A VOLUME OF FLUID APPROACH TO MODEL BANK EROSION

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(1)MOTIVATIONS

By using a fixed-mesh approach, morphodynamic models have some difficulty predicting realistic equilibrium hydraulic geometries with vertical banks. In order to properly account for bank erosion without resorting to a complicated moving mesh algorithm, an immersed boundary approach is needed that handles lateral bank retreat through fix computational cells.

(2)LINK TO FESD: A DELTA DYNAMICS

One of the main goals of the FESD Delta Dynamics Collaboratory is to develop a tested, high-resolution numerical model that predicts the coupled morphologic and ecologic evolution of deltas over engineering to geologic time scales. This involves the creation and destruction of numerous channels, mouth bars, and other channel-edge features, and we therefore require an approach that is able to accurately predict the lateral motion of steep banks.

(3) VOLUME OF FLUID (VOF) APPROACH: AN OVERVIEW

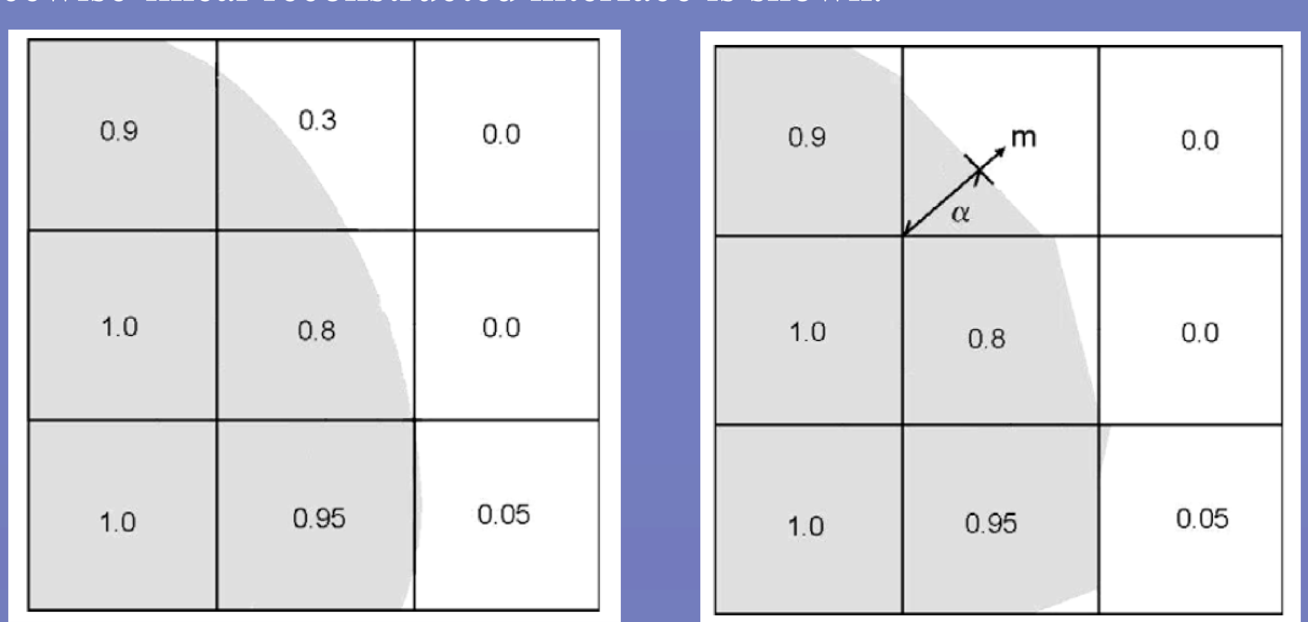
Usually used to capture the shape and the location of fluid-fluid interfaces (Hirt and Nicholas, 1981), the volume of fluid (VOF) method is here used to reconstruct bank interfaces. The domain is first covered by a colour function *c:*

$$c(\vec{x})=\begin{cases}1 \text{ if } & \vec{x}\in \text{'dark' fluid}\\ 0 \text{ if } & \vec{x}\in \text{'light' fluid}\end{cases}$$

The color function is then integrated in the each computation cell Ω to give:

$$f=\frac{\int_{\Omega} c(\vec{x})\,dv}{\Delta\Omega}$$

where *f* will be called a "proportion" (or "porosity") function. The figure below on the left depicts the real interface together with the values assumed by *f* at each cell. On the right the piecewise-linear reconstructed interface is shown.



