A Morphodynamic Explanation for the Shoreface Depth of Closure

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Abstract

This research aims to understand the evolution of the shoreface of sandy, wave-dominated coasts. Using energetics-based formulations for wave-driven sediment transport, we develop a robust methodology for estimating the morphodynamic evolution of a cross-shore beach profile. The derived cross-shore sediment flux formula enables the calculation of a steady state (or dynamic equilibrium) profile based on three components of wave influence on sediment transport: two onshore-directed terms (wave asymmetry and wave drift) and an offshore-directed slope term.

Equilibrium profile geometry depends on wave period and grain size. The profile evolution formulation yields a morphodynamic Péclet number. The diffusional, offshore-directed slope term dominates long-term profile evolution. A depth-dependent characteristic timescale of diffusion allows the estimation of an effective morphodynamic depth of closure for a given time envelope.

Theoretical modeled computations are compared to six field sites along the Eastern US coastline. Using characteristic wave quantities for each site, we compute the equilibrium profile and the morphodynamic depth of closure, showing reasonable similarities between the computed equilibrium profiles and the actual profiles. Overall, the methodology espoused in this paper can be used with relative ease for a variety of sites and with varied sediment transport equations.

What do we want to do?

- Describe the evolution of a sandy, wave-dominated shoreface using sediment transport relationships.
- Quantify shoreface response to environmental changes such as sea-level rise.

Approach (and novelty)

- Include onshore- and offshore-directed terms.
- Use linear wave theory (not shallow water wave assumptions).

What steps do we take?

1). Couple linear wave theory and energetics based sediment transport
2). Compute equilibrium profile
3). Link Exner Equation with evolution of the bed morphology
4). Calcluate Kinematic Celertiy & Diffusivity
5). Calcluate Morphodynamic Peclet Number
6). Compute Timescale of Morphodynamic Depth of Closure
7). Calculate characterstic wave parameters
8). Compare theoretical predictions to field sites

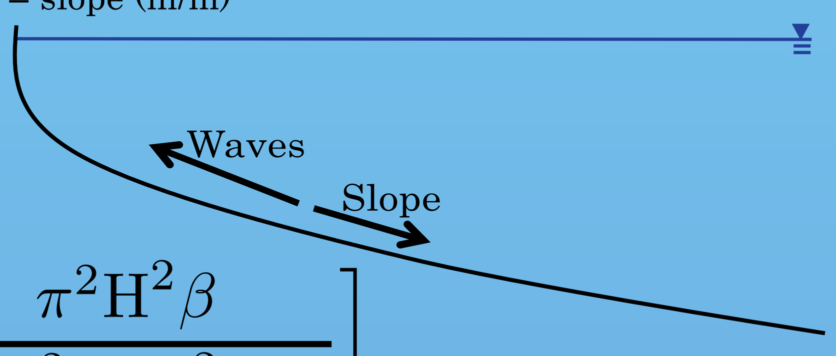
Theory

1). $$q_s = -K\frac{u_o^3}{w_s}\left[5u_1 + 3u_2 + \frac{\beta}{w_s}u_o^2\right]$$ (Bowen, 1980)

Drift — Asymmetry — Slope

$$K = \frac{16 e_s C_s \rho}{15\pi(\rho_s - \rho)g}$$

q_s= cross-shore sediment transport (m^3/s)
e_s= sediment transport efficiency factor (0.01)
C_s= friction factor
ρ, ρ_s= density
u_o= Wave orbital velocity (m/s)
u_1= Stokes wave drift velocity (m/s)
u_2= Wave asymmetry velocity (m/s)
w_s= fall velocity (0.008 - 0.16 m/s)
β = slope (m/m)

