel network connectivity and transport dynamics, and the response to change is lacking.
- The aim is to develop a framework within which a delta channel network can be studied for:
  (1) Understanding its **connectivity** structure and **flux** transport
  (2) Understanding the response of the system to change: **Vulnerability** Assessment
  (3) Quantifying **complexity**: Topologic and dynamic
  (4) Interrogating **spatially** a delta in terms of both its complexity and its shape.

Representation



Graph Notation:

- Channels → Links and Junctions → Nodes
- In-degree (d^{in}): Number of links entering a node
- Out-degree (d^{out}): Number of links leaving a node

Algebraic Representation:

- All the topologic information is encoded in the Adjacency matrix (e.g. $d^{in} = \sum_{i=1}^{N} A_{ij}$ and $d^{out} = \sum_{j=1}^{N} A_{ij}$)
- Out-Laplacian and In-Laplacian matrix

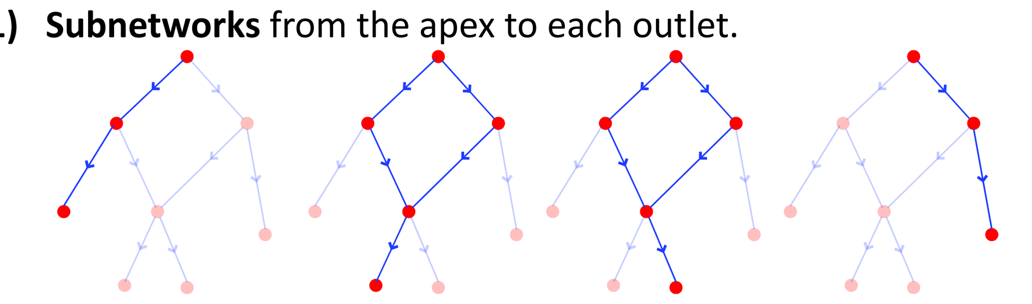
$$L^{out} = \mathbf{D^{out}} - A = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 2 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \end{pmatrix}$$

- Weighted Adjacency Matrix:
  Connectivity + strength of the connection.

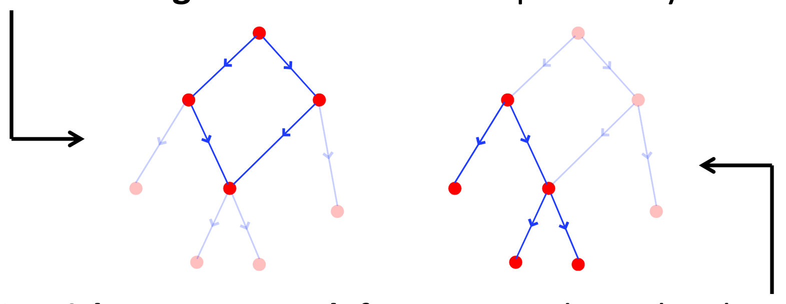
Spectral Graph theory

From the null space of the *proper* Laplacian matrix, we can compute:

(1) Subnetworks from the apex to each outlet.