REAL INTERFACE RECONSTRUCTED INTERFACE

The VOF approach consists of two main steps:

- Reconstruction of the interface
- Advection of the interface

(3) RECONSTRUCTION OF THE INTERFACE

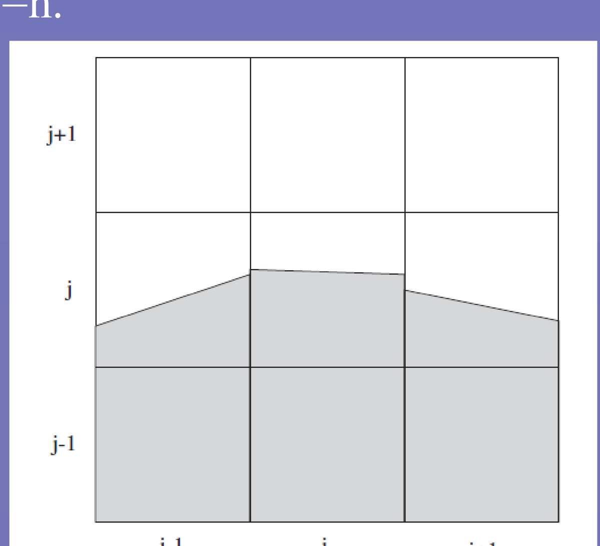
The interface can be described by the following equation:

$$m_x x + m_y y = \alpha$$

where **m**=(m_x,m_y) is the normal and α is the distance from the origin.

In order to reconstruct the interface, one must perform two sub-steps:

(a) First, the normal to the interface is found. This the most complicated part and numerous procedures have been proposed in the literature. Here Parker and Youngs' method is used to compute the normal (Pilliod and Puckett, 2004). Consider the 3×3 block of square cells shown in Figure, with Δx=Δy=h.



The normal **m** is first estimated at the four corners of the central cell (i,j) with a finite difference formula, for example the *x*-component m_x at the top-right corner is given by:

$$m_{x,i+1/2,j+1/2}=\frac{1}{2h}(C_{i+1,j+1}+C_{i+1,j}-C_{i,j+1}-C_{i,j})$$

and similarly for the *y*-component m_y and in the other three corners. Then the required cell-centred vector is obtained by averaging the four cell-corner values:

$$\mathbf{m}_{i,j}=\tfrac{1}{4}(\mathbf{m}_{i+1/2,j+1/2}+\mathbf{m}_{i+1/2,j-1/2}+\mathbf{m}_{i-1/2,j+1/2}+\mathbf{m}_{i-1/2,j-1/2})$$

For a non-uniform grid the formulation proposed in Bohluly et al. (2009) can be used.

(b) Second, the constant α must be computed given the normal ***m*** and the proportion function *f*. This problem is solvable analytically for the case of a grid made of rectangular cells (Scardovelli and Zaleski, 2000).