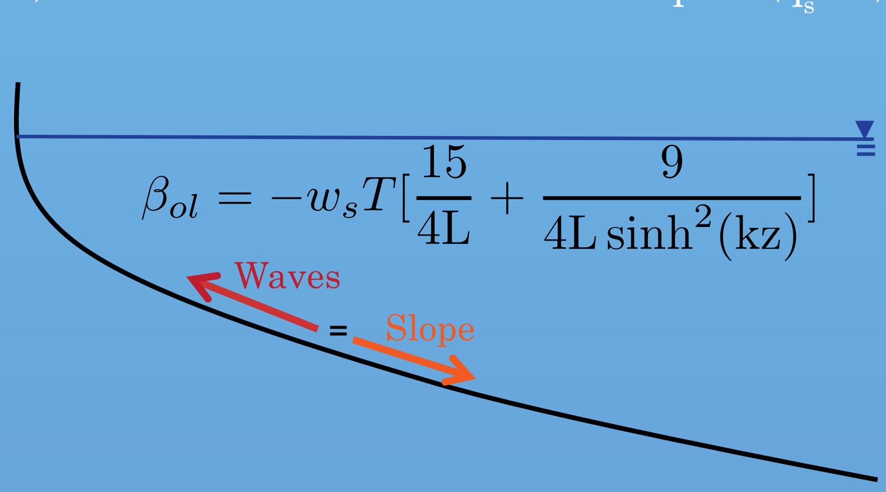


1). $$q_s = -K\frac{\pi^3 H^3}{T^3 L \sinh^3(kz)}\left[\frac{15\pi^2H^2}{4TL\sinh^2(kz)} + \frac{9\pi^2H^2}{4TL\sinh^4(kz)} + \frac{\pi^2H^2\beta}{w_sT^2\sinh^2(kz)}\right]$$