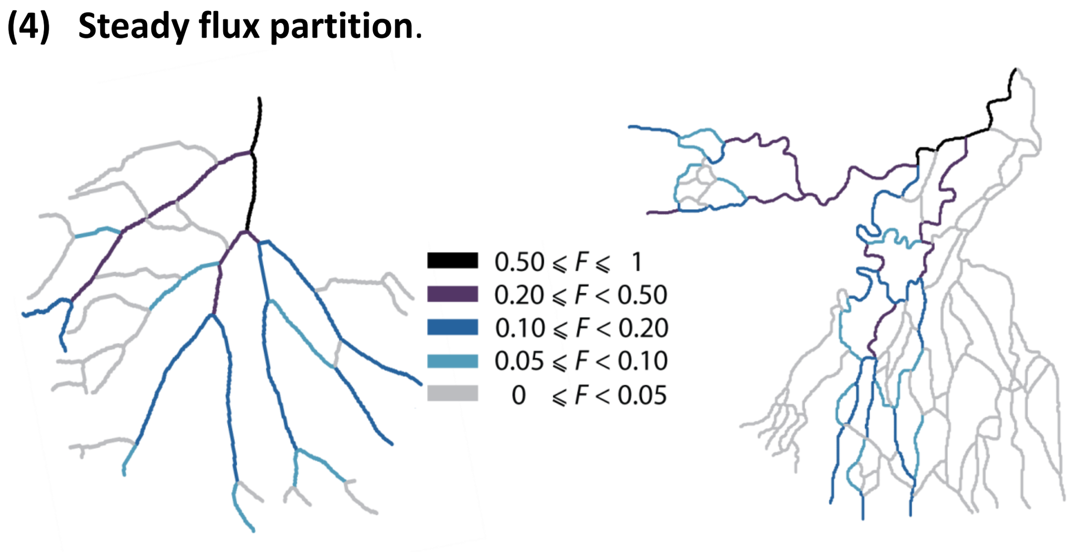


(2) Contributing network from the apex to any node.



(3) Nourishment network from any node to the shoreline.

(4) Steady flux partition.

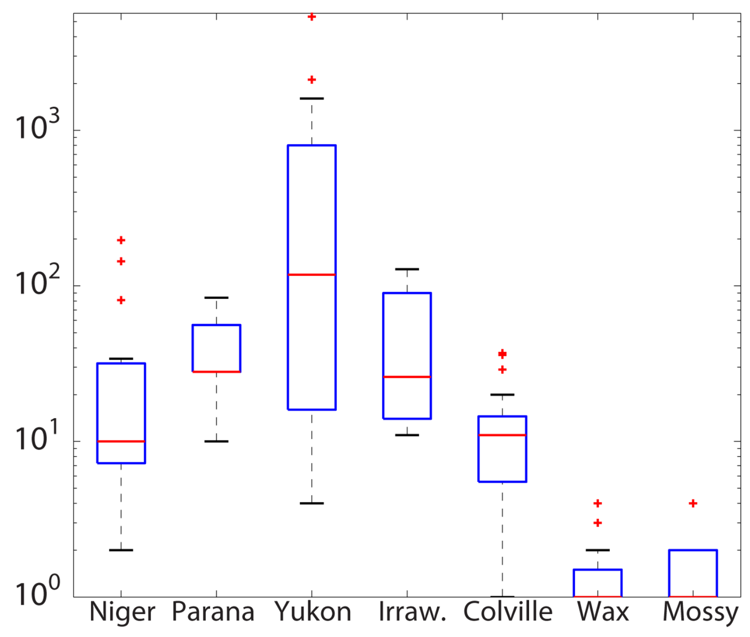


Quantifying Complexity

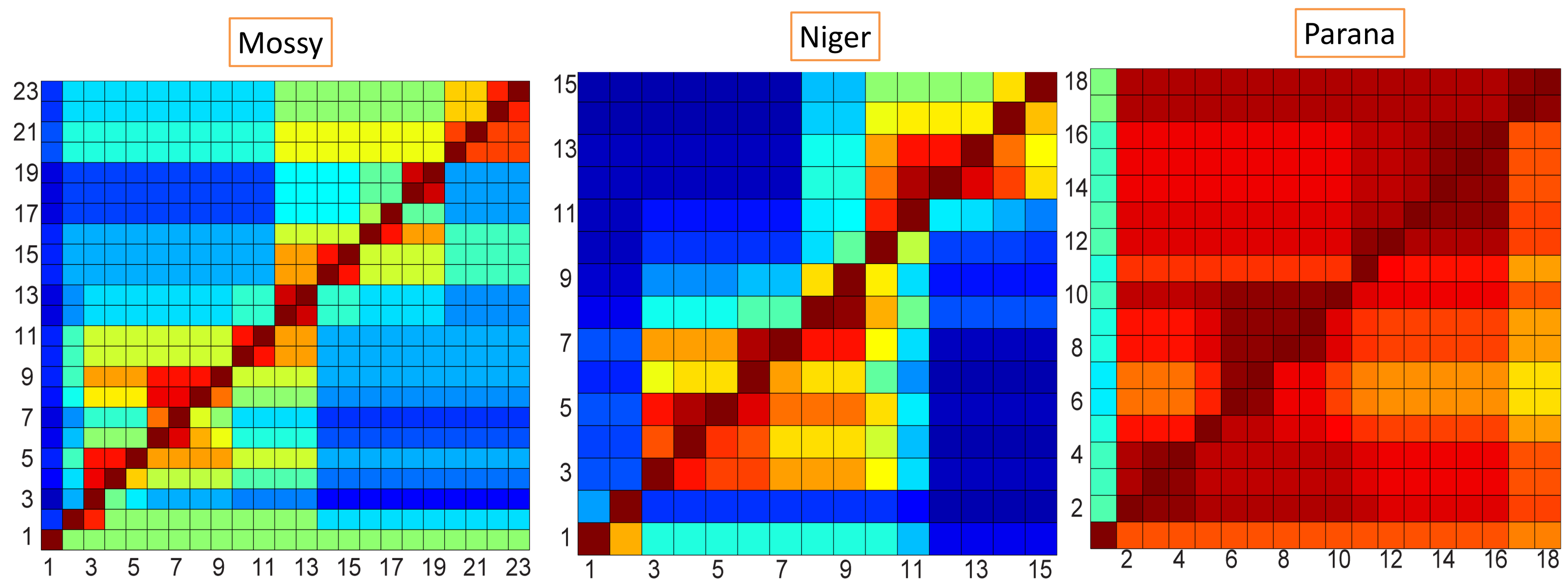
We define a suite of metrics that depicts the topologic (structure of pathways) and dynamic (flux) complexity of deltas.