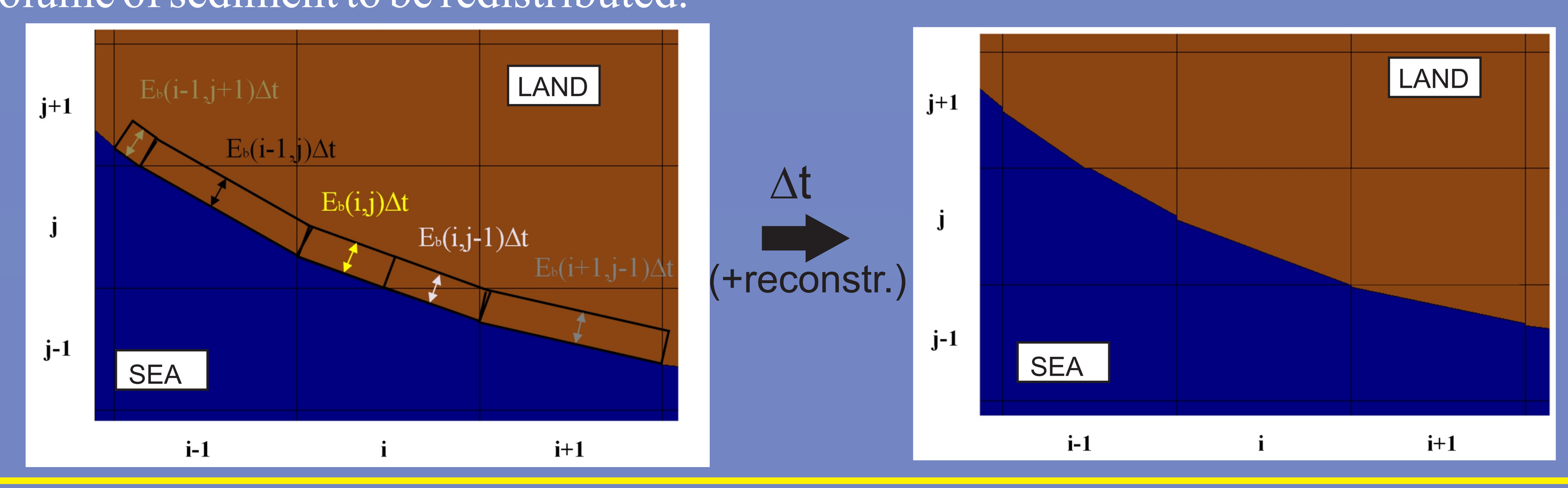
(4) ADVECTING THE INTERFACE

The speed of propagation of the interface E_b can be obtained through an erosion formulation as a function of the wall shear stress:

$$E_b = K_e\left(\frac{\tau}{\tau_c}-1\right)$$

where K_c is an erodibility coefficient, τ is the fluid shear stress on the wall and τ_c is the critical shear stress of the bank, in general a function of mud content and type of vegetation.

Erosion rate is computed in the following way: first the union of the eroding rectangles obtained by shifting the interface of $E_b\Delta t$ is computed. Then this union is intersected with the land region.The area of this intersection times the bank height provides the eroded volume of sediment to be redistributed.