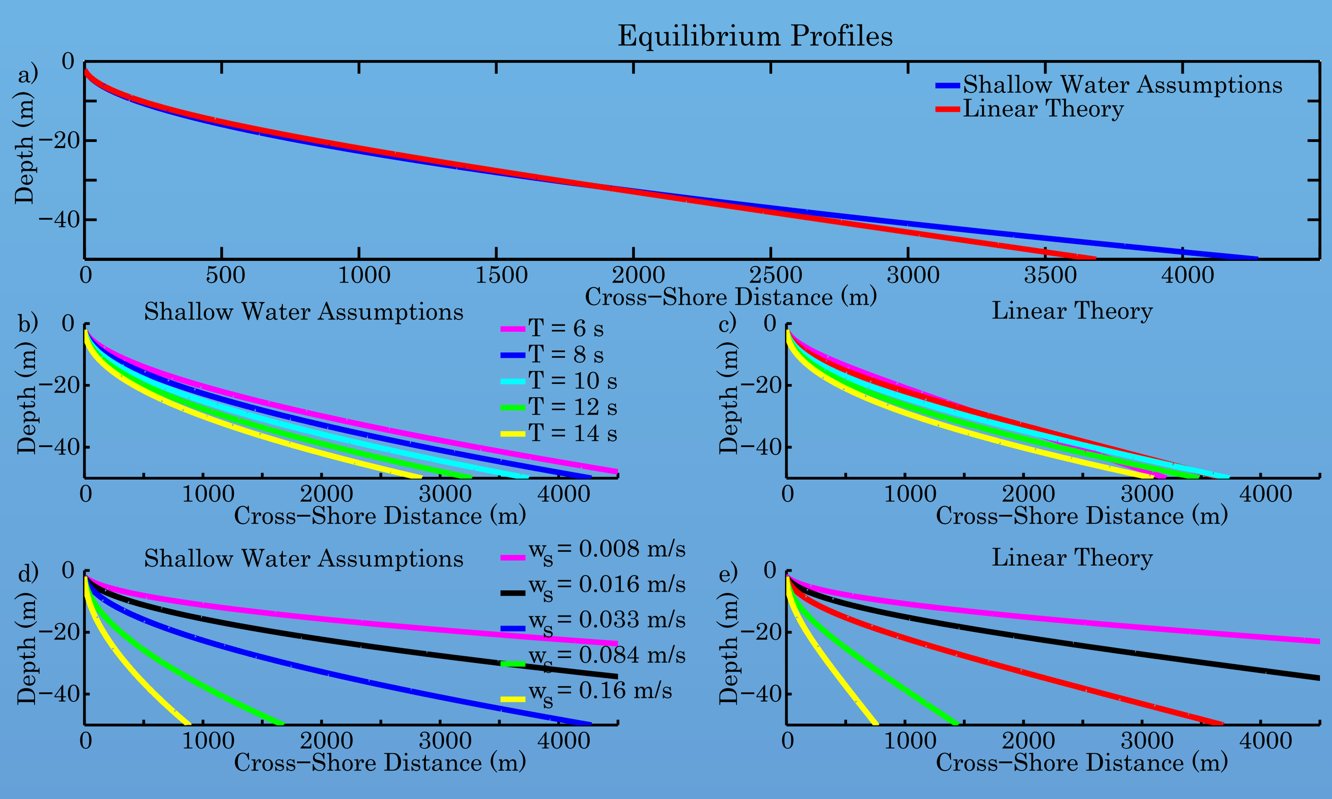
Drift — Asymmetry — Slope

2). Assume no net sediment transport (q_s = 0)

$$\beta_{ol} = -w_sT\left[\frac{15}{4L} + \frac{9}{4L\sinh^2(kz)}\right]$$



Given our sediment transport equation, we solve for an equilibrium slope assuming a balance of sediment transported onshore and offshore.