Topologic:

Number of alternative paths, N_{ap} from the apex to each outlet quantifies the **loopiness** of the subnetworks.

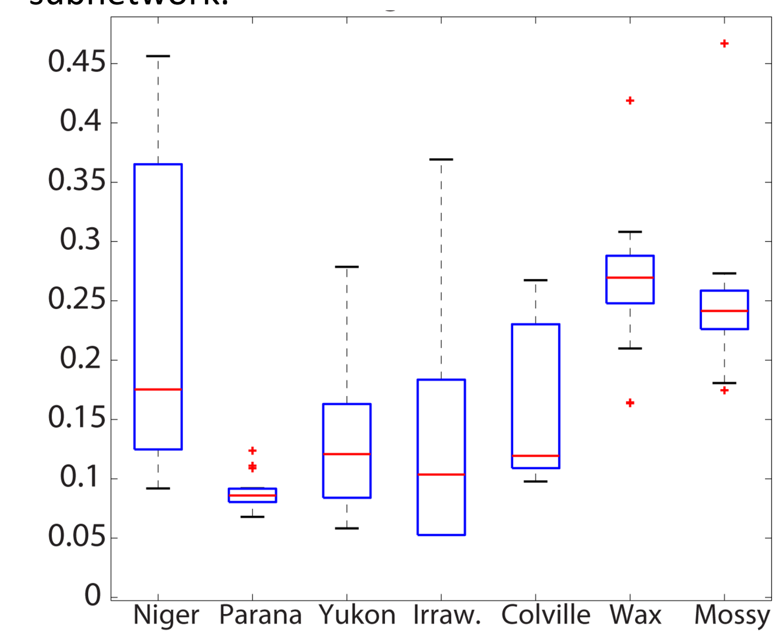


Topologic Dependence, TD measures the **overlapping** among subnetworks in terms of **links** (subnetwork to subnetwork)