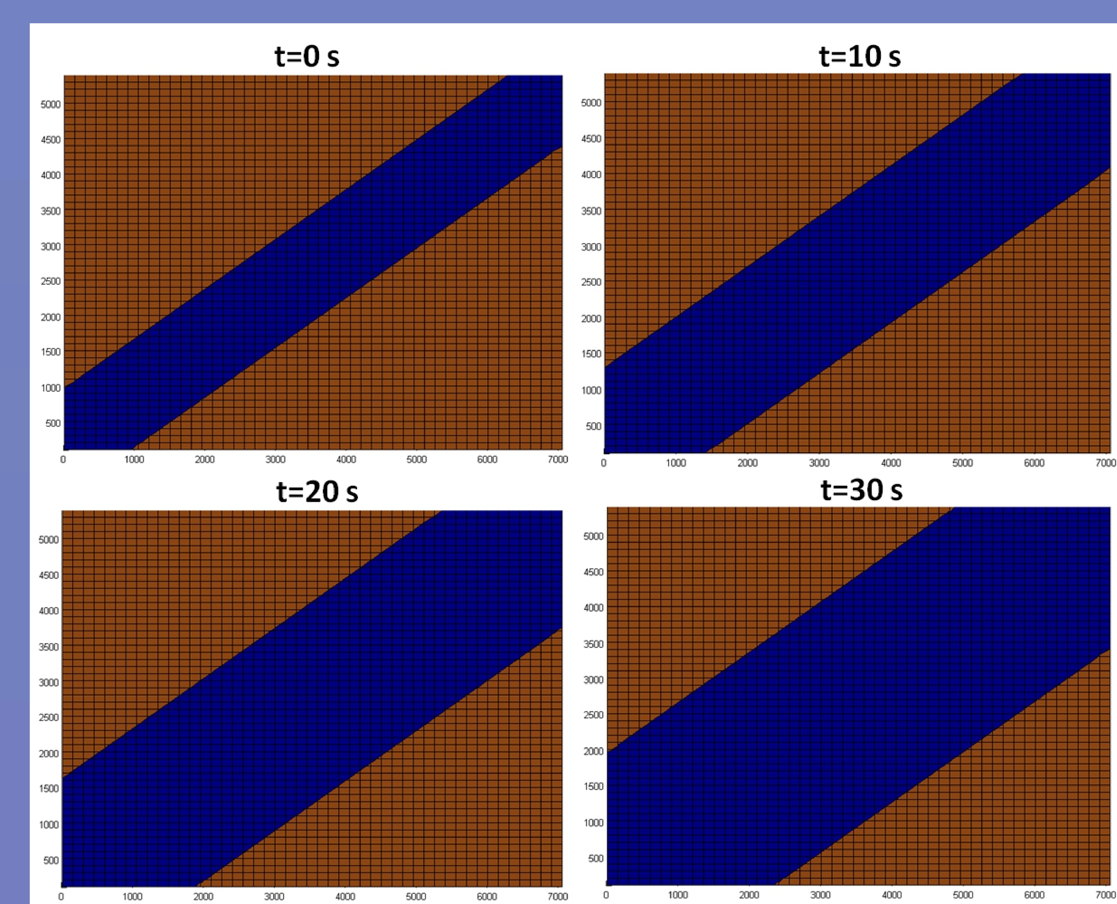
(5) TEST CASES

Here the results of some simple erosional test cases are shown. For simplicity the bank material is simply lost.