3).Exner Equation – Conservation of Mass

$$\frac{\partial z}{\partial t} = -\frac{1}{\varepsilon_o}\frac{\partial q_s}{\partial x} \quad q_s = -K\frac{u_o^3}{w_s}\left[5u_1 + 3u_2 + \frac{\beta}{w_s}u_o^2\right]$$

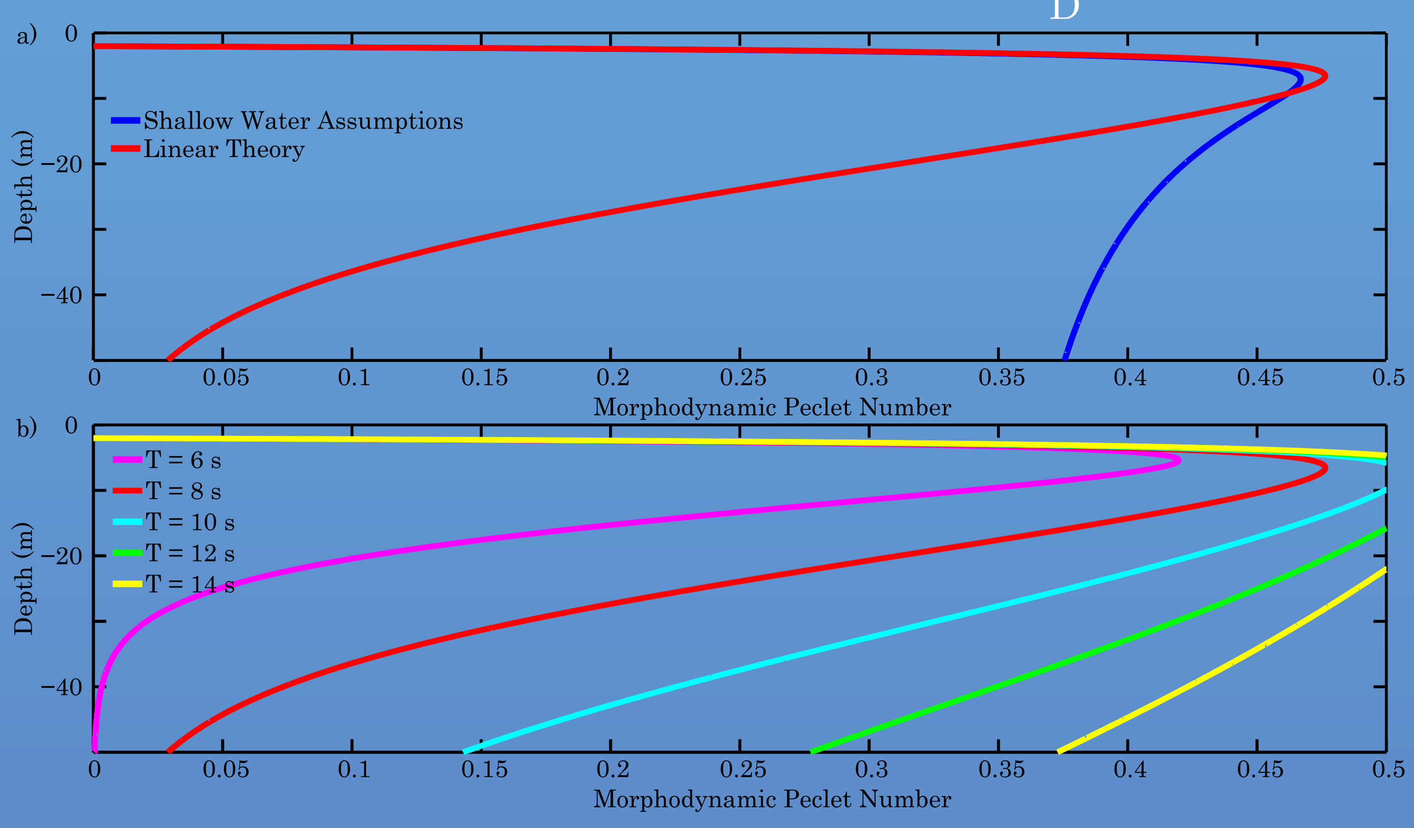
$$\frac{\partial q_s}{\partial x} = \frac{\partial q_s}{\partial z}\cdot\frac{\partial z}{\partial x}$$