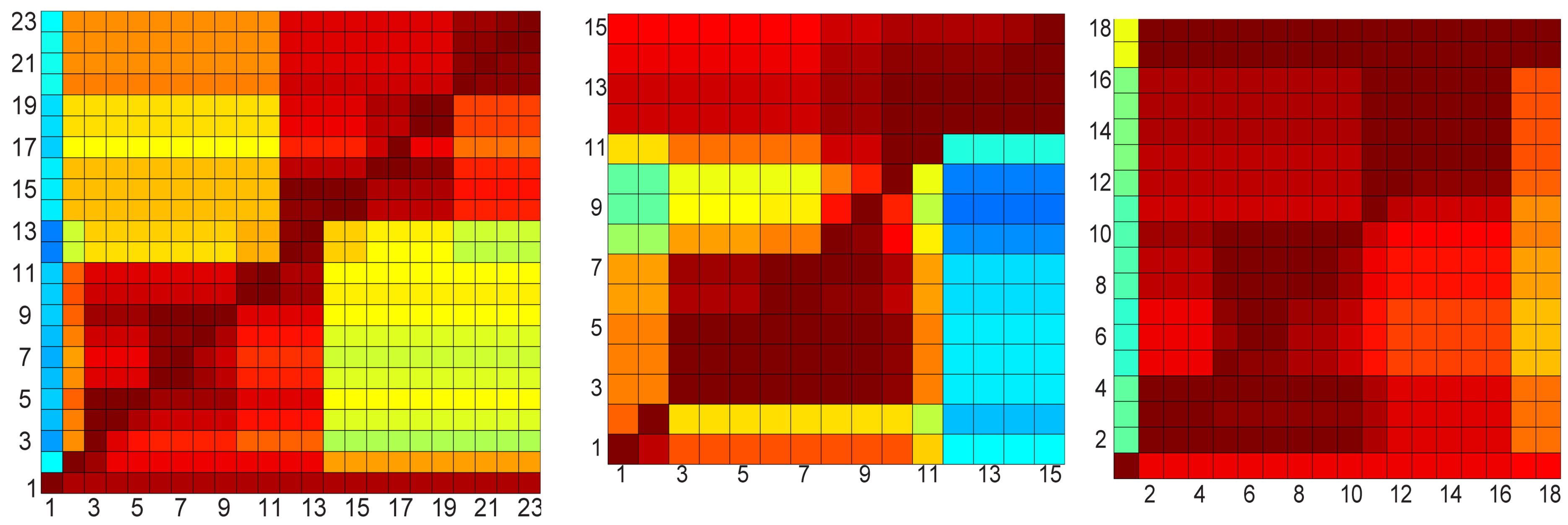


Dynamic:

Leakage Index, LI quantifies the **interaction** among subnetworks in terms of the proportion of flux *leaked out* by a subnetwork.

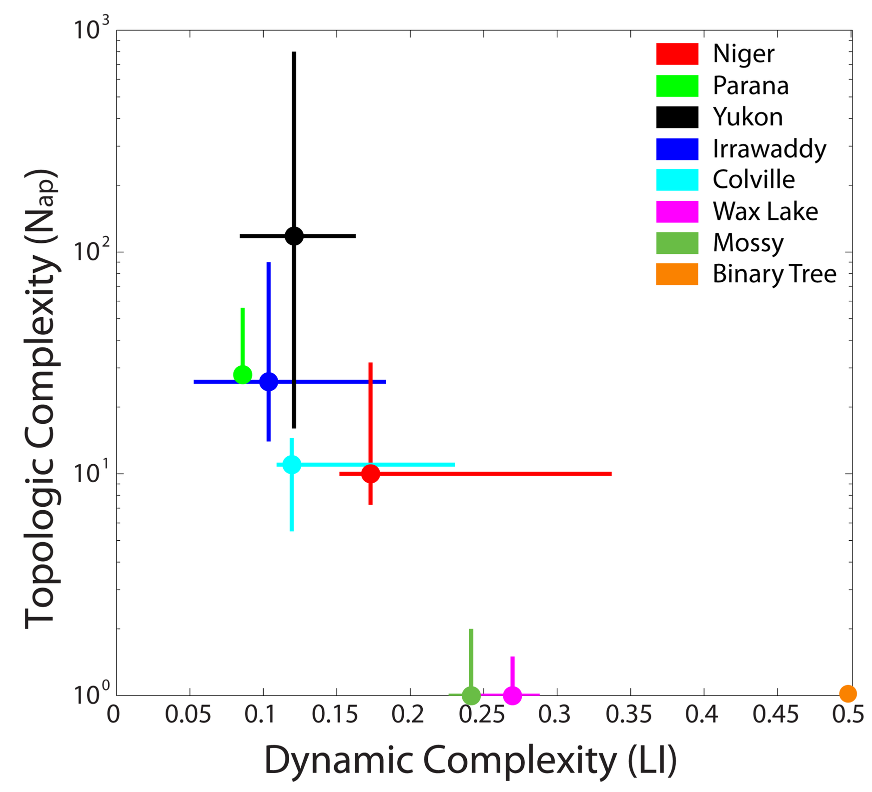


Dynamic Dependence, DD measures the **overlapping** among subnetworks in terms of **flux** (subnetwork to subnetwork)