Channel with constant width

τ_c=1 Pa

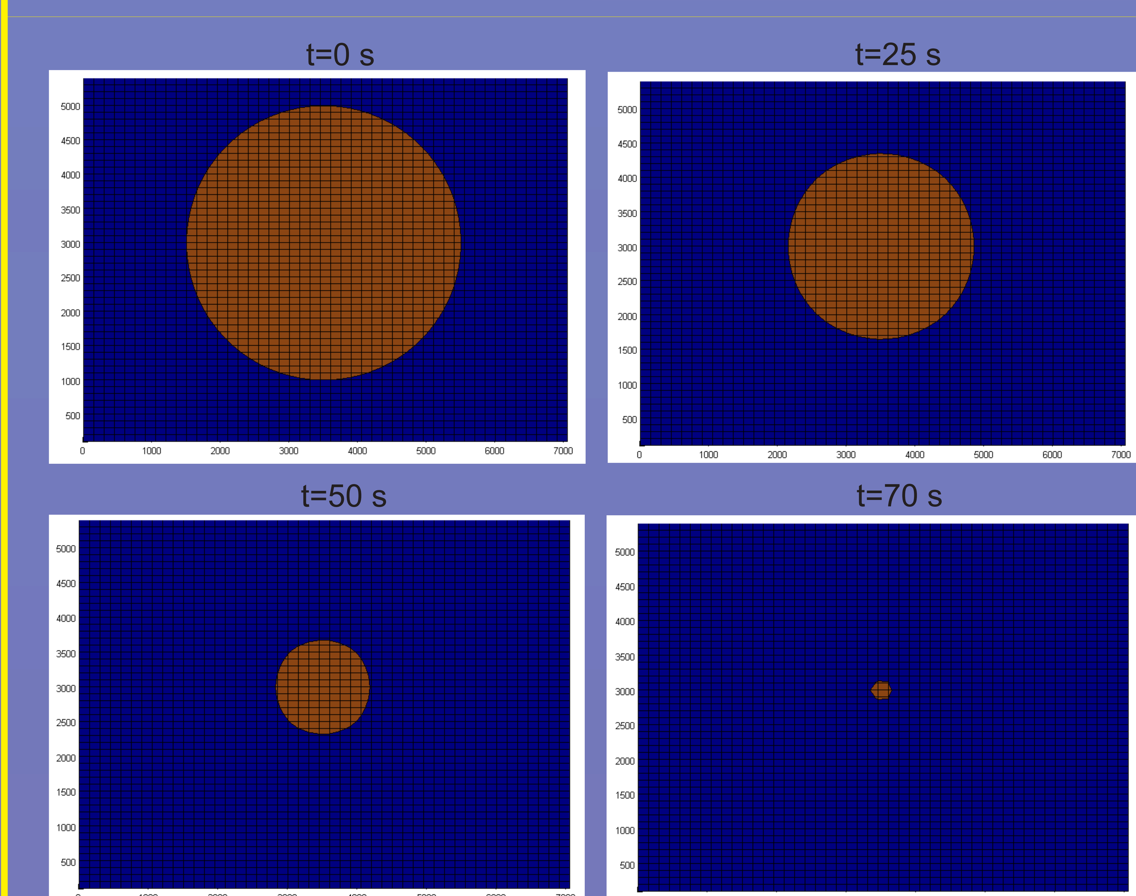
τ_b=10 Pa



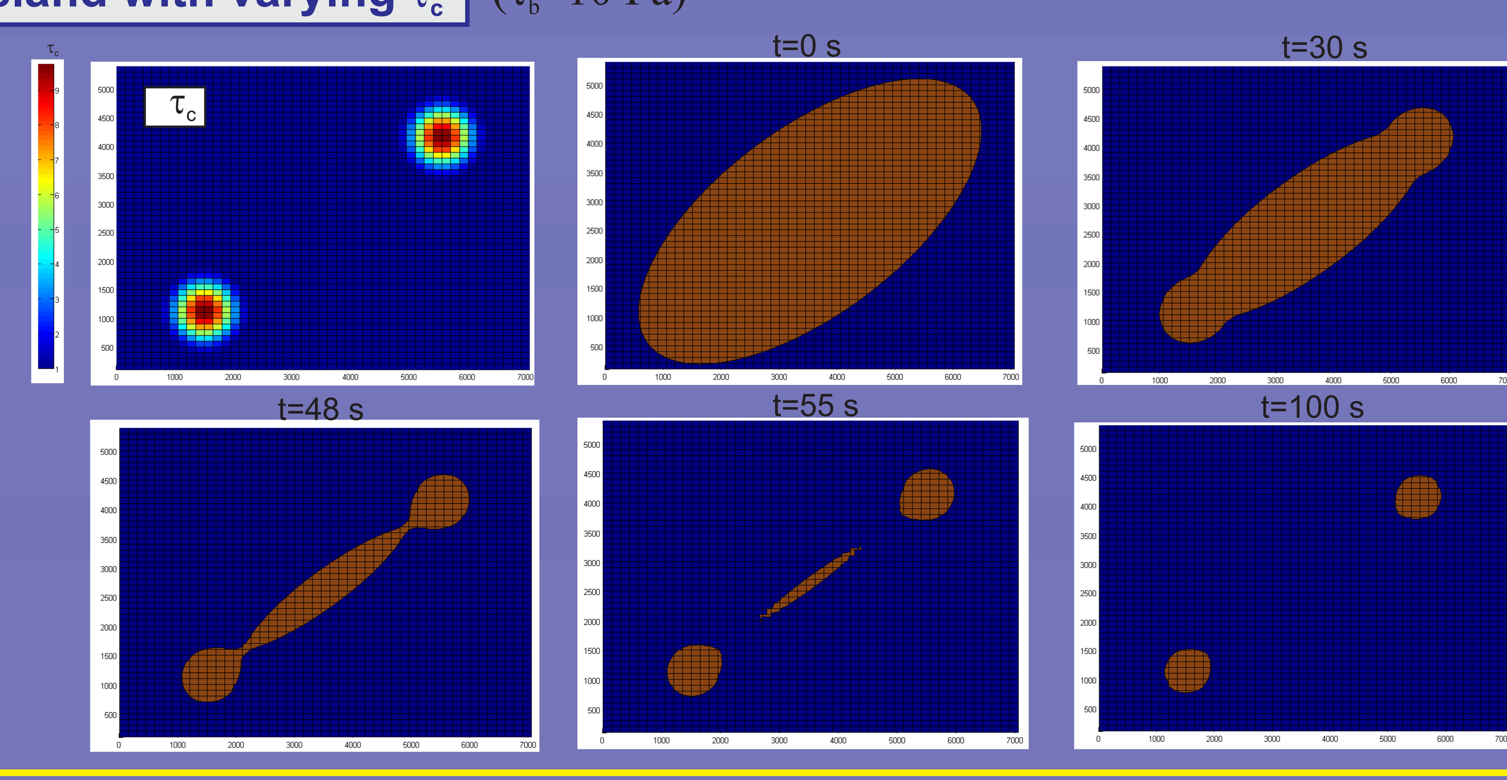
Circular island

τ_c=1 Pa

τ_b=10 Pa



Island with varying τ_c (τ_b=10 Pa)



(6) FUTURE WORK

In the next months, the following issues will be faced:

1. the proposed VOF algorithm will be coupled with Delft3D hydrodynamic, morphodynamic, and vegetation modules.
2. A ghost cell technique will be tested for solving the hydrodynamics in partially wet cells.
3. Different procedures for distributing material from the eroded banks to the adjacent cells will be implemented and compared.
4. The ability of the model to reproduce hydraulical geometries of real rivers with vegetated banks will be checked.

(7) ACKNOWLEDGEMENTS

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(8) REFERENCES

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Scardovelli and Zaleski, Analytical Relations Connecting Linear Interfaces and Volume Fractions in Rectangular Grids, Journal of Computational Physics 164, 228–237 (2000).