4). Advection-Diffusion Equation of bed evolution

$$\frac{\partial z}{\partial t} = \left[(V)\frac{\partial z}{\partial x} + (D)\frac{\partial^2 z}{\partial x^2}\right]$$

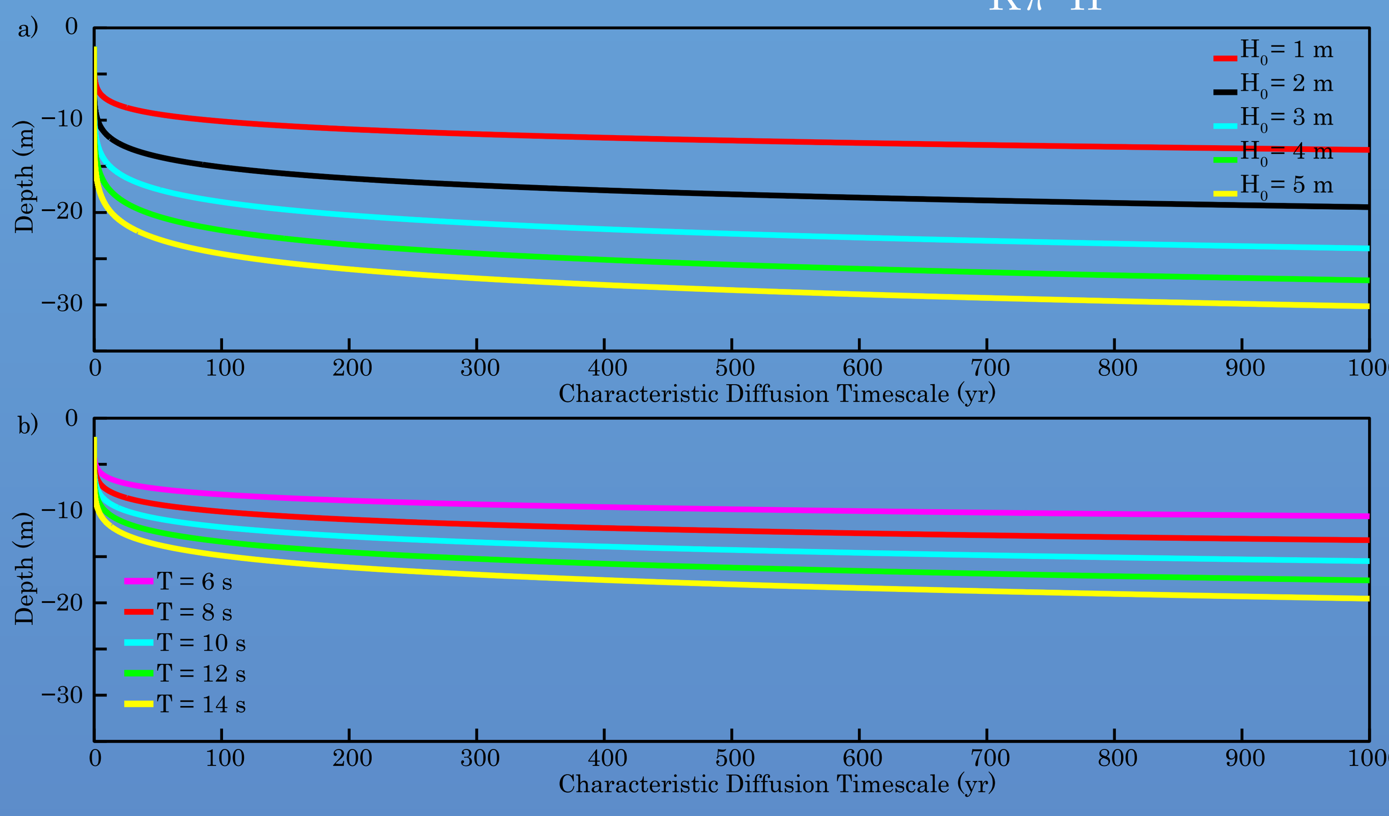
Using conservation of mass, we solve for the evolution of the bed over time. This simplies to an advection-diffusion equation that we then use to calculate kinematic celerity rates (advection) and diffusivity rates.