First steps to define a complexity delta space

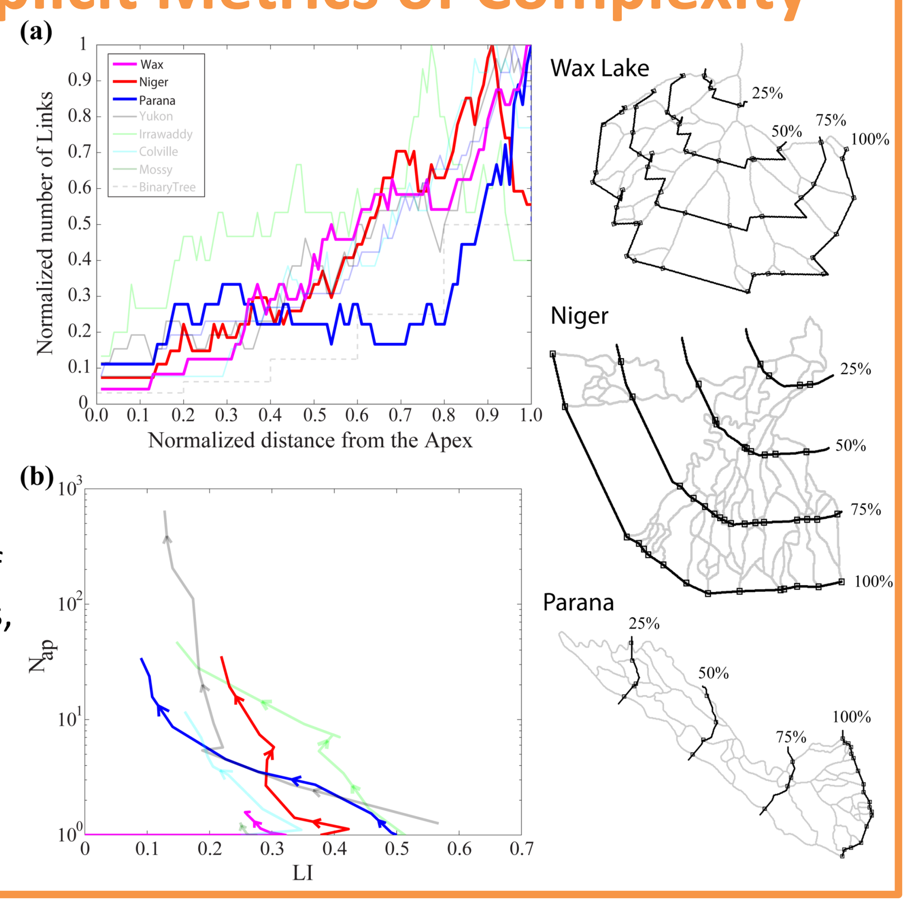
Different deltas exhibit different degrees of topologic and dynamic complexity. Using the metrics that we present in this work, we can define a *complexity delta space*. Defining and populating such a space are the first steps to establish a quantitative framework for better understanding of the underlying physics responsible for the emergent complexity, and to compare different deltaic systems.

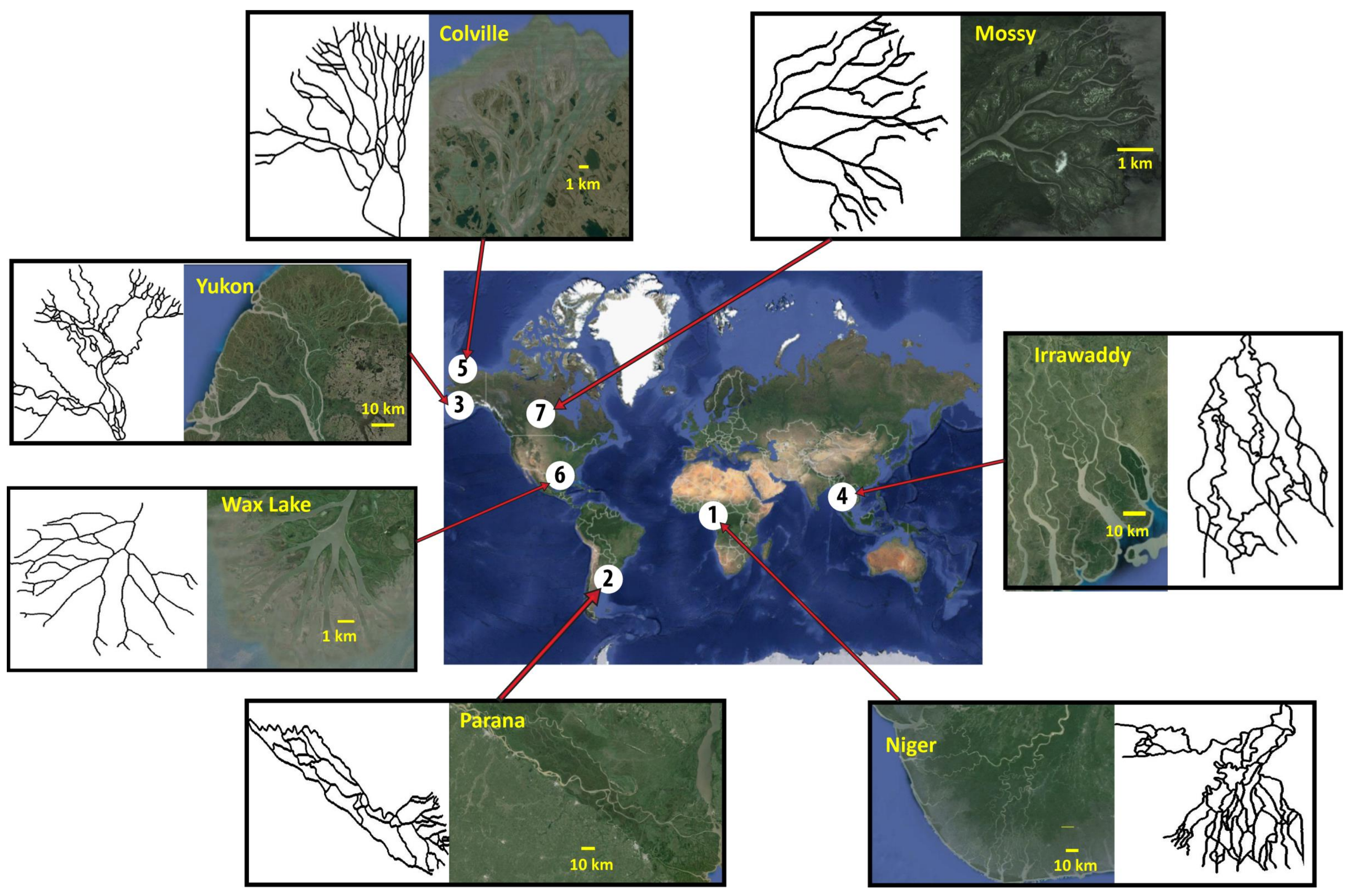


Spatially Explicit Metrics of Complexity

(a) The ***Delta With Function***, normalized number of links vs. normalized distance from the apex, reveals the shape of the delta (convergent, divergent or confined).

(b) An explicit spatial interrogation of delta complexity, in terms of the complexity metrics, can help us to understand the self-organization of deltaic systems.





Future Work

Under the premise that different morphodynamic processes leave different signatures on topology and dynamics of deltas, the developed metrics are expected to be used for:

1. Understanding delta physical processes from form
2. Temporal evolution

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References:

1. Tejedor, A., Longjas, A., Zaliapin, I. and Foufoula-Georgiou, E. (2015), Delta channel networks: 1. A graph-theoretic approach for studying connectivity and steady state transport on deltaic surfaces. Water Resour. Res.. Accepted Author Manuscript. doi:10.1002/2014WR016577
2. Tejedor, A., Longjas, A., Zaliapin, I. and Foufoula-Georgiou, E. (2015), Delta channel networks: 2. Metrics of topologic and dynamic complexity for delta comparison, physical inference, and vulnerability assessment. Water Resour. Res.. Accepted Author Manuscript. doi:10.1002/2014WR016604