5). Calculate Morphodynamic Peclet Number: $Pe = \frac{Vl}{D}$



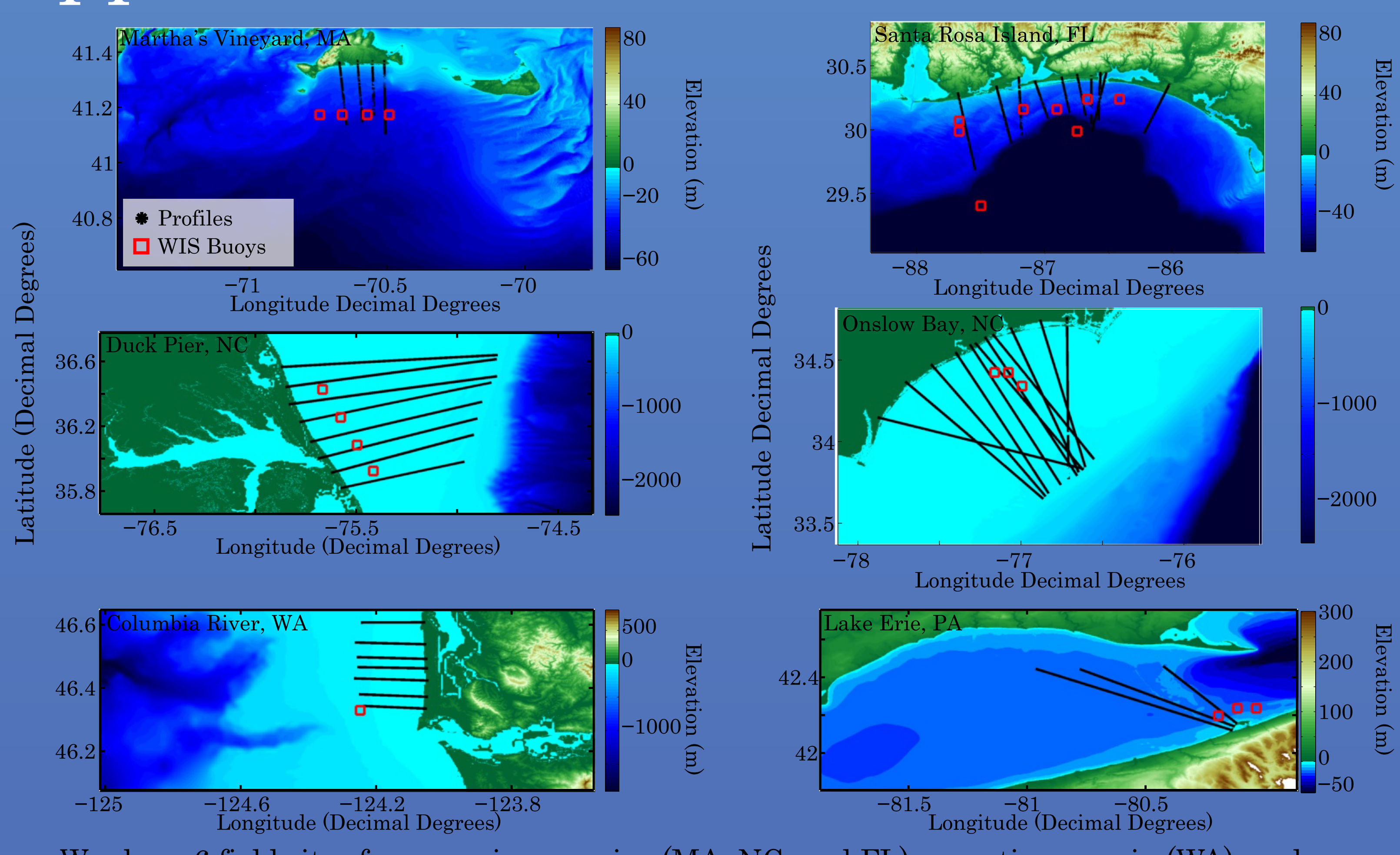
Given the advection and diffusivity rates over depth, we calculate a morphodynamic Peclet number that varies with wave period and depth. The system is dominantly diffusive (Pe < 1). We therefore compute the morphologic timescale using the depth-dependent diffusivity and a characteristic lengthscale, which is the distance to the shoreline for an equilibrium shoreface.

6). Calculate Diffusive Timescale: $$T_{Diff} = -\frac{x_{eq}^2 w_s^2 T^5 \sinh^5(kz)}{K\pi^5 H^5}$$



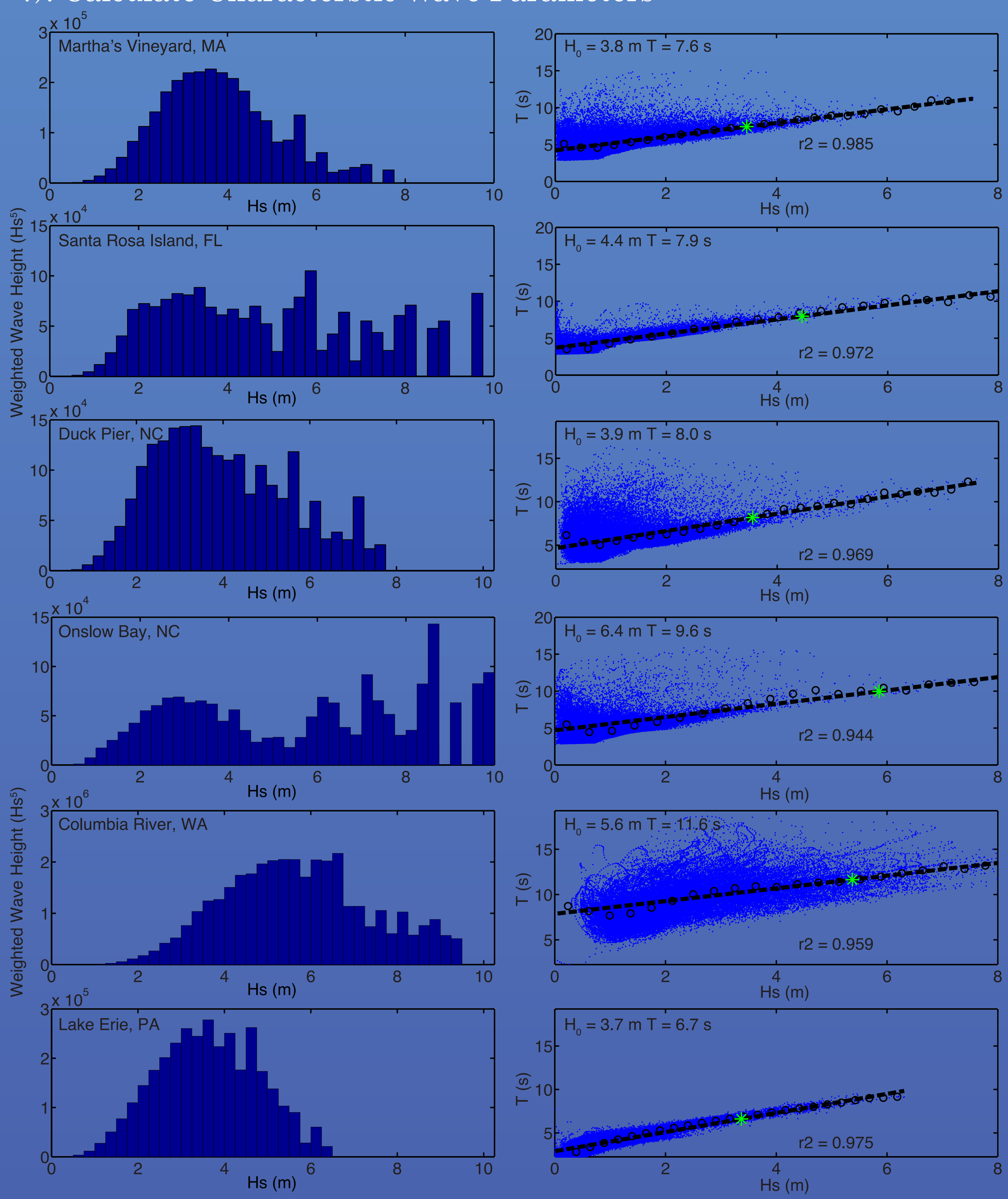
For typical values of deep-water wave height and wave period, shoreface response timescales get significantly large (over a 1,000 years) at depths between 10 and 30 meters, suggesting a type of morphodynamic "depth of closure". In other words, profile evolution and, in particular, sediment transport may continue beyond this depth but evolution of the shoreface shape becomes geologically slow and response to environmental changes is virtually non-existent.

Application



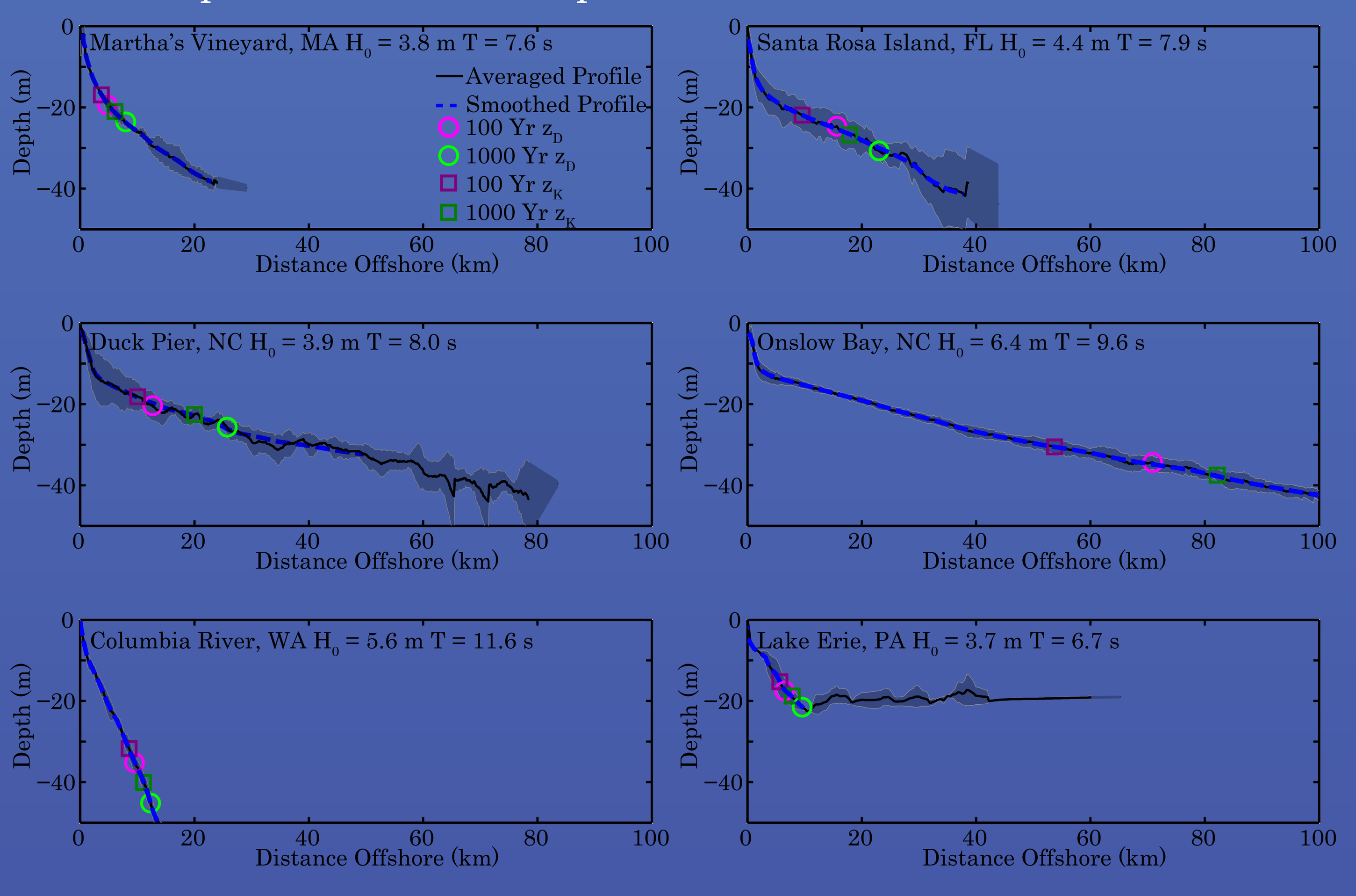
We chose 6 field sites from passive margins (MA, NC, and FL), an active margin (WA), and from a fetch-limited lake (Lake Erie). We calculate characterstic wave height and wave period for each field site.

7). Calculate Characterstic Wave Parameters



We calculate a weighted histogram of the significant wave height to the fifth power for each buoy. The cross-shore sediment flux, qs, is relative to local wave height to the fifth power based on our equation of cross-shore sediment flux. Weighting wave height to the fifth power accentuates the importance of extreme wave events in a record. We plot wave heights versus wave periods and use linear regression to predict a characteristic wave period for a given characteristic wave height.

8). Compare theoretical compuations to field sites



We use the representative wave height and wave period for each field site with the advection-diffusion equation to calculate a morphologic depth of closure from the timescales of kinematic celerity and diffusivity. We then compare these morphologic depth of closure measurements to the actual profiles.

Conclusions

- Understanding equilibrium shoreface dynamics requires consideration of both onshore and offshore terms.
- Model predicts a morphodynamic depth of closure
- Theoretical predictions of the depth of closure can be compared to natural profiles using weighted wave height data

Acknowledgements & References

Bowen (1980), Simple models of nearshore sedimentation: beach profiles and longshore bars, paper presented at The Coastline of Canada, Halifax, Nova Scotia.